Numerical Reconstruction of Ionization Signal

L. Kurchaninov

- Calibration procedure
- History of reconstruction
- HEC Chain
- Quality of the fit
- NR method
- Reconstruction algorithm
- Signals in June-2000
- Accuracy of the reconstruction

Calibration Procedure

- Steps of calibration procedure:
  - Measurements (pulsing all channels, DAC scan, delay scan)
  - Reconstruction (recalculation from calibration pulse shape to ionization signal)
  - Output (weights, nonlinearity corrections, etc.)

- "Standard" approach to reconstruction (used in HEC in 1998, 1999, 2000) is the signal shape fit:
  - Fit the calibration signal by function derived for calibration signal.
  - Apply parameters to another function, that describes ionization shape

- Calibration and ionization functions:

$$Sc(t) = Ic(t) * Ha(t), \qquad Sp(t) = Ip(t) * Ha(t)$$

Ic(t) – calibration current on the pad level, *Ip(t)* – ionization current (triangular), *H(t)* – chain response,
that has simple expression in s-domain either as rational function (poles and zeros) or as a sum of poles:

$$H(s) = \sum_n \frac{d_n}{s + s_n} \qquad Ha(t) = \sum_n d_n \cdot \exp(-t \cdot s_n)$$

The similar expression in the case of repeated roots.

History of Reconstruction

- Test beam Setup 1998: Kosice calibration system, short cold cables.
  - *Ic(t)* – simple exponent;
  - *H(s)* expressed as the sum of poles. The knowledge of the chain was not precise, so many of the parameters d_n and s_n were free;

  ⇨ Fit was not stable and reproducible, free coefficients have no physical meaning

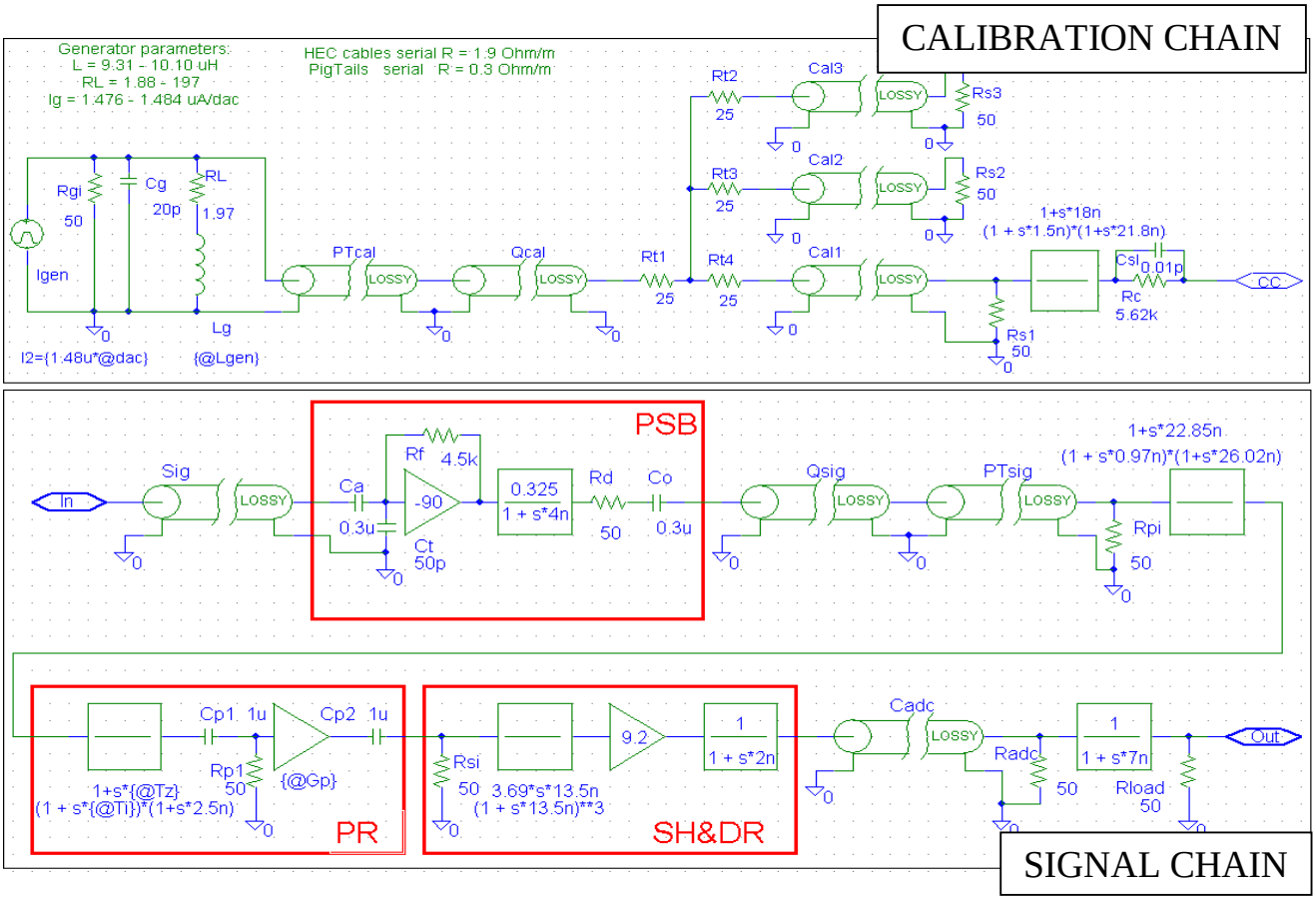
- Test beam Setup 1999: ATLAS calibration system, cold cables of final length, FEB-1.
  - Chain components have been studied (mainly in lab), parameters were fixed;
  - *Ic(s)* – 2 zeros and 3 poles (next transparency);
  - *H(s)* – 3 zeros, 8 poles and 1 triple pole; 2 time constants are free in the fit.
  - Simplified version of the function was used in order to have symbolic expression in t-domain;

