Tutorial 11: Measurements of the Higgs boson

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In the last tutorial, we reviewed the evidence for a new particle with $m \approx 125$ GeV observed at the LHC. The data are in strong disagreement with the "no-Higgs" hypothesis, at the level of more than 7σ. However, this in itself does not mean that a Higgs boson has been discovered; for this assertion many further measurements are required. The simple fact that the new particle can be discovered in the diboson channels (ZZ and $\gamma\gamma$) indicates that the particle itself must be a boson. Beyond this, its spin is not determined by the simple fact of discovery, nor is its intrinsic parity. For a full characterisation, the couplings of the new boson should also be measured, and compared to the SM predictions. If there is a deviation in this case, it could mean that there are multiple Higgs bosons, of which this is the first, or that the new particle is unrelated to electroweak symmetry breaking. In addition, searches for as-yet unseen decay modes continue, in case they yield further information. Today, we will look at some of the most up-to-date measurements in this field.

Caution: This is a rapidly changing field of study. Some interpretations are neccessarily subjective and preliminary; it will take many years of further study to fully characterise and understand the new boson.

1 Mass of the boson

The mass is an important parameter to measure for any new particle. This is doubly true for the new boson, as the Higgs boson mass, m_H, was the last SM parameter with no direct measurement to constrain its value. With its discovery in two channels with excellent mass resolution ($H \to 4\ell$ and $H \to \gamma\gamma$), the measurement of the boson's mass is conceptually straightforward. The 4-vector of the boson is estimated as the sum of the 4-vectors of the decay products, and the mass of this 4-vector is constructed in the usual way. For example, in the $\gamma\gamma$ case, the diphoton mass is defined as

$$m_{\gamma\gamma} = \sqrt{\left(E_{\gamma 1} + E_{\gamma 2}\right)^2 - \left(\boldsymbol{p}_{\gamma 1} + \boldsymbol{p}_{\gamma 2}\right)^2}. \tag{1}$$

A similar relation holds for the 4ℓ channel. The difficulty in this measurement arises from the precise understanding of the energy and momentum

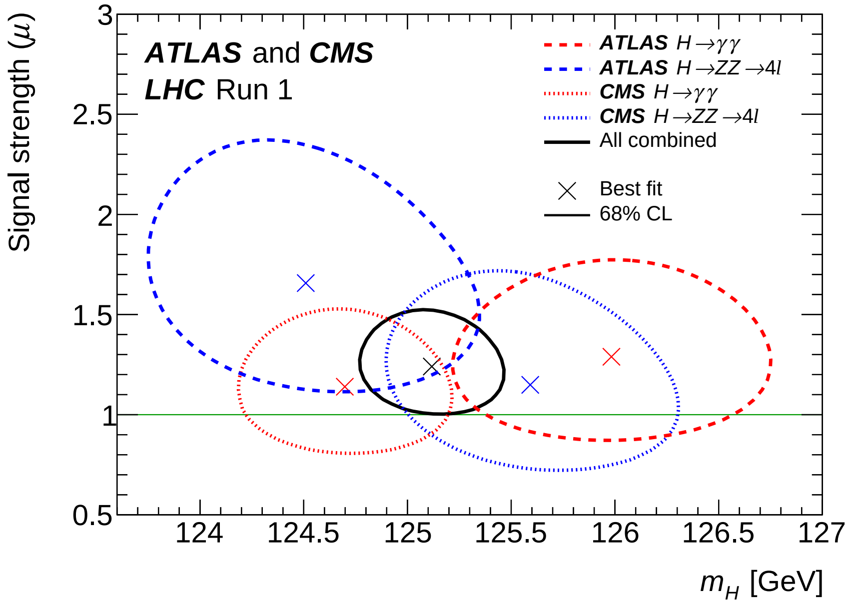


Figure 1: Constraints in the μ-m_H plane from ATLAS and CMS.

reconstruction of electrons, muons and photons. These are calibrated chiefly using $Z \to e^+e^-$ and $Z \to \mu^+\mu^-$ decays, using the known mass and width of the Z boson that was measured at LEP. Additional calibration information for leptons with low p_{T} ($\lesssim 30$ GeV) comes from heavy flavour quark decays, e.g. $J/\psi \to \mu^+\mu^-$. After calibration, the systematic uncertainty on the mass measurement in a single channel is reduced to 0.2% or better, smaller than the statistical uncertainty.

