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Determination of the single-tube resolution with tracks

Muon Calibration and Alignment Workshop

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Outline

The correct spatial drift-tube resolution $\sigma(r)$ is needed for the precise reconstruction of muon trajectories.

- It depends on the operating parameters of MDT chambers and background count rates in ATLAS cavern $\Longrightarrow$ it has to be repedeatly redetermined during detector operation.

Two approaches for the resolution determination using muon tracks:

- conventional iteration method
- χ^2-metod

Both methods tested for the performance in ATLAS environment:

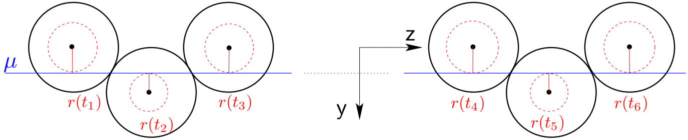
- tracks at large angles of incidence on the chamber
- curved tracks in a magnetic field
- additional hits caused by the background irradiation

Tests are done with the Monte-Carlo and the X5-testbeam data.

Track Reconstruction and Single-Tube Resolution

The (straight or curved) muon trajectory be described by

$$y^{(fit)} = \sum_{k=1}^{d(=2 \text{ or } 3)} \alpha_k \cdot z^{k-1}.$$



Parameters α_k are obtained by the χ^2-minimization:

$$\frac{\partial \chi^2}{\partial \alpha_k} = 0 \quad \text{for} \quad \chi^2 = \sum_{i=1}^{N_{hits}} w_i \left[r(t_i) - r_i^{(fit)} \right]^2$$

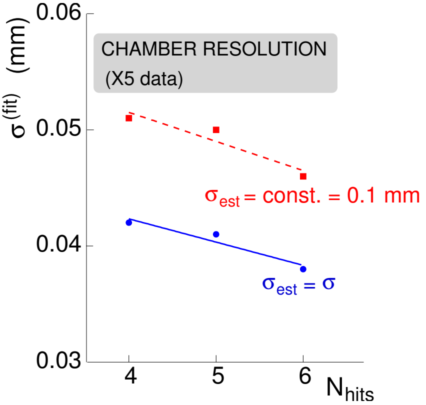
- $r_i^{(fit)} = f(\alpha_k)$ - shortest distance of the trajectory from the i^{th} wire
- $r(t_i) \equiv r_i$ - measured drift radius of the i^{th} track point
- w_i - weight of the i^{th} track point, $w_i = 1/\sigma^2_{est}(r_i)$

Track Reconstruction Accuracy

Averaging over a sample of identical muon tracks gives the variance of the track fit, σ^2_{fit}:

$$\sigma^2_{fit} = \left\langle \left(\sum_{k=1}^{d} (\alpha_k - \langle \alpha_k \rangle) \cdot z^{k-1} \right)^2 \right\rangle = F(w_i, \sigma(r_i))$$

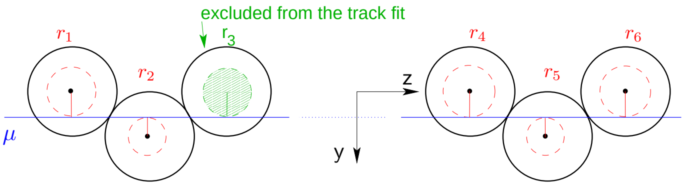
Best accuracy achieved with $w_i = 1/\sigma^2(r_i)$, i.e. $\sigma_{est}(r_i) = \sigma(r_i)$



The reconstruction of muon trajectories relies on the accurate knowledge of the spatial single-tube resolution $\sigma(r)$.

- It can be determined using muon tracks.

Iteration Method for Resolution Determination



Spatial resolution $\sigma(r_m)$ of the tube m:

- Hit r_m excluded from the track fit.
- Track reconstruction through the remaining hits $r_{i,i\neq m}$ predicts the position $r_m^{(fit)}$ in tube m.
- Repeated over a sample of identical muon tracks.

Comparing r_m and $r_m^{(fit)}$ gives: $\sigma^2(r_m) = \sigma^2(r_m^{(fit)} - r_m) - \sigma^2_{fit}(r_m^{(fit)})$

The track fit variance $\sigma^2_{fit}(r_m^{(fit)})$ depends on the resolution $\sigma(r_{i,i\neq m})$.

- $\rightarrow$ starting with estimate σ_{est} $\rightarrow$ obtain the new resolution curve $\rightarrow$
- Iterating until the obtained resolution agrees with estimated one.

