q^2-Abhängigkeit der Eichkopplungen:

$$\alpha_i(q^2) = \frac{\alpha_i(q_0^2)}{1 + b_i \frac{\alpha_i(q_0^2)}{4\pi} \ln\left(\frac{q^2}{q_0^2}\right)}$$

$$\Longleftrightarrow \frac{1}{\alpha_i(q^2)} = \frac{1}{\alpha_i(q_0^2)} + \frac{b_i}{4\pi} \ln\left(\frac{q^2}{q_0^2}\right)$$

mit (N_G = Zahl der Fermion-Generationen $= N_q/2$):

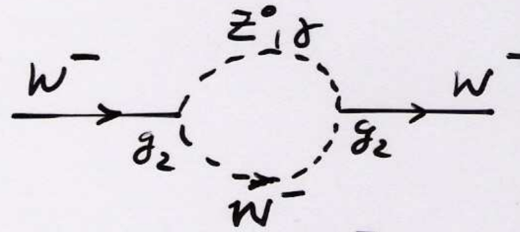
$$\underbrace{11}_{(g)} \quad \underbrace{\tfrac{4}{3}N_G}_{(q)}$$

$b_3 = 11 - \frac{4}{3}N_G > 0$ (für $N_G \leq 8$) $=$ QCD $\rightarrow \alpha_s(q^2)$

$SU(3) \subset SU(5)$

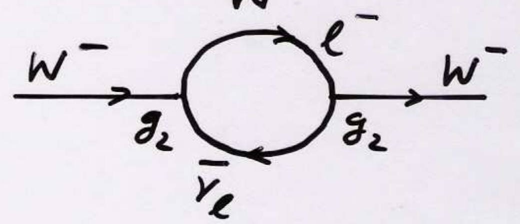
(a) (b)

$b_2 = \frac{22}{3} - \frac{4}{3}N_G > 0$ (für $N_G \leq 5$)



$SU(2) \subset SU(5)$

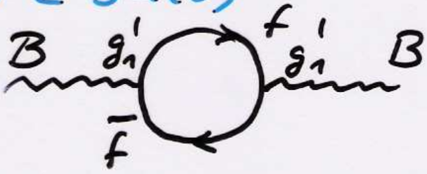
GSW $\rightarrow \alpha_w(q^2) = \frac{g_2^2}{4\pi}$



$b_1' = \quad - \frac{4}{3}N_G < 0$

$U(1) \subset SU(5)$

wie QED $\rightarrow \alpha(q^2)$



z.B.: $\qquad \alpha_2(M_{GUT}^2) \equiv \alpha_3(M_{GUT}^2) = \alpha_5,$ d.h.

$$\frac{1}{\alpha_3(q_0^2)} + \frac{b_3}{4\pi} \ln\left(\frac{M_{GUT}^2}{q_0^2}\right) = \frac{1}{\alpha_2(q_0^2)} + \frac{b_2}{4\pi} \ln\left(\frac{M_{GUT}^2}{q_0^2}\right),$$

$$\frac{1}{\alpha_3(q_0^2)} - \frac{1}{\alpha_2(q_0^2)} = -\frac{11}{12\pi} \ln\left(\frac{M_{GUT}^2}{q_0^2}\right).$$

Ebenso:

$$\underbrace{\frac{1}{\alpha_3(q_0^2)} - \frac{1}{\alpha_1(q_0^2)}}_{=\frac{1}{\alpha_3(q_0^2)}-\frac{3}{5}\left(\frac{1}{\alpha(q_0^2)}-\frac{1}{\alpha_2(q_0^2)}\right)} = -\frac{11}{4\pi}\ln\left(\frac{M_{GUT}^2}{q_0^2}\right)$$

$$\text{mit} \quad \frac{1}{\alpha_1} = \frac{1}{c^2\alpha'} = \frac{3}{5}\cdot\frac{1}{\alpha'}$$

$$\text{und} \quad \frac{1}{\alpha} = \frac{1}{\alpha'} + \frac{1}{\alpha_2}, \quad \text{da} \quad e^2 = \frac{g^2 g'^2}{g^2 + g'^2}.$$

$$\Longrightarrow \frac{8}{3}\cdot\frac{1}{\alpha_3(q_0^2)} - \frac{1}{\alpha(q_0^2)} = -\frac{11}{2\pi}\ln\left(\frac{M_{GUT}^2}{q_0^2}\right).$$

Mit $q_0^2 = M_Z^2$, $\alpha_3(M_Z^2) = \alpha_s(M_Z^2) = 0.119$ und $\alpha(M_Z^2) = 1/128.9$ Vorhersage für die GUT-Skala möglich:

$$\ln\left(\frac{M_{GUT}^2}{M_Z^2}\right) = 30.4 \quad \text{und}$$

$$M_{GUT} = 1.5\cdot 10^{15}\ \text{GeV} \ll M_{\text{Planck}} \approx 10^{19}\ \text{GeV}.$$

3.2.3 Vorhersage von $\sin^2\theta_W(M_W^2)$:

$$\sin^2\theta_W = \frac{g_1^2}{g_1^2+g_2^2} = \frac{\alpha_1}{\alpha_1+\alpha_2} = \frac{1}{1+\frac{\alpha_2(q^2)}{\alpha_1(q^2)}}.$$

$$\sin^2\theta_W(M_{GUT}^2) = \frac{1}{1+c^2} = \frac{3}{8}\,.$$

Rückentwicklung von M_{GUT}:

$$\begin{aligned}
\cdot\quad & \frac{1}{\alpha'(M_W^2)} = \frac{5}{3}\frac{1}{\alpha_1(M_W^2)} \\
= \quad & \frac{5}{3}\left[\frac{1}{\alpha_1(M_{GUT}^2)} + \frac{b_1'}{4\pi}\ln\left(\frac{M_W^2}{M_{GUT}^2}\right)\right] \\
\equiv \quad & \frac{5}{3}\left[\frac{1}{\alpha_2(M_{GUT}^2)} - \frac{4N_G}{12\pi}\ln\left(\frac{M_W^2}{M_{GUT}^2}\right)\right] \\
= \quad & \frac{5}{3}\left[\frac{1}{\alpha_2(M_W^2)} + \frac{22-4N_G}{12\pi}\ln\left(\frac{M_{GUT}^2}{M_W^2}\right) - \frac{4N_G}{12\pi}\ln\left(\frac{M_W^2}{M_{GUT}^2}\right)\right] \\
= \quad & \frac{5}{3}\left[\frac{1}{\alpha_2(M_W^2)} - \frac{11}{6\pi}\ln\left(\frac{M_W^2}{M_{GUT}^2}\right)\right].
\end{aligned}$$

$$\Longrightarrow \frac{\alpha_2}{\alpha_1}(M_W^2) = \frac{5}{3}\left[1 - \frac{11\alpha_2(M_W^2)}{6\pi}\ln\left(\frac{M_W^2}{M_{GUT}^2}\right)\right]$$
$$= \frac{5}{3}\left[1 - \frac{11 G_F M_W^2}{3\pi^2\sqrt{2}}\ln\left(\frac{M_W^2}{M_{GUT}^2}\right)\right] = 3.79,$$

unabhängig von N_G. Mit $M_W = 80.33$ GeV ist

$$\Longrightarrow \sin^2\theta_W(M_W^2) = \frac{1}{1+\frac{\alpha_2}{\alpha_1}(M_W^2)} = 0.209.$$