The mass measurement results from both ATLAS and CMS are shown in Figure 1. The constraints are illustrated in the 2D plane of mass and signal strength. Both collaborations observe signal strengths compatible with unity, and the combined mass measurement is

$$m_H = 125.09 \pm 0.21(\text{stat.}) \pm 0.11(\text{syst.}) \text{ GeV}. \tag{2}$$

Exercise: Why is it important to consider the signal strength simultaneously with the mass? *Hint:* consider a coarsely-binned m_H distribution, with an excess in a single bin. Remember that the SM production cross-section for the Higgs boson decreases rapidly with mass (see last tutorial).

2 Spin and parity of the boson

In the Standard Model, the Higgs field is a scalar field, i.e. it has a spin-parity (J^P) of 0^+. This means that the physical Higgs boson decays isotropically

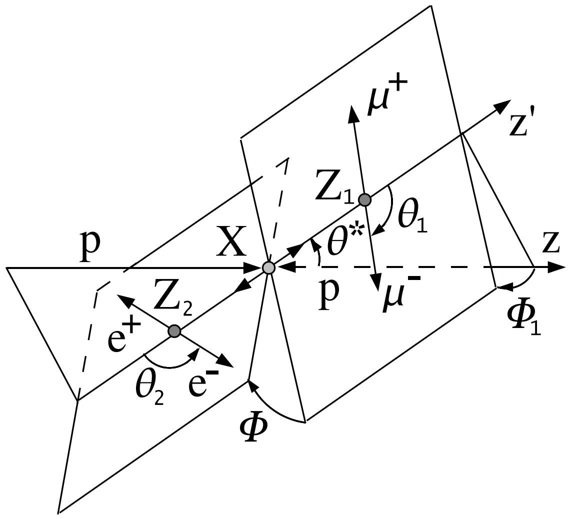


Figure 2: Illustration of the 5 measurable angles in an $X \to Z_1 Z_2 \to e^+e^-\mu^+\mu^-$ decay. The angles θ^*, Φ_1 and Φ are shown in the X rest frame, while the θ_i angles are shown in the Z_i rest frames.

in its own rest frame. For other, hypothetical, particles with a different J^P assignment, this is not necessarily true. In particular, if a particle with $J \neq 0$ is produced in pp collisions at the LHC, the different polarisation states $0 \leq |m| \leq J$ will in general be produced with different amplitudes, and therefore different rates.[1] The decays of this particle will therefore not, on average, be isotropic. The parity of the particle can also influence the decay angles in more subtle ways. Therefore, it is possible, in principle, to differentiate experimentally between different spin-parity hypotheses by analysing the angular distributions of the outgoing particles.

Three channels are used to investigate the spin and parity of the new boson: $H \to \gamma\gamma$, $H \to ZZ \to 4\ell$ and $H \to WW \to e\nu\mu\nu$. The $\gamma\gamma$ final state is a relatively simple two-body final state that, for a given mass m_H, can be described entirely by two angles in the new boson's rest frame. The 4ℓ final state is more complex, described by two masses (those of the intermediate Z bosons) and five angles, illustrated in Figure 2. As long as the correct association of lepton pairs can be made, these masses and angles can all be measured in every $H \to 4\ell$ candidate event. The additional degrees of freedom in this decay (in particular θ_1 and θ_2) allow different parity states to be distinguished. The case of $H \to WW$ is similar, except that only two of the four leptons can be directly observed.

The discrete nature of J^P leads to an interesting problem related to hypothesis testing. Normally, we wish to test some hypothesis H_1 (say, the

[1] Note that the polarisation is usually described with respect to the direction of motion in the laboratory frame.

