Tutorial 12: Two-Higgs doublet models

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1 Theoretical overview

In the SM, boson and fermion masses are generated through the action of one complex scalar field ϕ. As an SU(2) doublet, this field has four real degrees of freedom. When the field acquires a vacuum expectation value (vev), three degrees of freedom become associated with the longitudinal polarisation modes of the W and Z bosons, leaving the fourth degree of freedom to become the Higgs boson H.

This is a minimal prescription for electroweak symmetry breaking, however it is not unique. The simplest non-minimal approach would be to introduce a second scalar SU(2) doublet, ϕ_2.[1] This is called a *two Higgs doublet model* (2HDM). In the general case, its description is rather complicated, so it is conventional to assume that the Higgs sector conserves CP and that certain terms are absent from the Lagrangian density due to discrete symmetries. In this case, the combined potential of both Higgs doublets can be written as follows:[2]

$$\begin{aligned} V_\phi = {} & m_{11}^2 \phi_1^\dagger \phi_1 + m_{22}^2 \phi_2^\dagger \phi_2 - m_{12}^2 \left(\phi_1^\dagger \phi_2 + \phi_2^\dagger \phi_1 \right) \\ & + \frac{\lambda_1}{2} \left(\phi_1^\dagger \phi_1 \right)^2 + \frac{\lambda_2}{2} \left(\phi_2^\dagger \phi_2 \right)^2 \\ & + \lambda_3 \phi_1^\dagger \phi_1 \phi_2^\dagger \phi_2 + \lambda_4 \phi_1^\dagger \phi_2 \phi_2^\dagger \phi_1 \\ & + \frac{\lambda_5}{2} \left[\left(\phi_1^\dagger \phi_2 \right)^2 + \left(\phi_2^\dagger \phi_1 \right)^2 \right] . \end{aligned} \tag{1}$$

In principle, what we should do next, as in the SM case, is to minimise this potential function, requiring that at least one Higgs doublet obtains a non-zero vev, and study the oscillations around this minimum. In practice, this function can have multiple minima, including ones that produce charge- and/or CP-violating vacua. Excluding these solutions, the position of the

[1] From this point on, we will call the first scalar doublet ϕ_1.

[2] We assume that all coefficients are real.

minimum can be written in this form:

$$\phi_1 = \begin{pmatrix} 0 \\ \frac{v_1}{\sqrt{2}} \end{pmatrix}; \qquad \phi_2 = \begin{pmatrix} 0 \\ \frac{v_2}{\sqrt{2}} \end{pmatrix}. \tag{2}$$

Exercise: Consider adding a term $\begin{pmatrix} w_2/\sqrt{2} \\ 0 \end{pmatrix}$ to the minimum of ϕ_2. Find some of the additional mass-like terms that would arise from the Lagrangian density. What kinds of interactions would these allow? *Hint:* Re-read tutorial 3 if you are not sure how to do this.

Note that each doublet obtains its own vev.[3] As we will see later, the quadrature sum of these vevs is constrained by electroweak measurements (and must be equal to the SM vev), and so it is useful to introduce a parameter β to describe how the vev is shared between the two doublets:

$$v^2 = v_{\text{SM}}^2 = v_1^2 + v_2^2, \tag{3}$$

$$\tan\beta = \frac{v_2}{v_1}$$

$$\Rightarrow \sin\beta = \frac{v_2}{v}; \quad \cos\beta = \frac{v_1}{v}. \tag{4}$$

At the minimum point, the derivatives of V_ϕ must all be zero, which leads to the following constraints on the model parameters:

$$v_1\left(m_{11}^2 + \frac{\lambda_1}{2}v_1^2 + \frac{\lambda_3}{2}v_2^2\right) = v_2\left(m_{12}^2 - \frac{(\lambda_4+\lambda_5)}{2}v_1v_2\right) \quad \text{from } \frac{\partial V_\phi}{\partial \phi_1^\dagger} = 0,$$

$$v_2\left(m_{22}^2 + \frac{\lambda_2}{2}v_2^2 + \frac{\lambda_3}{2}v_1^2\right) = v_1\left(m_{12}^2 - \frac{(\lambda_4+\lambda_5)}{2}v_1v_2\right) \quad \text{from } \frac{\partial V_\phi}{\partial \phi_2^\dagger} = 0. \tag{5}$$

Naturally, if these equations cannot be satisfied for any values of v_1 and v_2, then such a vacuum is impossible.

