ATLAS
HEC Note-171
3 June 2009

Electron Induced Signal in LAr gap

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Abstract. In this Note I describe a model developed to calculate electron induced signal in LAr gap containing known ion space charge and electric field strength. Some demonstrative examples of induced current waveforms and shaped signals are presented for HEC at sLHC and at Protvino beam conditions.

1. Introduction

The model to calculate space charge, electric field, and anode voltage drop in LAr gap under high ionization rate conditions is described in my previous Note [1]. This model provides distribution of ions $n_i(z)$ in the gap volume and electric field $E(z)$ which is a superposition of the field produced by external bias (through impedance Z) and the field of built-in positive ions. As soon as both functions $n_i(z)$ and $E(z)$ are known, the electron drift in the gap can be calculated. This drift defines signal induced in external circuit.

An example of ion density distribution and electric field in the gap is shown in Fig.1 for nominal set of LAr parameters (see [1] for details). Such distributions, calculated for various values of LAr parameters are used as an input for calculations of induced signals.

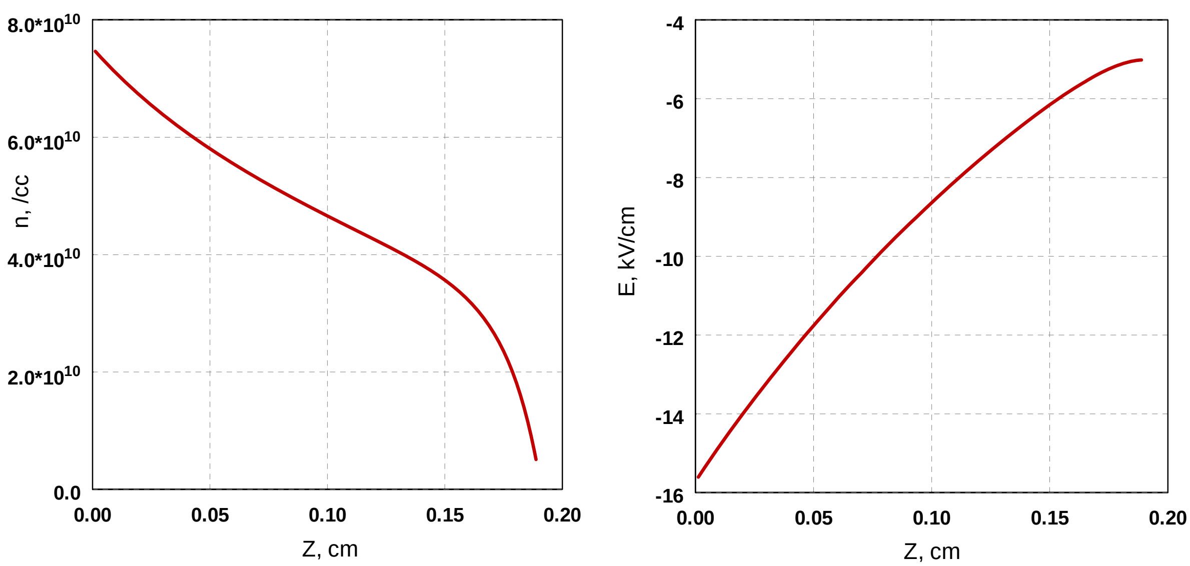

***Figure 1**: Ion charge in LAr gap (left) and electric field (right).*

This Note describes the details of signal waveform calculations and some results obtained for conditions studied in [1]. To evaluate the effects of high ionization rate on shaped and digitized signal, a simplified model of HEC electronics chain is employed.

2. Current induced by point charge

The geometry considered is shown in Fig.2. The gap of width a is filled with LAr. The ion density $n_i(z)$ and electric field $E(z)$ are assumed to be known. At time $t=0$ a small-size (point) electron cloud with charge Q_0 is generated at position z_0. Electrons will drift from $z=z_0$ to anode $z=a$ with drift velocity v_e which is not constant because electric field is not constant throughout the gap. Since $E(z)$ is known, the drift velocity can be calculated for any z, following parameterization introduced in [1]

$$v_e(z) = v_m \cdot \frac{E(z)}{E_T} \cdot \left(1 + \left(\frac{E(z)}{E_T}\right)^{1/3}\right)^{-3} \qquad (1)$$

with E_T=840 V/cm and v_m=1.34 $cm/\mu s$.