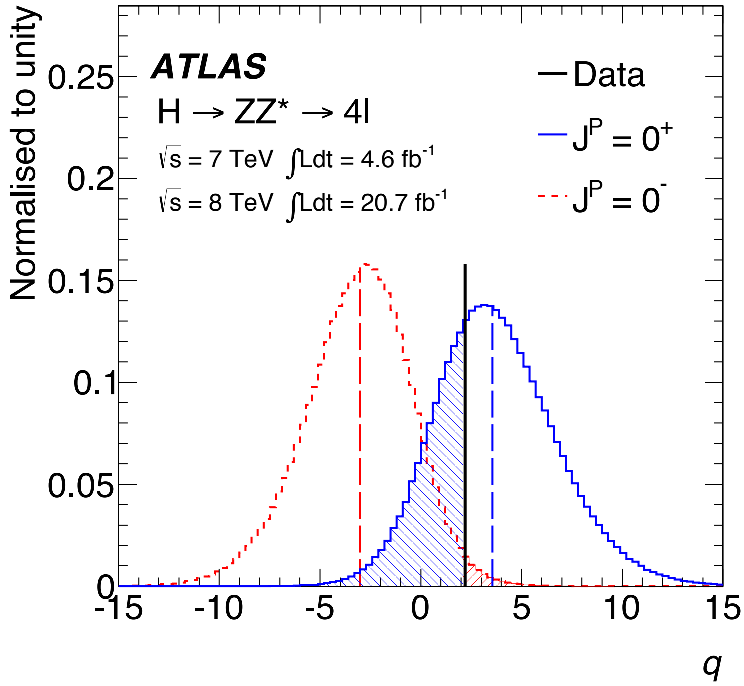


Figure 3: Comparison of the $J^P = 0^+$ and $J^P = 0^-$ hypotheses in the 4ℓ channel. The variable q is defined in Equation (3). The probability density functions obtained assuming each hypothesis is shown, along with the observation from data ($q \approx 2$).

Standard Model with a Higgs boson mass of $m_H = 125$ GeV) by comparing it to some null hypothesis H_0 (in this case, the "no-Higgs" hypothesis). Then, by comparing the predictions in each case to experimental data, we can either reject H_0 or set constraints on one or more parameters in H_1 (e.g. the signal strength). In this case, however, we can only compare one J^P hypothesis against another. In practice, we wish to ask whether this new particle could be the SM Higgs boson, and so usually $J^P = 0^+$ is compared to one of the other possibilities, effectively taking the place of H_0.

Figure 3 shows one example of such a comparison. The variable q is defined in this case as[2]

$$q = \log \frac{\mathcal{L}(0^+)}{\mathcal{L}(0^-)} = \log \frac{\prod_{\text{channels}} P(N^{\text{data}}_{\text{channel}}|0^+)}{\prod_{\text{channels}} P(N^{\text{data}}_{\text{channel}}|0^-)}$$
$$= \sum_{\text{channels}} \log \frac{P(N^{\text{data}}_{\text{channel}}|0^+)}{P(N^{\text{data}}_{\text{channel}}|0^-)}, \quad (3)$$

[2]Several dependencies on other model parameters, experimental uncertainties, etc., have been dropped for clarity.

where $\mathcal{L}$ denotes a *likelihood*, which can be thought of as the probability of the given data observation under some particular model assumption. Thus, if the observation in a particular channel is likely under both hypotheses, it contributes little to q as the ratio of probabilities will be close to 1. If, instead the observation in a channel favours the 0^+ hypotheses, $P(N^{\text{data}}_{\text{channel}}|0^+)/P(N^{\text{data}}_{\text{channel}}|0^-)$ will be greater than 1, and this channel will contribute positively to q. Similarly, an observation favouring the 0^- hypothesis will contribute negatively to q. This can be seen in Figure 3, where the probability density functions for each hypothesis are shown, with possible results for the 0^+ hypothesis favouring positive values of q. Statistical and systematic uncertainties prevent the complete separation of the two distributions, potentially allowing for some ambiguity in the result.

The actual experimental result (shown as a vertical black line at $q \approx 2$) is clearly more consistent with the 0^+ hypothesis than the 0^- hypothesis. For each distribution, the probability (*p-value*) of getting a value of q larger than the observed value is calculated. Thus, the red shaded area corresponds to $p(0^-)$, and the blue shaded area to $1 - p(0^+)$. The degree of disagreement with the 0^- hypothesis is then given by the so-called CL_s value, defined as

$$\text{CL}_s = \frac{p(0^-)}{1 - p(0^+)}. \tag{4}$$

If CL_s is small, then it can allow the 0^- hypothesis to be excluded at the appropriate confidence level (CL). In this case, it is excluded at the 97.8% CL, in favour of the 0^+ hypothesis. Similarly, the LHC data allow the 1^+, 1^- and 2^+ hypotheses to be excluded at the 99.7% CL or better. Thus, by elimination, the data strongly suggest that the new particle is indeed scalar.