  ⇨ Typical residual for calibration signal was ±0.7% and for particle shape ±2%. Weights underestimated systematically the particle amplitude on 0.5 – 1.5% (up to 3%)

- Setup-2000 is the same as in 1999.
  - Additional studies of calibration generators in MPI
  - Technical run in Febr.2000
  - Additional lab measurements of cables

  ⇨ Improved model parameters. Residual for calibration signal was ±0.5% and for particle shape ±1%.

HEC Chain



Calibration current:

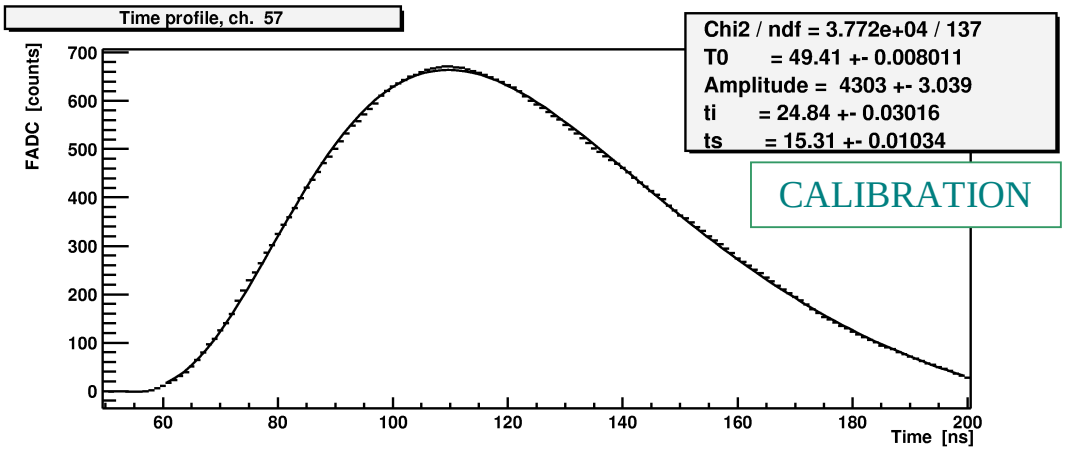
$$Ic(s) = Ig \cdot \frac{Rl}{Rc + Rin}\left(\frac{1}{s}\frac{\alpha + s\cdot\tau c}{1 + s\cdot\tau c}\right)\cdot\frac{ac\cdot(1 + s\cdot\tau zc)}{(1 + s\cdot\tau oc)(1 + s\cdot\tau pc)}$$