χ^2-Method for Resolution Determination

The track fit variance has a known dependence on the spatial resolution, $\sigma^2_{fit}(r_m^{(fit)}) = F(\sigma(r_{i,i\neq m})) \Longrightarrow$

Measured distribution $\sigma^2(r^{(fit)} - r)$ can be described analytically:

$$\sigma^2(r_m^{(fit)} - r_m) = \sigma^2(r_m) + \sigma^2(r_m^{(fit)})$$
$$= \sigma^2(r_m) + F(\sigma(r_{i,i\neq m})) = \Sigma_m(\sigma(r))$$

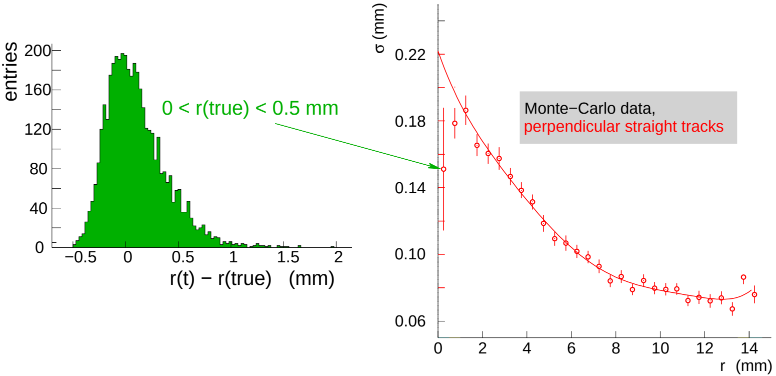
- $\sigma^2(r_k^{(fit)} - r_k) \equiv \sigma_k^2$ determined from pool of n_k tracks at each r_k bin
- Resolution curve parametrized by Legendre polynomials, $\sigma^2(r) = \sum_l c_l \cdot P_l(r)$.
- Parameters c_l obtained from the χ^2-minimization:

$$\chi^2 = \sum_k \left[\frac{n_k}{\sigma_k^2}(\sigma_k - \Sigma_k)^2\right]$$

The method is independent of the initial resolution estimate $\Longrightarrow$ no need for iterations.

Special Treatment Near the Anode Wire

- The $(r(t) - r(true))$-distribution is asymmetrical close to the wire, but the tracking methods assume a gaussian distribution $\Longrightarrow$ the $\left(r - r^{(fit)}\right)$-measurement in the first bin is excluded.



- Instead, the resolution near the wire is obtained from the drift-time spectrum: $\sigma(r=0) = a \cdot T_0 \cdot v(r=0)$.

T_0 - rise time, v - drift velocity

Monte-Carlo Studies

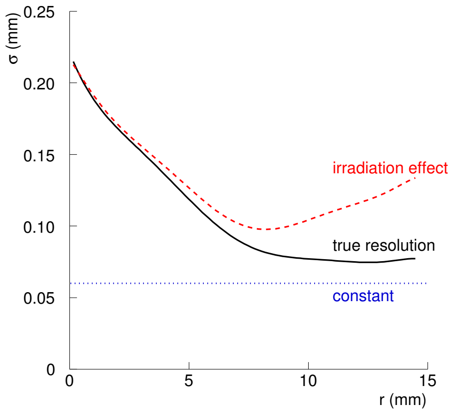
MTGEANT-4 simulation of muon tracks traversing a 6-layer chamber (BOS chamber):

- Straight tracks at different angles of incidence
  $\theta = 0^o, 13^o, 24^o, 30^o, 41^o, 45^o, 51^o$
  - test of both methods with a straight track fit
  - test of both methods with a curved track fit (curvature = 0)

- Curved tracks with different curvatures, at 0^o angle of incidence
  curvature = 20, 45, 90 μm
  (p_T = 80, 40, 20 GeV/c for B=0.4 T)
  - test of both methods with a curved track fit

Monte-Carlo Studies: Initial Resolution Estimate

Starting with three different initial resolution estimates:

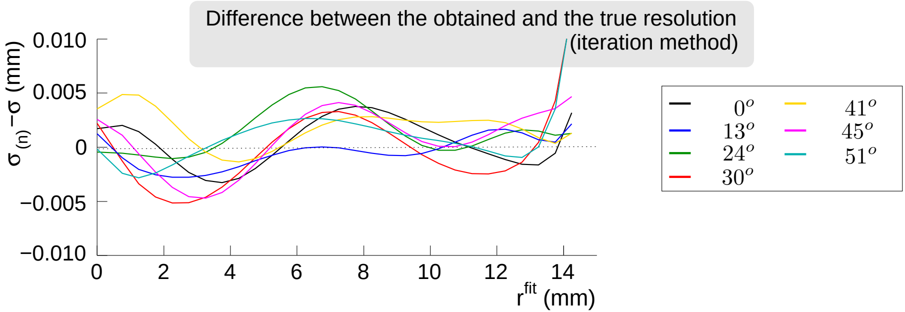


- i) true resolution
  - convergence expected already after the first step
  - test the limits on the accuracy
- ii) background irradiation effect
  - representative case for ATLAS
- iii) constant resolution
  - extreme case for the test of convergence

Accuracy with the Straight Track Fit

i) True resolution as the initial estimate:

- Both methods converge in one step.
- Similar accuracy is obtained for both methods.



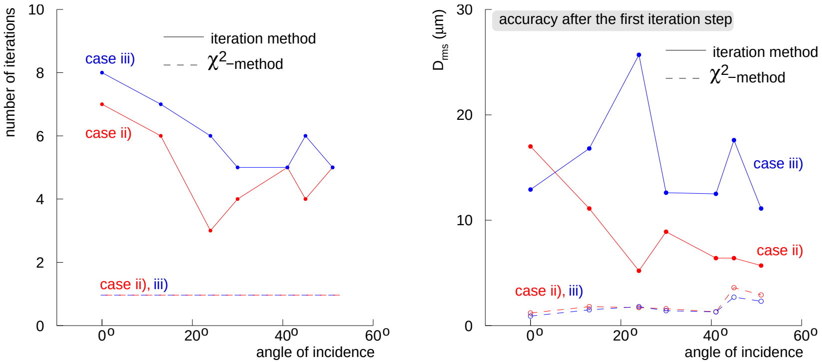
- The width of the difference:

$$D_{rms} = \sqrt{\left[\sum_{bin} (\sigma_n - \sigma)^2\right] / N_{bins}} \approx 2\ \mu m \text{ (with} \sim\text{20 000 tracks)}$$

Convergence with Straight Track Fit

ii), iii) Initial estimate different from true resolution:

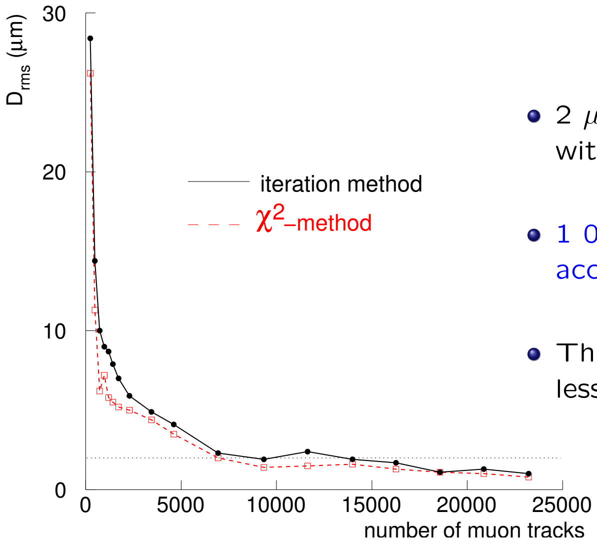
- Iteration method needs several iteration steps to converge.
- χ^2-method converges in one step



- Final accuracy is independent of the initial estimate, similar accuracy of $\sim 2\ \mu$m obtained with both methods.

Dependence on the Available Statistics

Number of muon tracks needed for a given accuracy:

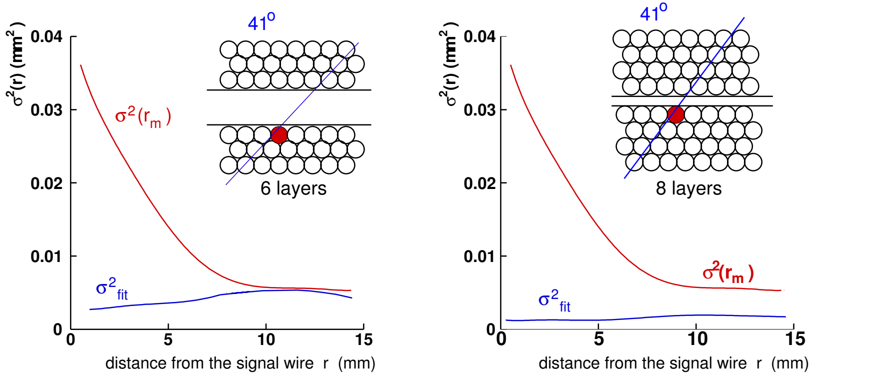


- 2 μm accuracy is achievable with 5 000 muon tracks.
- 1 000 tracks provide an accuracy of better than 10 μm.
- The method is not reliable with less than 500 tracks.