Exercise: Consider what conclusions would be drawn for observed q values of -5, 0 and 5, given the probability density distributions in Figure 3.

3 Couplings of the boson

If the couplings of the new boson are not the same as those of the SM Higgs boson, there will be deviations in the observed rate of signal events in different channels. These deviations can be conveniently expressed in terms of scaling factors κ_i, applied to the SM couplings. For example, the coupling g_f of the new boson to an SM fermion f can be expressed in terms of the SM Yukawa coupling y_f as

$$g_f = \kappa_i y_f, \tag{5}$$

with similar relations for the SM bosons. Effective couplings can also be calculated for massless SM particles, allowing κ_g and κ_γ to be defined in terms of the other κ_i. For example, $\kappa_\gamma^2 \approx 1.59\kappa_W^2 - 0.66\kappa_W\kappa_t + 0.07\kappa_t^2$.

With this definition, the partial width of the new boson to the ZZ final state, for example, is proportional to κ_Z^2. This observation allows us to define a scale factor for the total width of the boson in terms of the κ_i parameters:

$$\kappa_H^2 = \sum_{i=b,W,Z,\text{etc.}} \frac{\kappa_i^2 \Gamma^{\text{SM}}_{H\to ii}}{\Gamma^{\text{SM}}_H}. \tag{6}$$

However, in any LHC measurement, the production mechanism must also be taken into account. The production cross-section will also be scaled by κ_i^2, where i depends on the flavours of the incoming particles. For example, consider the 4ℓ channel proceeding via gluon fusion. In this case, the total rate[3] for the $gg \to H \to ZZ \to 4\ell$ process can be written as

$$\frac{\sigma(gg \to H)\text{BR}(H \to ZZ \to 4\ell)}{\sigma_{\text{SM}}(gg \to H)\text{BR}_{\text{SM}}(H \to ZZ \to 4\ell)} = \frac{\kappa_g^2 \kappa_Z^2}{\kappa_H^2}. \tag{7}$$

Exercise: Why does κ_H^2 appear in the denominator of Equation (7)?

There is little possibility, in a hadron collider, to measure each of the κ_i individually, at least in the near future. To progress further, some additional assumptions are needed. For example, one could assume that all fermions have the same scaling factor κ_f, and all bosons have the same scaling factor κ_V, and that κ_H can be calculated only from these. This simple two-parameter model can be tested in multiple final states, as shown in Figure 4. The results are consistent with the SM prediction, with best-fit values of

$$\begin{aligned} \kappa_V &= 1.09 \pm 0.07 \\ \kappa_F &= 1.11^{+0.17}_{-0.15} \end{aligned} \tag{8}$$

The possibility of a negative phase between the couplings (with $\kappa_F \sim -1$) is disfavoured at the level of about 4σ. Other coupling models have also been tested, with similar results. In particular, the ratio of κ_W/κ_Z cannot deviate far from 1 if the boson is to play any role in electroweak symmetry breaking. Assuming a common fermion factor κ_F, that ratio is measured to be $0.92^{+0.14}_{-0.12}$, showing very good compatibility with the SM prediction.

4 Searches for rare channels

In addition to carefully measuring observed decay channels of the new boson, it is also potentially fruitful to search for production and decay modes that are rare in the SM. If observed with current data, they would imply non-SM

[3] Written here as $\sigma \times \text{BR}$, the product of a production cross-section σ and a decay branching fraction BR.