For calculation of induced current as a function of time, the drift velocity vs. time must be known. It is obtained from low of motion $z(t)$ which is a solution of obvious equation:

$$\frac{dz}{dt} = v_e(z); \qquad \text{or} \qquad dt = \frac{dz}{v_e(z)} \qquad (2)$$

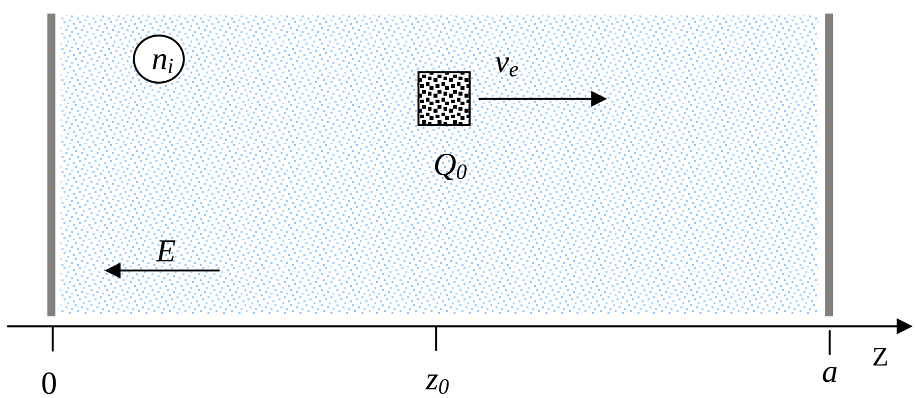

***Figure 2**: LAr gap geometry.*

The solution of (2) is simple integral

$$t(z;z_0) = \int_{z_0}^{z} \frac{dz'}{v_e(z')} \qquad (3)$$

which is calculated numerically for given z_0 and $z(t;z_0)$ is obtained by inverting function (3). Then $v_e(t;z_0)$ can be calculated either by differentiating $z(t;z_0)$ or by using (1). The drift time for given z_0 is readily obtained from (3):

$$T_d(z_0) \equiv t(a;z_0) = \int_{z_0}^{a} \frac{dz'}{v_e(z')} \qquad (4)$$

Current induced by the charge motion is calculated with Ramo's theorem:

$$I_p(t;z_0) = \frac{Q(t;z_0)\cdot v_e(t;z_0)}{a} \qquad \text{for} \qquad 0 < t < T_d(z_0) \tag{5}$$

Here charge Q depends on time because electrons recombine with bulk ions in the course of their drift. This dependence can be expressed as

$$\frac{dQ}{dt} = -R\cdot n_i(t)\cdot Q \tag{6}$$

where ion concentration depends on time through electron motion $z(t;z_0)$. Solution of (6) is

$$Q(t;z_0) = Q_0\cdot\exp\left(-R\cdot\int_0^t n_i(z(t';z_0))\cdot dt'\right) \equiv Q_0\cdot r(t;z_0) \tag{7}$$

and now both functions in r.h.s of (5) are defined.

In the low ionization rate limit, the electric field in the gap is uniform and there is no ion charge. In this case, the electron drift velocity is constant and there is no recombination, so the induced current (5) is constant, i.e. has rectangular shape.

3. Current induced by extended track

In the case of track produced by charged particle, ionization is uniform over gap width and induced signal can be calculated as superposition (integral) of waveforms (5). If density of energy deposition is constant over gap, the electron charge density is not because of initial recombination is field dependent. If Q_e is the total electron charge deposited in the gap prior initial recombination, the free charge density can be written as:

$$q_e(z_0) = \frac{Q_e}{a}\cdot\left|\frac{E(z_0)}{E_T}\right|\cdot\ln\left(1+\left|\frac{E_T}{E(z_0)}\right|\right) \equiv \frac{Q_e}{a}\cdot j(z_0) \tag{8}$$

where initial recombination is parameterized by Thomas-Imel function as in [1].