1.1 Higgs mass eigenstates

Unlike the BEH theory in the SM, there are a number of possible excitations around the minimum of Equation (2), rather than just one. This means that there are multiple observable Higgs bosons. It turns out that these excitations can be written in the following way:

$$\phi_i = \begin{pmatrix} H_i^+ \\ \frac{v_i + H_i^0 + iA_i^0}{\sqrt{2}} \end{pmatrix} \quad \text{for } i = 1, 2. \tag{6}$$

[3] At this point, there is ambiguity in how the two doublets are defined. In fact, they can be rotated into each other arbitrarily without changing the phenomenology. Later, when we discuss fermion interactions, we will see how this ambiguity might be resolved.

To determine the masses and properties of these states, we need to compute the Lagrangian terms involving them. We will start by finding the mass eigenstates of the charged Higgs bosons. These can be found by replacing Equation (6) into Equation (1) and selecting only those terms proportional to $H_i^- H_j^+$. These terms can be written in the following matrix form:

$$V_\pm = \begin{pmatrix} H_1^- & H_2^- \end{pmatrix} \begin{pmatrix} m_{11}^2 + \frac{1}{2}\lambda_1 v_1^2 + \frac{1}{2}\lambda_3 v_2^2 & -m_{12}^2 + \frac{\lambda_4+\lambda_5}{2} v_1 v_2 \\ -m_{12}^2 + \frac{\lambda_4+\lambda_5}{2} v_1 v_2 & m_{22}^2 + \frac{1}{2}\lambda_2 v_2^2 + \frac{1}{2}\lambda_3 v_1^2 \end{pmatrix} \begin{pmatrix} H_1^+ \\ H_2^+ \end{pmatrix}. \tag{7}$$

Exercise: Derive Equation (7) explicitly.

This can be simplified considerably by using Equation (5):

$$V_\pm = \left(m_{12}^2 - \frac{\lambda_4 + \lambda_5}{2} v_1 v_2 \right) \begin{pmatrix} H_1^- & H_2^- \end{pmatrix} \begin{pmatrix} \frac{v_2}{v_1} & -1 \\ -1 & \frac{v_1}{v_2} \end{pmatrix} \begin{pmatrix} H_1^+ \\ H_2^+ \end{pmatrix} \tag{8}$$

The two eigenvalues of the mass matrix in Equation (8) are 0 and $\sec\beta \csc\beta = v^2/v_1^2 v_2^2$. The first eigenvalue corresponds to a Goldstone particle (absorbed by the W), while the second gives a physical, charged Higgs boson $H^\pm$. By convention, this state is defined as

$$\begin{aligned} H^\pm &= -H_1^\pm \sin\beta + H_2^\pm \cos\beta \\ \text{with } m_{H^\pm}^2 &= \left[\frac{m_{12}^2}{v_1 v_2} - \frac{\lambda_4 + \lambda_5}{2} \right] v^2. \end{aligned} \tag{9}$$

A similar analysis of the pseudoscalar states A_i^0 yields a similar result – one Goldstone boson that is absorbed by the Z, and one physical eigenstate rotated from the original basis by an angle of β (with a squared mass that depends on $\frac{m_{12}^2}{v_1 v_2} v^2$).

Exercise: Complete the analysis to find an explicit form for the physical A^0 state and its mass.

The scalar fields H_i^0 have a more complex behaviour. As in the SM case, these are not absorbed during electroweak symmetry breaking, and thus two physical states are obtained. Conventionally, h and H denote the less massive and more massive states, respectively. They are an admixture of the original H_i^0 states, with a mixing angle α that is in general not equal to β. The neutral Higgs boson states are defined as follows:

$$\begin{aligned} h &= \sqrt{2} \left(H_1^0 \sin\alpha - H_2^0 \cos\alpha \right) \\ H &= -\sqrt{2} \left(H_1^0 \cos\alpha + H_2^0 \sin\alpha \right) \\ A &= \sqrt{2} \left(A_1^0 \sin\beta - A_2^0 \cos\beta \right). \end{aligned} \tag{10}$$

The masses for h and H are difficult to write in closed form, however if m_H is sufficiently larger than m_h, it scales with $m_{H^\pm}$ and m_A forming a near-degenerte set of Higgs bosons at high mass.