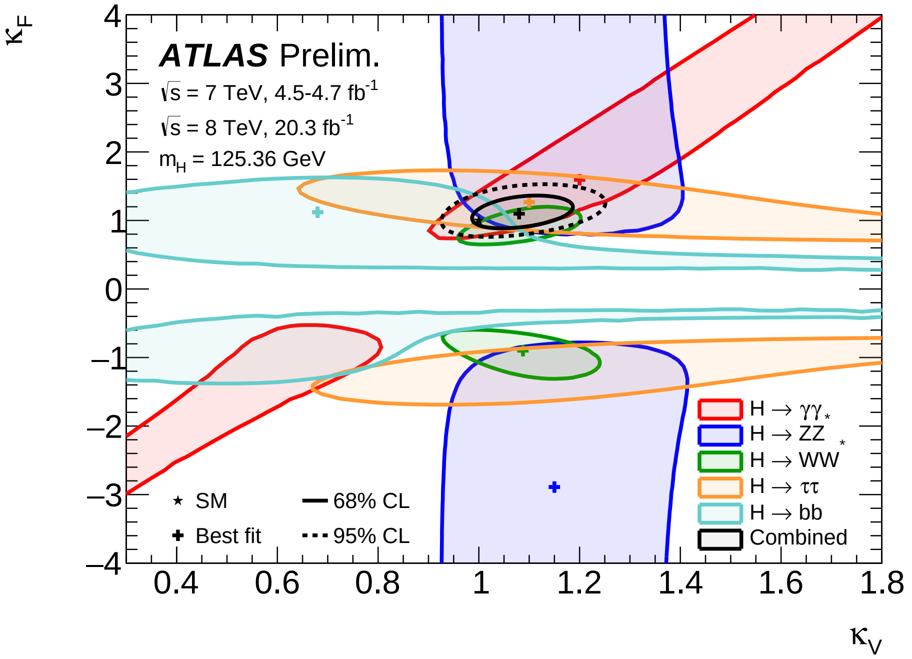


Figure 4: Constraints on the κ_i scaling factors, assuming a universal fermion factor $\kappa_F = \kappa_t = \kappa_b = \kappa_\tau = \ldots$ and a universal boson factor $\kappa_V = \kappa_W = \kappa_Z$. The 68% and 95% CL contours for the combination of all channels are shown, and the SM prediction is marked at (1,1).

phenomena. It could be that the boson itself is not the SM Higgs boson, e.g. with non-standard couplings, or it could be that yet more new particles alter the apparent behaviour of the particle, to name two possibilities. Here, we take a very brief look at a few of the studies that have been performed to date.

4.1 Searches for $t\bar{t}H$ production

Of the four principal production modes, the $t\bar{t}H$ mode has the smallest cross-section (see last tutorial). In addition, the most common decay mode is $t\bar{t}H \to (bq\bar{q})(\bar{b}q\bar{q})(b\bar{b})$, leading to eight jets in the final state, and few distinctive features to aid the separation of the signal from background multijet events. As with the initial discovery of the boson, sensitivity can be improved by also looking at other channels that may have fewer events, but that benefit from improved signal-to-background ratios. These include the $H \to \gamma\gamma$ and $H \to \tau\tau$ decay modes, in addition to "multi-lepton" channels. The $H \to 4\ell$ decay is too rare to be useful (BR $\sim 10^{-4}$), but the semi-leptonic decay of the top quark ($t \to bW^+ \to b\ell^+\nu_\ell$) opens the possibility

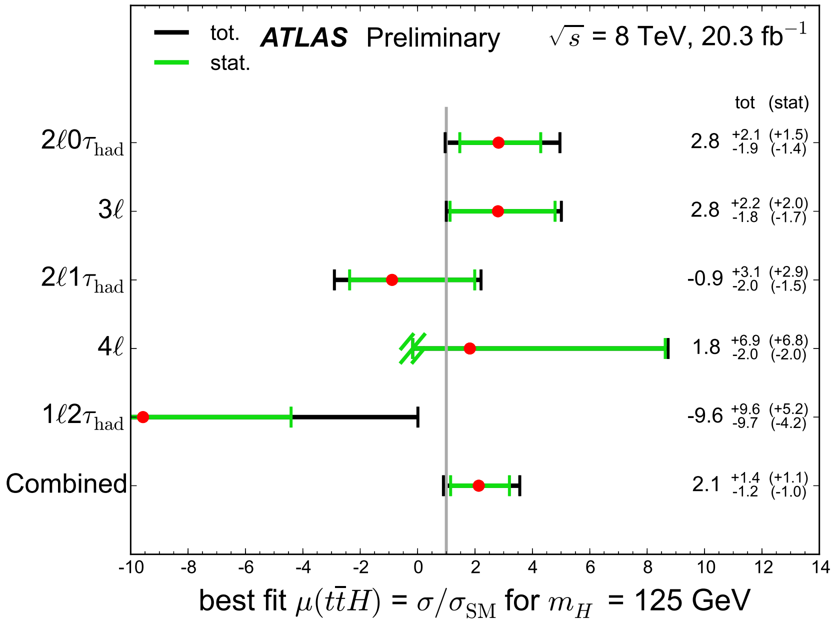


Figure 5: Results of the search for $t\bar{t}H$ production in leptonic channels with ATLAS.