Consider small layer Δz_0 at point z_0. The ionization charge $q_e(z_0)\cdot\Delta z_0$ can be considered as a point charge with contribution to signal waveform described by (5). Combining (5), (7), and (8), this contribution takes the form:

$$\Delta I_p(t) = \frac{Q_e}{a^2}\cdot j(z_0)\cdot r(t;z_0)\cdot v_e(t;z_0)\cdot\Delta z_0 \qquad \text{for} \qquad 0 < t < T_d(z_0) \tag{9}$$

The final signal waveform is obtained by integrating (9) over gap width:

$$I_e(t) = \frac{Q_e}{a^2}\cdot\int_0^a j(z_0)\cdot r(t;z_0)\cdot v_e(t;z_0)\cdot\theta(T_d(z_0)-t)\cdot\theta(t)\cdot dz_0 \tag{10}$$

with $\theta(t)$ being conventional step function. In the limit of low ionization rate, when all three functions in (9) are constants and each $\Delta I_p(t)$ is a rectangular of width $(a\text{-}z_0)/v_e$, the result of integration (10) is well known triangular shape.

In actual calculations the gap is subdivided into N sub-gaps, then waveforms (9) are calculated for each of N points and integral (10) is replaced by discrete sum of those N waveforms. In examples presented further, I use N=80 and some small step-like discontinuities can be seen on the calculated waveforms.

4. HEC electronics chain

Practically it is interesting to predict degradations of shaped waveforms measured at the output of electronics chain. In this Note I use HEC chain model developed for Protvino High-Luminosity experiment because the Protvino data will be initially used to tune LAr parameters. The details of this model can be found in [2]. The transfer function is described in frequency domain as a set of poles as:

$$Ha(s) = Ra \cdot \frac{1}{1 + s \cdot \tau a} \cdot \frac{s \cdot \tau s}{(1 + s \cdot \tau s)^3} \cdot \frac{1}{1 + s \cdot \tau o} \qquad (11)$$

It represents amplifier, shaper, and cable; all time constants are (will be) adjusted with calibration pulses. In further calculations I use the set of parameters shown in Tab.1. This set represents a typical transfer function and does not correspond to any actual HEC channel of Protvino setup.

The chain response function is written in time domain as a set of exponential terms with coefficients calculated analytically. The predicted response is then calculated as convolution of this function with induced current (10). This convolution is performed numerically with MCAD, from which the output waveform is exported as a vector of 1-ns samples. Fig.3 shows an example of such calculations for the case of triangular induced current (no space charge in the LAr gap) of total ionization charge of 1 pC.