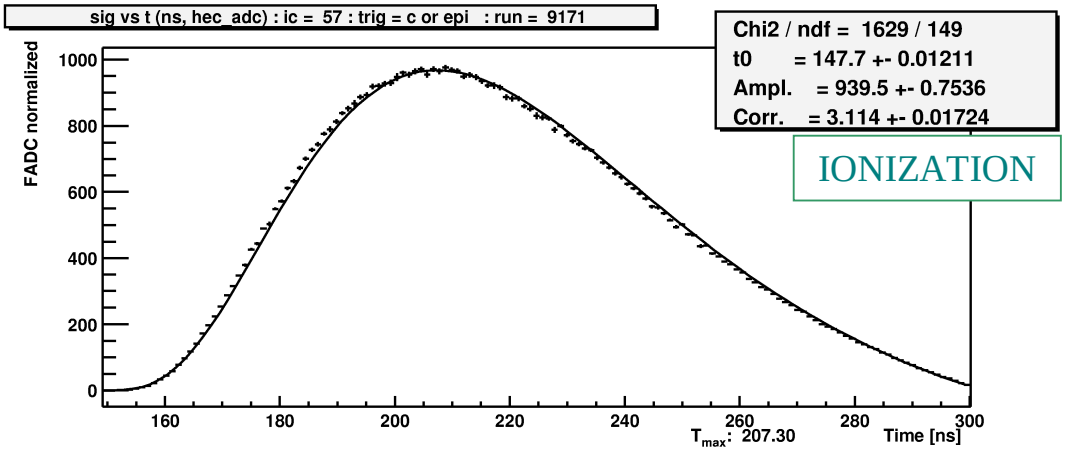
Signal chain:

$$Ha(s) = \frac{Rp}{(1 + s\cdot\tau a)(1 + s\cdot\tau d)}\cdot\frac{as\cdot(1 + s\cdot\tau zs)}{(1 + s\cdot\tau os)(1 + s\cdot\tau ps)}$$

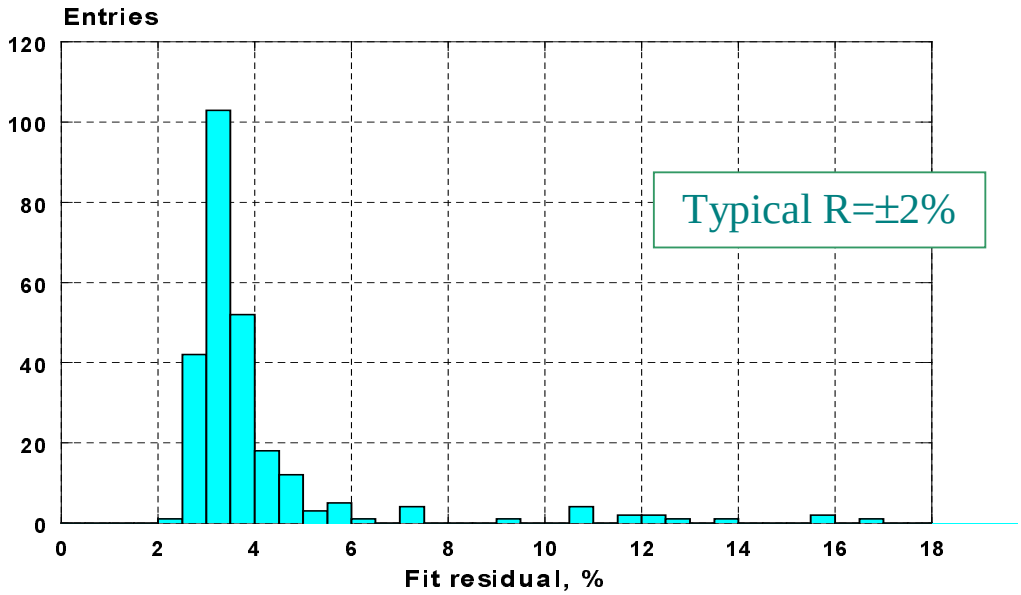
$$\cdot\frac{Gp\cdot(1 + s\cdot\tau pz)}{(1 + s\cdot\tau i)(1 + s\cdot\tau o)}\cdot\frac{Gs\cdot s\cdot\tau s}{(1 + s\cdot\tau s)^3}\frac{1}{(1 + s\cdot\tau fd)(1 + s\cdot\tau ac)}$$