for other event signatures to be included in this category, for example:

- $t\bar{t}H \to (b\ell^+\nu_\ell)(\bar{b}q\bar{q})(\tau^+_{\rm lep}\tau^-_{\rm had})$, where two leptons have the same charge.[4]
- $t\bar{t}H \to (b\ell^+\nu_\ell)(\bar{b}q\bar{q})(\ell^+\ell^- q\bar{q})$, with three charged leptons.
- $t\bar{t}H \to (b\ell^+\nu_\ell)(\bar{b}\ell^-\bar{\nu})(\ell^+\ell^-\nu\bar{\nu})$, with four charged leptons.[5]

In some cases, hadronically decaying tau leptons may also be included in the target signature. There is insufficient data to conclusively observe $t\bar{t}H$ production, but both ATLAS and CMS observe excess events beyond what would be expected if this process did not occur (i.e. $\mu(t\bar{t}H) = 0$). The excesses have a significance of 1.8σ and 3.4σ, respectively. Example constraints on the $t\bar{t}H$ signal strength from the ATLAS experiment are shown in Figure 5.

4.2 Invisible Higgs boson decays

In the SM, it is possible for the Higgs boson to decay completely invisibly, via $H \to ZZ \to \nu\bar{\nu}\nu\bar{\nu}$. This has a branching fraction of about 0.1% if $m_H = 125$ GeV, and can be neglected for most practical purposes. If,

[4] $\tau_{\rm lep}$ and $\tau_{\rm had}$ indicate leptonically and hadronically decaying τ leptons, respectively.
[5] Here the Higgs decay may proceed via WW or ZZ.

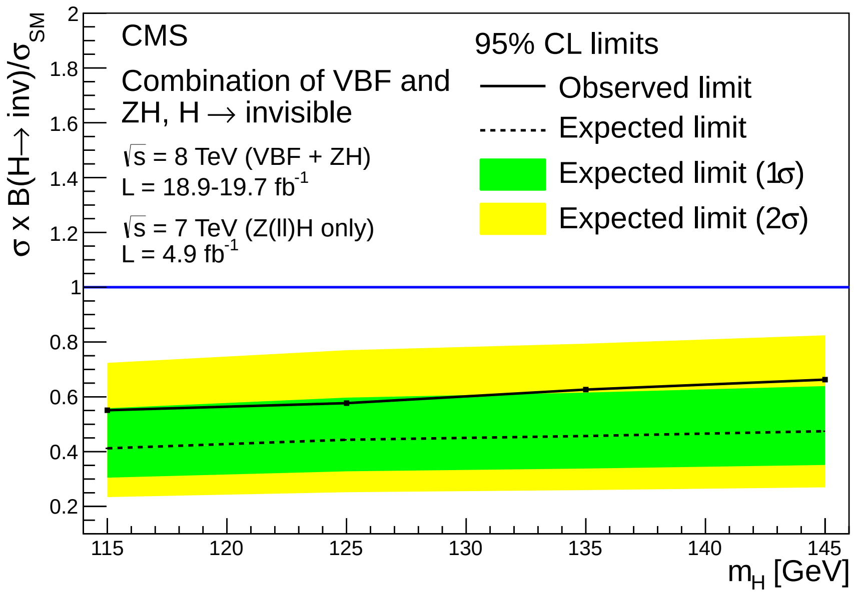


Figure 6: CMS limit on the "invisible" branching fraction of the new boson, expressed as the ratio $\sigma \times \mathrm{BR}(H \to \mathrm{invisible})/\sigma_{\mathrm{SM}}$.

however, dark matter is made of elementary particles (χ) that obtain their mass in the same way as the SM particles, then the invisible decay channel $H \to \chi\chi$ may also be open, if $m_\chi < m_H/2$. In these circumstances, the "invisible width" of the Higgs boson may be substantially larger than in the SM.