Table 1: *Chain parameters.*

Parameter	Value
Ra, kΩ	20.1
τa, ns	12
τs, ns	13.9
τo, ns	9

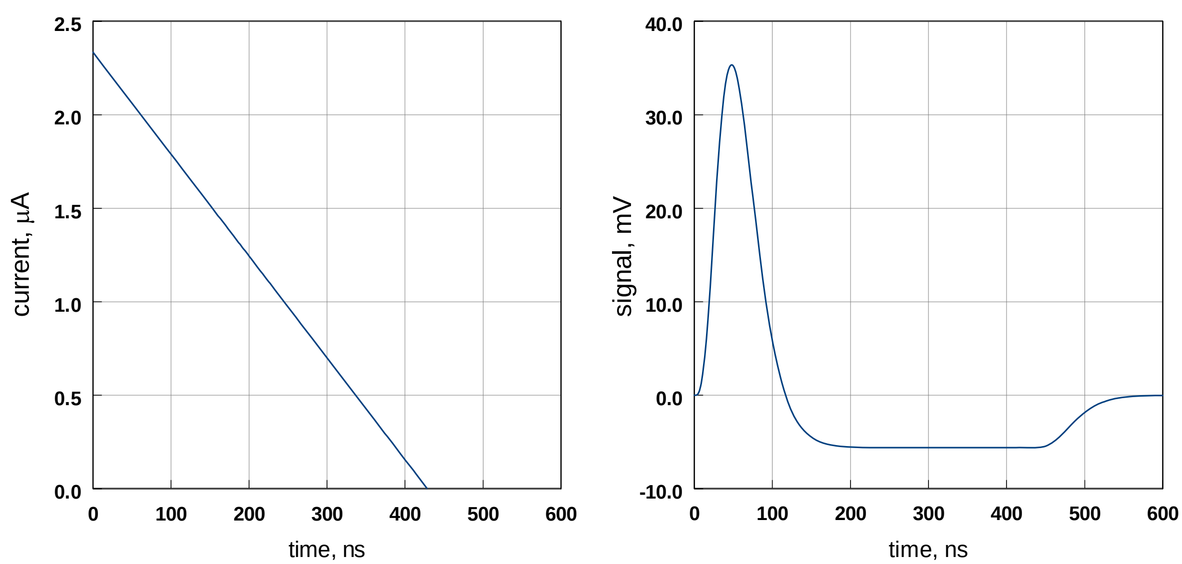

Figure 3*: Left: triangular ionization pulse. Right: its convolution with chain transfer function (11).*

5. Examples of signal waveforms

Fig. 4 and 5 demonstrate waveforms of ionization current and electronics response calculated for different values of LAr recombination constant and ion mobility. For comparison, the reference signal corresponding to zero space charge is also shown. All plots are calculated for ionization signal of 1 pC and space charge corresponds to the hottest HEC cell at sLHC conditions (1 MeV/BC in 2-mm gap and area of 100 cm^2). The bold red curves represent nominal values of LAr constants (R=10^4 cm^3/s and μ_i = $6.5{\cdot}10^{-4}$ cm^2/V·s). All signals correspond to stationary limit, when process of charge build-up has completed.

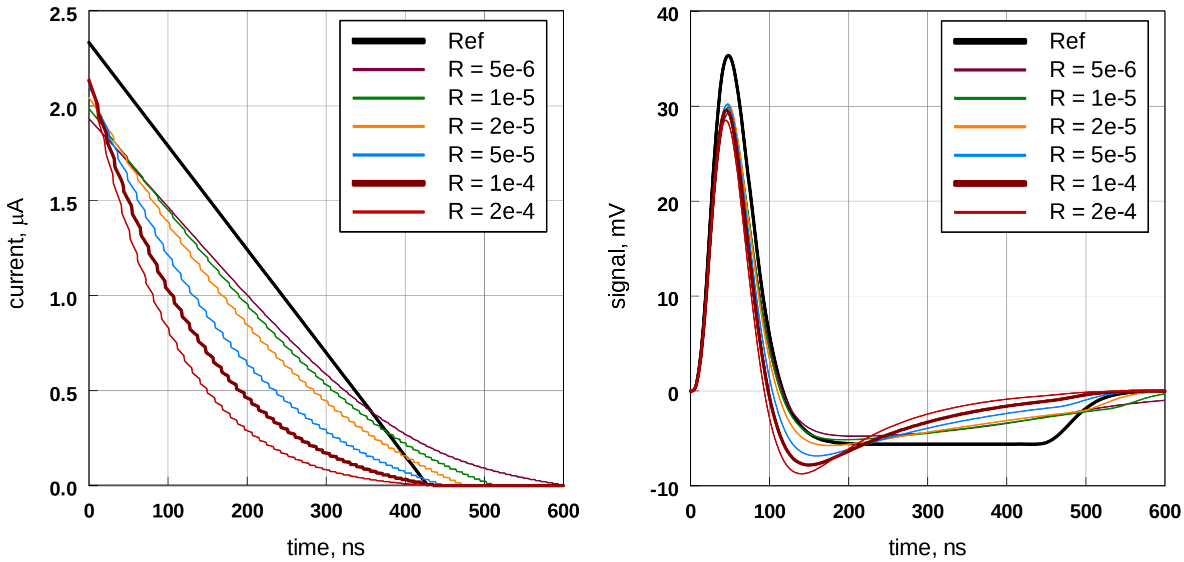

***Figure 4**: Ionization pulse (left) and shaped signal (right for different values of LAr recombination rate constant.*