Quality of the fit in 1999



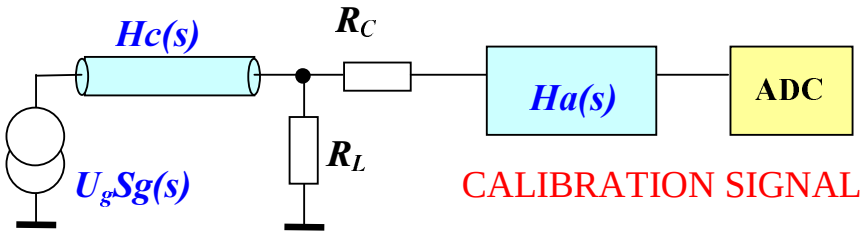


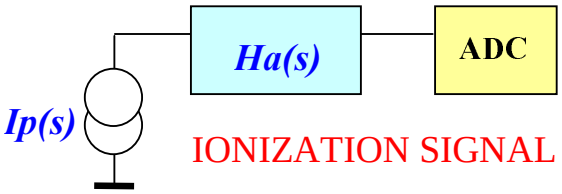
Particle shape fit residual $R = (max\text{-}min)/amplitude$



NR Method

- Very general diagram of the chain:





- Calibration and ionization signal in s-domain:

$$Sc(s) = Ig \cdot Hg(s) \cdot Hc(s) \cdot Ha(s)$$

$$Sp(s) = Ip(s) \cdot Ha(s)$$

- The goal is to express ***Sp*** in terms of ***Sc***. In s-domain:

$$\frac{Sp(s)}{I_P} = \left[\frac{Tr(s)}{Hg(s) \cdot Hc(s)}\right]\frac{Sc(s)}{I_C}$$

- In time domain it can be done by convolution:

$$\frac{S_P(t)}{I_P} = \frac{1}{I_C}\int_{-\infty}^{\infty} S_C(x)R(t-x)dx$$

with function *R(t)* which is determined in s-domain

$$R(s) \equiv \frac{1}{Hg(s) \cdot Hc(s)} \cdot Tr(s) \rightarrow R(t) \equiv \int_{-\infty}^{\infty} Rc(t-x)Tr(x)dx$$

$$Rc(t) \leftarrow \frac{1}{Hg(s) \cdot Hc(s)}$$

- Functions *Hg(s)*, *Hc(s)* are simple combination of poles and zeros. In time domain *Rc(t)* is:

$$Rc(t) = r0 \cdot \delta(t) + r1 \cdot \delta^{I}(t) + r2 \cdot \delta^{II}(t) + r3 \cdot \Theta(t) \exp\left(-\frac{t}{\tau_Z}\right) + r4 \cdot \Theta(t) \exp\left(-\frac{\alpha \cdot t}{\tau_C}\right)$$

and coefficients *r* are expressed in terms of calibration chain parameters.

- Convolution with triangular calculated analytically. The result is:

$$R(t) = Ri(t) + Rd(t) + Ro(t)$$

where three R-functions have very simple analytical form:

$$Ri(t) = \delta^{I}(t) \cdot r2 + \delta(t) \cdot \left(r1 - \frac{r2}{\tau_{DR}}\right) + \delta(t - \tau_{DR}) \cdot \frac{r2}{\tau_{DR}}$$

$$Rd(t) = r0 \cdot \left(1 - \frac{t}{\tau_{DR}}\right) - \frac{r1}{\tau_{DR}} + r3 \cdot \tau_Z Rdz(t) + r4 \cdot \frac{\tau_C}{\alpha} Rdc(t)$$

$$Ro(t) = r3 \cdot \tau_Z \cdot Roz(t) + r4 \cdot \frac{\tau_C}{\alpha} Roc(t)$$

- Functions *Rdz(t), Rdc(t), Roz(t)* and *Roc(t)* are also very simple:

$$Rdz(t) = \left(1 + \frac{\tau_Z}{\tau_{DR}}\right) \cdot \left(1 - \exp\left(-\frac{t}{\tau_Z}\right)\right) - \frac{t}{\tau_{DR}}$$