Despite the lack of visible particles from the Higgs boson decay, it is possible to detect this channel by searching for Higgs production channels (vector boson fusion and Zh associated production) where the associated jets and/or leptons may be detected, in association with substantial *missing transverse momentum* from the invisible particles (usually denoted $E_{\mathrm{T}}^{\mathrm{miss}}$). This quantity is based on momentum conservation in the plane perpendicular to the proton beams (the "transverse" plane), where the sum of momenta of all reconstructed objects (leptons, jets, etc.) should be zero within the detector resolution. Substantial deviations from zero can be associated with the presence of undetected particles such as neutrinos. This momentum balance cannot be assumed in the z direction (along the beams), due to the unknown momenta of the initial state partons (see tutorial 5). The $E_{\mathrm{T}}^{\mathrm{miss}}$ is, however, subject to large uncertainties, mainly arising from the limited p_{T} resolution of jets, making the unambiguous identification of events with large $E_{\mathrm{T}}^{\mathrm{miss}}$ very challenging.

Figure 6 shows the current best limits on the invisible branching fraction of the new boson, as a function of its assumed mass. The normalisation assumes that the boson is produced with the SM cross-section; under this assumption, the branching fraction of a 125 GeV boson to undetectable particles is less than 58% at the 95% confidence level.

Exercise: What other rare processes might it make sense to search for, that should be unobservable in the current ATLAS and CMS data if the boson is the SM Higgs boson? Use the plots of the Higgs boson production and decay modes from the last tutorial to guide your choices.

4.3 Off-shell production and the Higgs boson width

According to the Standard Model, a Higgs boson with a mass of 125 GeV should have a natural width of about 4 MeV. This is too narrow to be directly measurable at the LHC, given the typical energy resolution of leptons and photons of $\sim 1-2$ GeV. However, it is possible to constrain the Higgs boson width indirectly by combining on-shell and off-shell measurements. To see why this is so, recall the Breit-Wigner distribution (for example, in the 4ℓ channel):

$$\frac{\mathrm{d}\sigma(pp \to H \to 4\ell)}{\mathrm{d}m_{4\ell}} \propto \frac{g_g^2 g_Z^2}{(m_{4\ell}^2 - m_H^2)^2 + m_H^2\Gamma_H^2}. \tag{9}$$

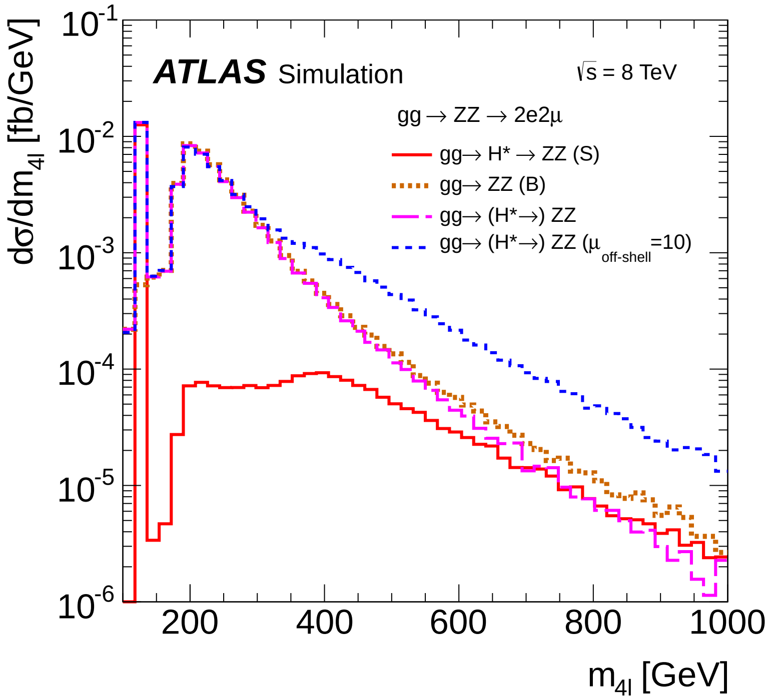
The on-shell and off-shell limits of Equation (9) are as follows:

$$\begin{aligned} \text{On-shell:} \quad & \left.\frac{\mathrm{d}\sigma(pp \to H \to 4\ell)}{\mathrm{d}m_{4\ell}}\right|_{m_{4\ell}=m_H} \propto \frac{g_g^2 g_Z^2}{m_H^2\Gamma_H^2}, \\ \text{Off-shell:} \quad & \left.\frac{\mathrm{d}\sigma(pp \to H \to 4\ell)}{\mathrm{d}m_{4\ell}}\right|_{m_{4\ell}>>m_H+\Gamma_H} \propto \frac{g_g^2 g_Z^2}{(m_{4\ell}^2 - m_H^2)^2}. \end{aligned} \tag{10}$$

If the same mixture of production and decay modes are probed in both regimes, and the couplings do not run significantly with $m_{4\ell}$, then the ratio of these two cross-sections allow Γ_H to be extracted if m_H is known. In reality the $m_{4\ell}$ differential cross-section is more complex than a simple Breit-Wigner, primarily due to reduced kinematic suppression of $H^* \to ZZ^* \to 4\ell$ for $m_{4\ell} > 2m_Z = 182$ GeV. This is shown in Figure 7 (left), along with the distribution for inclusive $ZZ^{(*)}$ production. While statistically limited, the rates are such that values the Higgs boson width larger than about four times the SM expectation can be excluded (see Figure 7 (right)), a result that is two orders of magnitude more precise than the direct measurement.

5 Summary

Today, we have looked at just a few of the measurements performed so far on the new boson. So far, there is no convincing evidence against the hypothesis of an SM Higgs boson with mass $m_H \approx 125$ GeV. In particular, the coupling of the boson to other SM particles is clearly related to the masses of those particles, and for this reason there is now general agreement that it is related to electroweak symmetry breaking, and can therefore be called a Higgs boson. There are still significant uncertainties, however – many measurements are still statistically limited, and the LHC experiments are not yet sensitive to several important production and decay channels. Over the coming 15 years, the LHC experiments plan to collect $\mathcal{O}(100)$ times their current dataset, in an attempt to improve on the quality of these measurements, and future colliders are being planned with further high-precision measurements of the Higgs boson in mind. We will return to these future projects later in the course.



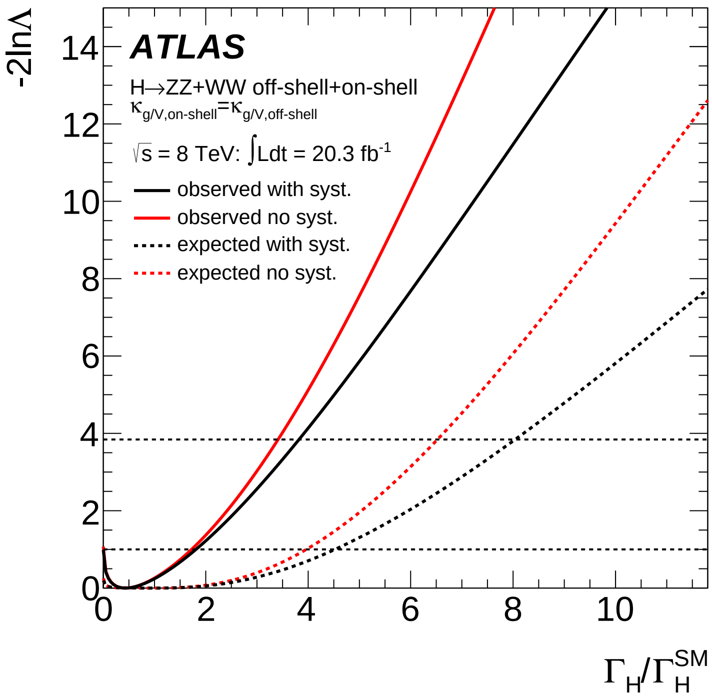


Figure 7: Left: Differential four-lepton cross-section, for ZZ and Higgs boson production. Right: Expected and observed likelihood values as a function of $\Gamma_H/\Gamma_H^{\rm SM}$.