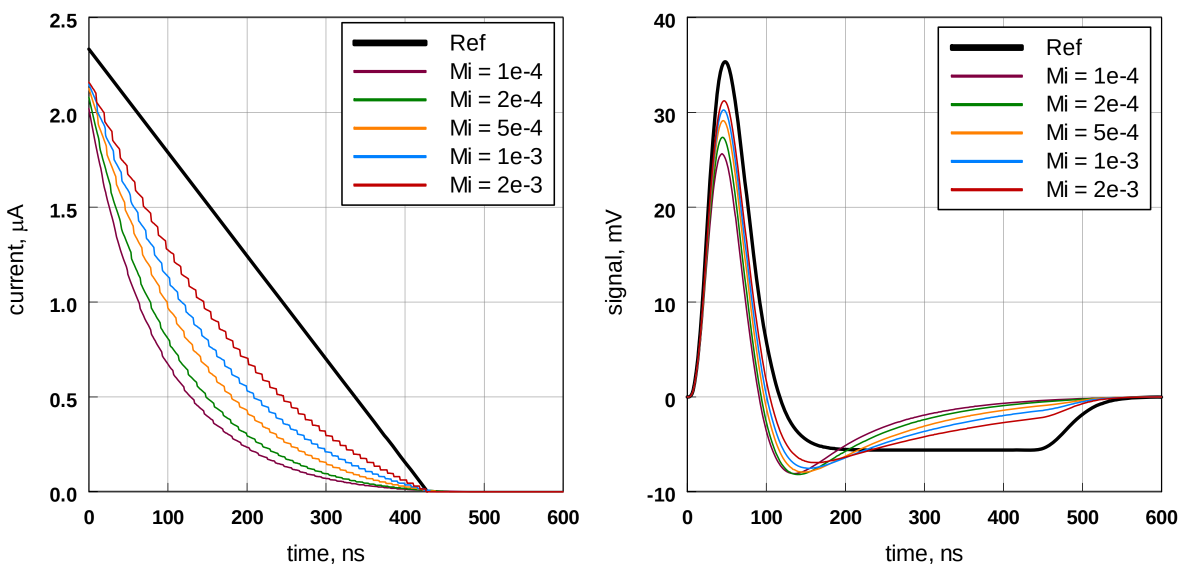

***Figure 5**: Ionization pulse (left) and shaped signal (right) for different values of LAr ion mobility.*

It can be seen that the signal amplitude is more sensitive to ion mobility than to recombination rate constant while the signal tail is sensitive to both. For higher *R* the signal is shorter because fewer electrons drift over entire gap width. In the case of a lower *R*, there are more ions built-up that causes increase of recombination (that's why amplitude is not sensitive to *R* value) and reduce (in average) electric field, that makes drift longer. Decreasing the ion mobility leads to their higher density and therefore to higher recombination rate (R constant is kept unchanged).

In any case, the space charge affects the signal tail changing its flat shape to exponential-like fall. The effects of *R* and μ_I on the signal shape are (anti)correlated. It seems to be very difficult to reconstruct them simultaneously from the shape analysis alone. Some supplementary information like evolution of signals during spill or/and their HV-dependence must be used.

6. Summary

The model described here is based on Ramo's expression for induced current, which is calculated for given electric field and ion density distribution. Calculation of ionization signal is simple and fast process, so any slow calculator like MCAD can be used.

References

1. L. Kurchaninov, *High rate effects in the ATLAS HEC calorimeter*, ATLAS-HEC Note XXX

2. L. Kurchaninov, *Model for Protvino electronics*, http://kurchan.home.cern.ch/kurchan/Higl/hec-chain-model.pdf