$$Rdc(t) = \left(1 + \frac{\tau_C}{\alpha \cdot \tau_{DR}}\right) \cdot \left(1 - \exp\left(-\frac{\alpha \cdot t}{\tau_C}\right)\right) - \frac{t}{\tau_{DR}}$$

$$Roz(t) = \left(\frac{\tau_Z}{\tau_{DR}} \exp\left(\tau_{DR} / \tau_Z\right) - \left(1 + \frac{\tau_Z}{\tau_{DR}}\right)\right) \exp\left(-\frac{t}{\tau_Z}\right)$$

$$Roc(t) = \left(\frac{\tau_C}{\alpha \cdot \tau_{DR}} \exp\left(\frac{\alpha \cdot \tau_{DR}}{\tau_C}\right) - \left(1 + \frac{\tau_C}{\alpha \cdot \tau_{DR}}\right)\right) \exp\left(-\frac{\alpha \cdot t}{\tau_C}\right)$$

Reconstruction Algorithm

- For normalized shapes:

$$T_C(t) = \frac{S_C(t)}{I_C} \quad \text{and} \quad T_P(t) = \frac{S_P(t)}{I_P}$$

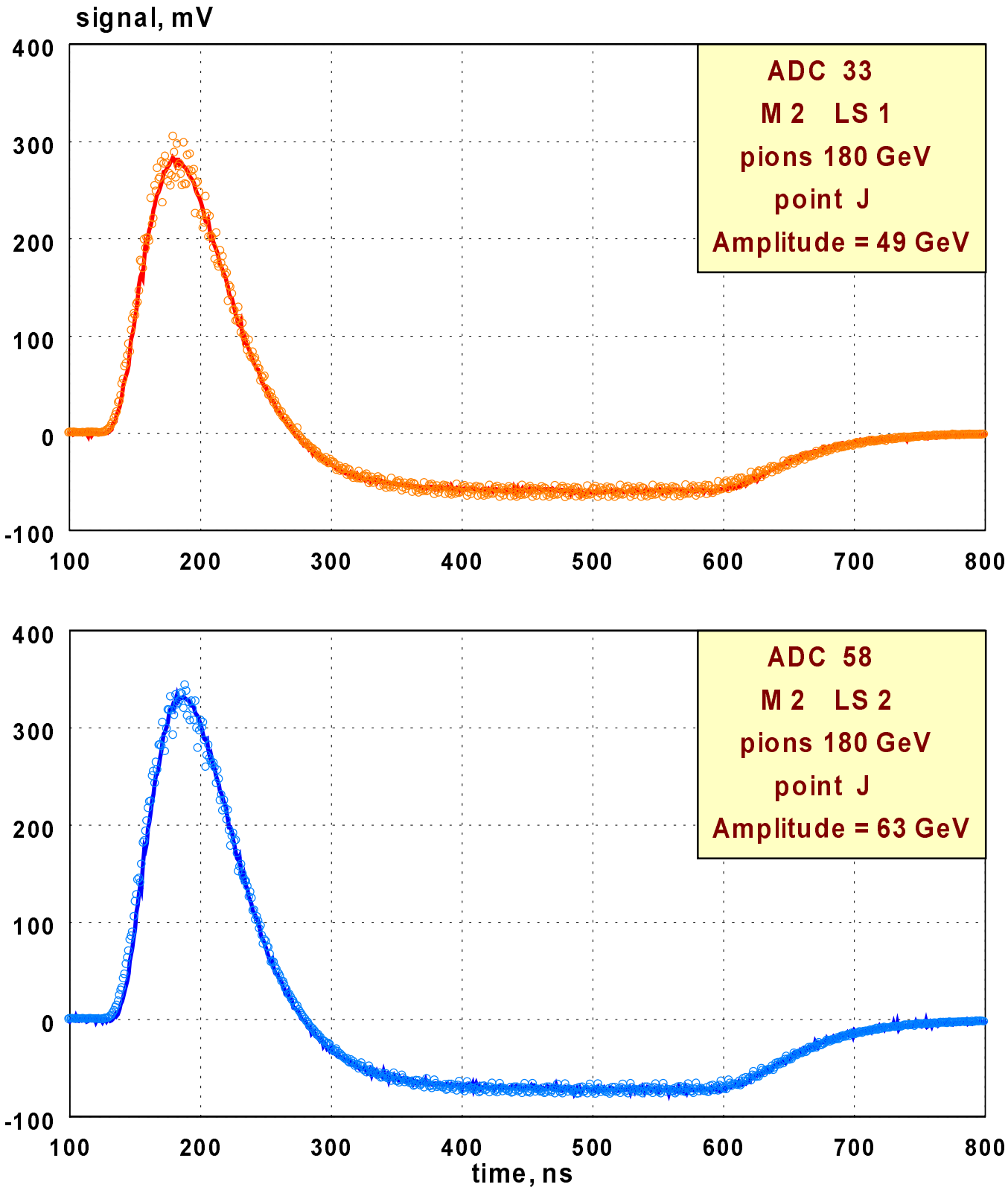
the algorithm is:

$$T_P(t) = T_C^I(t) \cdot r2 + T_C(t) \cdot \left(r1 - \frac{r2}{\tau_{DR}} \right) + T_C(t - \tau_{DR}) \cdot \frac{r2}{\tau_{DR}} + \\ + \int_{t-\tau_{DR}}^{t} T_C(x) Rd(t-x) dx + \int_{-\infty}^{t-\tau_{DR}} T_C(x) Ro(t-x) dx$$

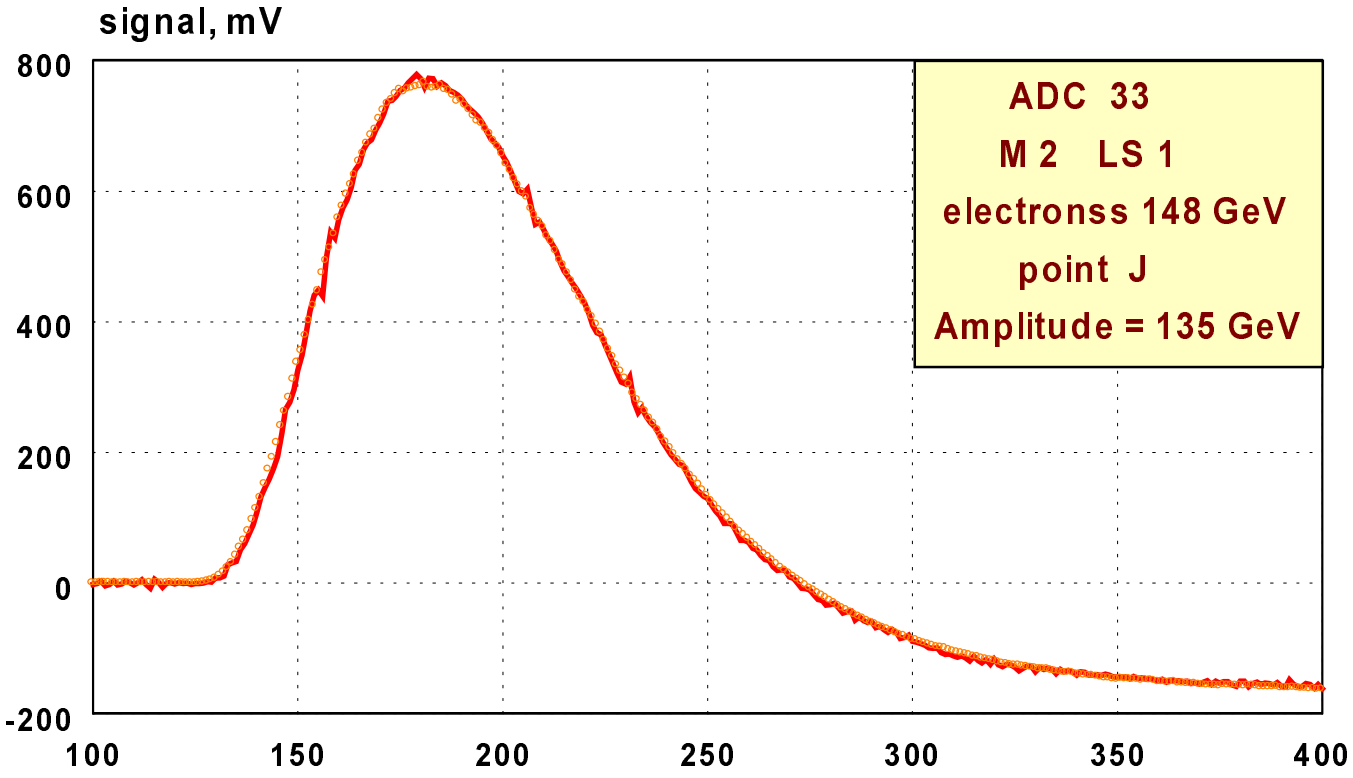
- So, the method of prediction of ionization signal consists of 2 steps:
  1. Calculate coefficients *r0 .. r4*
  2. Calculate $T_P(t)$ on the basis of measured calibration signal $T_C(t)$