A Comment on the Size of the Track Fit Variance

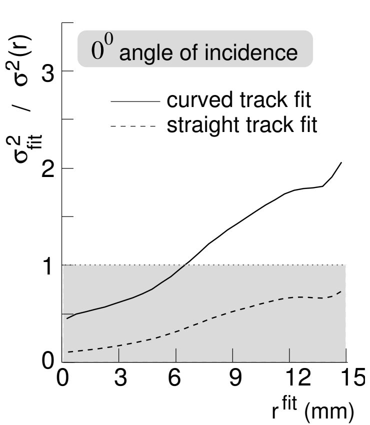
- Iteration method is sensitive to the initial resolution estimate.
- This sensitivity increases with the size of the track fit variance.
- The variance of the track fit through an 8-layer chamber is small, due to the larger number of track points $\Longrightarrow$

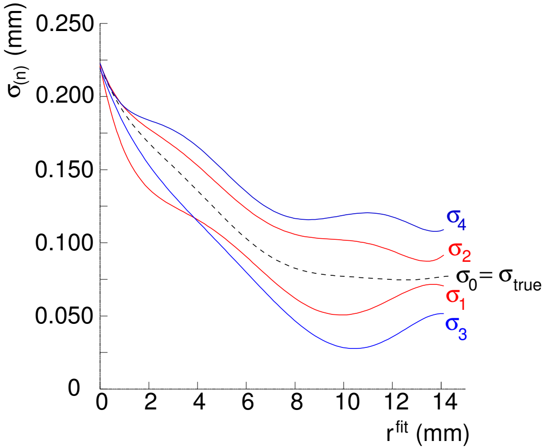
  the resolution can be determined more accurately.



Iteration Method with Curved Track Fit

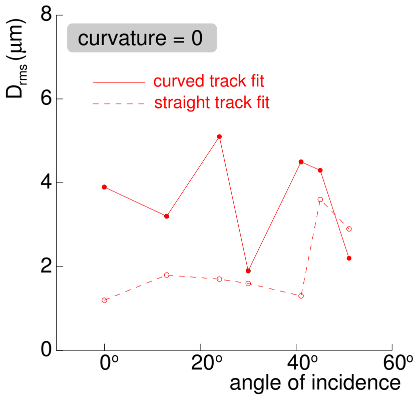
- $\sigma_{fit}(y = \alpha_0 + \alpha_1 \cdot z + \alpha_2 \cdot z^2) \;>\; \sigma_{fit}(y = \alpha_0 + \alpha_1 \cdot z)$
- the variance of the curved track fit becomes comparable to the single-tube resolution $\Longrightarrow$ iteration method highly sensitive, the convergence is lost!

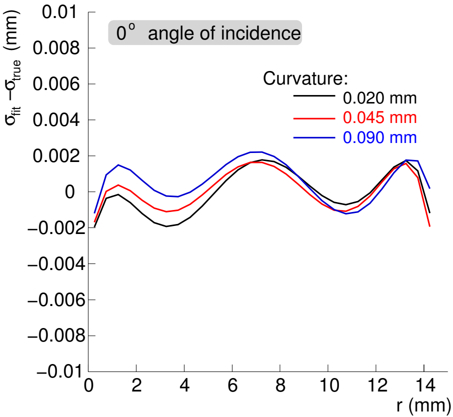




χ^2-Method with Curved Track Fit

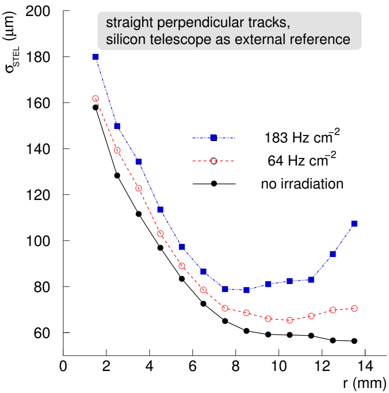
- The track fit variance is described by an exact formula $\Longrightarrow$ the convergence is not lost!
- Initial resolution estimate defines only the selection of hit points for the track reconstruction (second order effect).
- Accuracy is slightly worse than for the straight track fit.