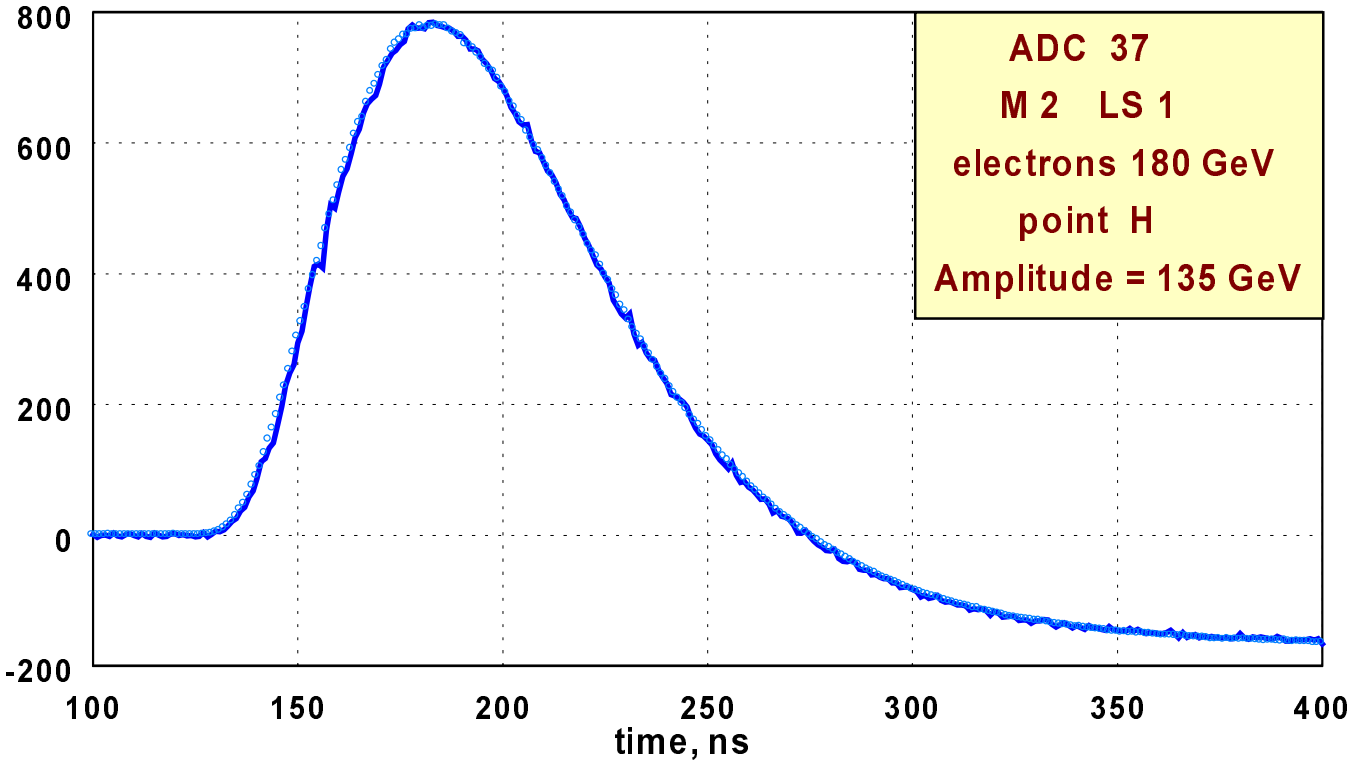
- Parameters of the calibration chain are known from lab measurements and from technical run
  - see "Summary of Technical Run-2000", presented in HEC/FCAL TB Analysis Meeting in LARG week April-2000, or
  - *www.mppmu.mpg.de/HEC_notes/hec096*

June-2000. Pions Signals



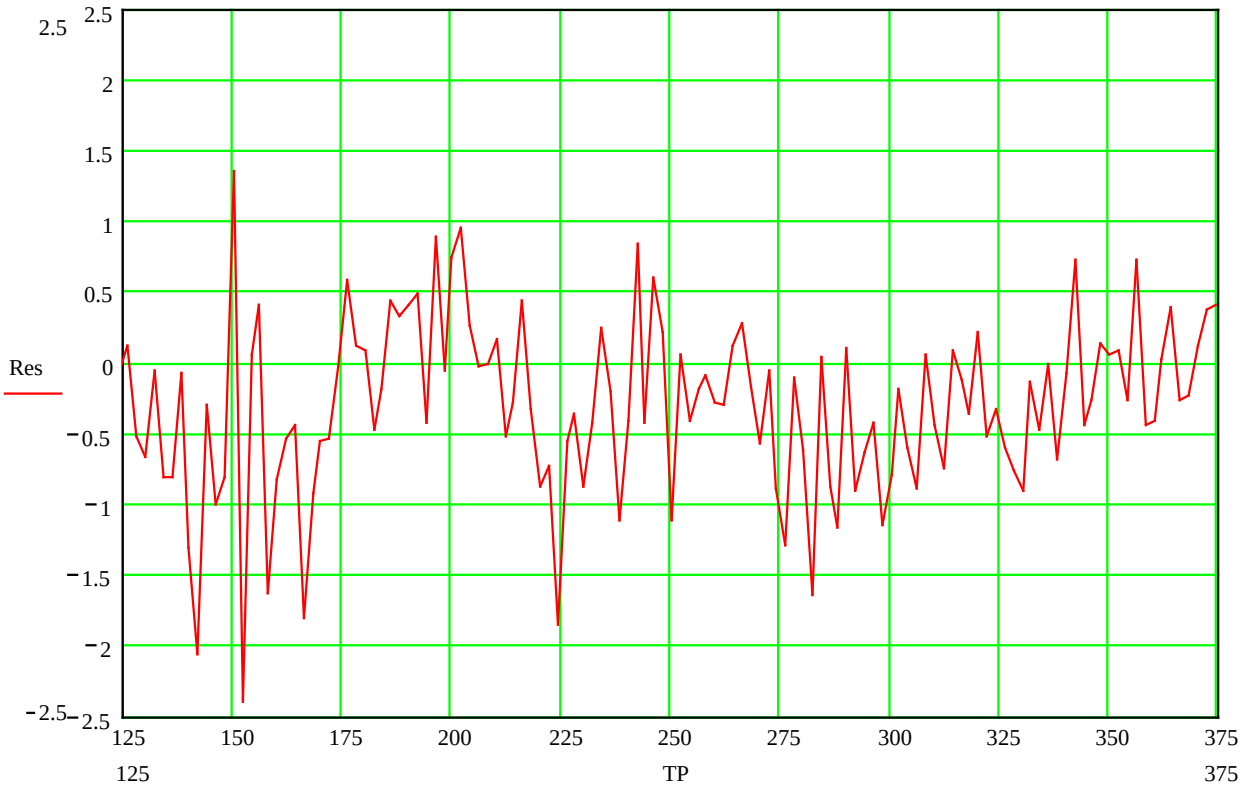
June-2000. Electron Signals





Accuracy of the Reconstruction

- Typical residual for electrons is below ±1% (fluctuations come from statistics).



- The further improvement of the predictions (if necessary) requires additional efforts:
  - To increase statistics of the calibration signals (at least for one DAC level) or to find out a smoothing procedure
  - There is small dependence of the calibration shape on the DAC level. (difference between shape for DAC 1000 and DAC=10000 is 1.5%) It has to be studied.
  - Better model the signal distortion in cables.

Summary

- NR method is developed and checked using TB data.

- First check (by P.Strizenec) shows that the NR method is faster than the fitting procedure:

Method	Free parameters	CPU sec/ch.
NR	--	0.25
	to	6.0
	to, a	9.7
Fit	*to, a, τi*	22
	to, a, τi, τs	40

- Accuracy of the ionization signal reconstruction is quite good, typically below ±1%. Further improvement is possible.

- The method requires more statistics in calibration pulsing.