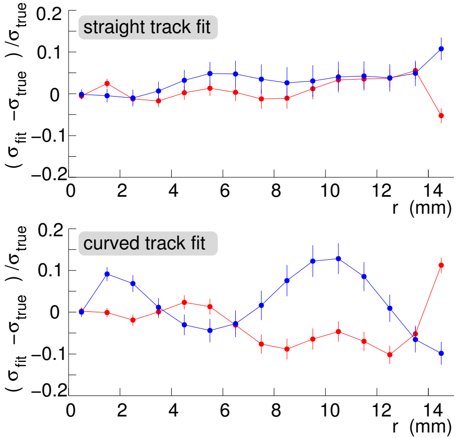




Influence of the γ-irradiation

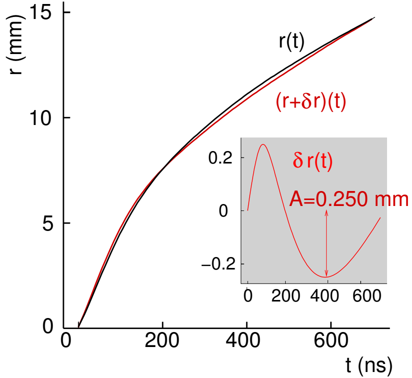
- χ^2-method applied on the X5-testbeam data taken in 2002.
- The resolution depends strongly on the background count rate.
- Additional background hits increase
  the variation of the track fit variance within a given *r*-bin.





Dependence on the r(t)-Relation

- Previous results are obtained with the accurate r(t)-relation.
- Assume a wrong r(t)-realation, $r(t) \rightarrow (r + \delta r)(t)$
- Take $\delta r = A \cdot \frac{2\pi r}{R}$, R - outer tube diameter
  (the residuals of the track fit still remain equal to zero).



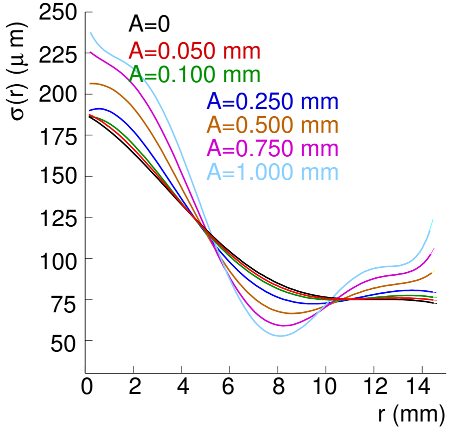
Effect on the resolution determination:

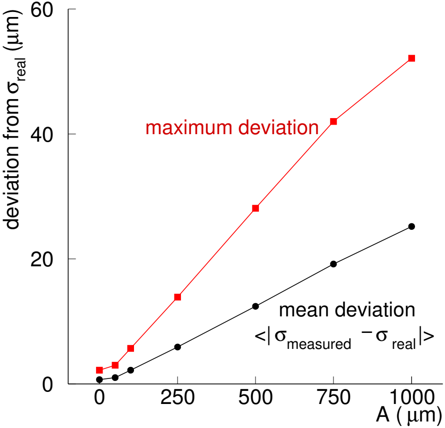
$$\left[\sigma^2(r_k - r_k^{(fit)})\right]'$$

$$= \sigma^2(r_k + \delta r_k - r_k^{(fit)} - \delta r_k^{(fit)})$$

$$= \left[\sigma^2(r_k - r_k^{(fit)})\right] \cdot \left(1 + \left(\frac{d(\delta r)}{dr}\right)^2_{r^{(fit)}}\right)$$

Dependence on the r(t)-Relation





- The deviation from the true resolution as expected.
- With an r(t)-accuracy of 100 μm
  the resolution can be determined with an accuracy of 5 μm.

Summary

Two methods for the determination of the spatial drift-tube resolution using straight or curved muon tracks have been tested for the performance in the ATLAS environment.

χ^2-method .vs. iteration method:

- Faster and more robust.
- No need for the iterations, i.e. re-tracking.

Accuracy of both methods:

- ~1 000 muon tracks needed for a reliable performance, accuracy of better than 10 μm.
- Accuracy of 2 μm with 5 000 tracks.

- Distortion of the r(t)-relation up to 100 μm (initial r(t)-accuracy) introduces a resolution error of less than 10 μm.