1.2 Couplings to gauge bosons

As in the SM, the gauge boson masses and couplings to the Higgs field are all described by the kinetic terms for the Higgs fields in the Lagrangian. From tutorial 3, we recall the form of the covariant derivative for the Higgs field:

$$\mathcal{D}_\mu^{\phi_i}\phi_i = \begin{pmatrix} \partial_\mu + ieA_\mu + i\frac{g\cos 2\theta_\mathrm{W}}{2\cos\theta_\mathrm{W}}Z_\mu & i\frac{g}{\sqrt{2}}W_\mu^+ \\ i\frac{g}{\sqrt{2}}W_\mu^- & \partial_\mu - i\frac{g}{2\cos\theta_\mathrm{W}}Z_\mu \end{pmatrix} \begin{pmatrix} H_i^+ \\ \frac{v_i + H_i^0 + iA_i^0}{\sqrt{2}} \end{pmatrix}. \tag{11}$$

Upon expansion of $(\mathcal{D}_\mu^{\phi_i}\phi_i)^\dagger \mathcal{D}^{\mu\phi_i}\phi_i$, and re-expression in terms of the physical Higgs boson eigenstates, the interactions of the gauge bosons with the Higgs fields can be determined. We will not attempt a complete overview of these interactions, but simply select a few of the most interesting terms for study.

The mass of the Z boson is determined by those terms proportional to $Z_\mu Z^\mu$ and contain no other fields (vevs are allowed). The only such terms arise from the product of the lower right element of $\mathcal{D}_\mu^{\phi_i}$ with v_i, which is then squared to give

$$\mathcal{L}_{m_Z} = \frac{g^2}{8\cos^2\theta_\mathrm{W}}v_1^2 Z_\mu Z^\mu + \frac{g^2}{8\cos^2\theta_\mathrm{W}}v_2^2 Z_\mu Z^\mu = \frac{g^2}{8\cos^2\theta_\mathrm{W}}v^2 Z_\mu Z^\mu. \tag{12}$$

This is the same term that arises in the SM Lagrangian, as anticipated in Equation (3). The same conclusion holds for the W boson, namely that the mass of each boson depends not on v_1 and v_2 individually, but only the combination v (at leading order). As in the SM, the photon remains massless, and only interacts with $H^\pm$ through its electric charge.

Next, we examine the interactions of the Z boson with the scalar fields h and H. Again, only the lower right element of $\mathcal{D}_\mu^{\phi_i}$ is relevant, and (apart from a combinatorial factor of two) we simply replace one vev in each term of Equation (12) with the corresponding scalar field:

$$\begin{aligned} \mathcal{L}_{ZH} &= \frac{g^2}{4\cos^2\theta_\mathrm{W}}\left\{v_1 H_1^0 + v_2 H_2^0\right\} Z_\mu Z^\mu \\ &= -\frac{g^2 v}{4\sqrt{2}\cos^2\theta_\mathrm{W}}\left\{h\sin\beta\cos\alpha + H\sin\beta\sin\alpha \right. \\ &\qquad\qquad \left. -h\cos\beta\sin\alpha + H\cos\beta\cos\alpha\right\} Z_\mu Z^\mu \\ &= -\frac{g^2 v}{4\sqrt{2}\cos^2\theta_\mathrm{W}}\left\{h\sin(\beta-\alpha) + H\cos(\beta-\alpha)\right\} Z_\mu Z^\mu \end{aligned} \tag{13}$$

Again, similar results hold for the W boson.

Exercise: Compute the mass terms and interaction terms for the W boson, equivalent to Equations 12 and 13.

The couplings to bosons thus depend on the relative alignment of the angles α and β. An interesting possibility is the case where $\sin(\beta - \alpha) = 1$. In this case, called the *decoupling limit*, h behaves as the SM Higgs boson, and H does not couple to the W or Z at all. Their roles are reversed if $\cos(\beta - \alpha) = 1$, although this is regarded as a less plausible scenario as it requires the lighter state h to remain unobserved at LEP and the LHC.

1.3 Couplings to fermions

Unlike the bosons, whose masses and couplings are determined by the electroweak symmetry breaking itself, the fermions are assigned couplings to the SM Higgs field in an ad-hoc manner. Nevertheless, with a single Higgs doublet, the assignment is unique. In 2HDM models, either or both doublet can be assumed to couple to each fermion. However, if the couplings are completely arbitrary, large flavour-changing neutral currents would be expected, as the mass eigenstates could not be diagonal in the interaction bases of both Higgs doublets simultaneously, in general. These effects have not been observed, and so most models assume that each fermion couples to just one of the doublets.

By convention, it is assumed that the up-type quarks (u, c, t) couple only to ϕ_2. In fact, this defines what we mean by ϕ_2, resolving the ambiguity between the two doublets mentioned in footnote 3. For the down-type quarks and the leptons, a number of different choices can be made, which are classified as follows:

Type I: All fermion fields couple to ϕ_2 only.

Type II: Down-type fermions couple to ϕ_1.

Type III or X: d-quarks couple to ϕ_2, charged leptons to ϕ_1.

Type IV or Y: d-quarks couple to ϕ_1, charged leptons to ϕ_2.

These assignments affect the fermion-Higgs couplings, and hence the phenomenology observed in Higgs boson production and decay, as given in Table 1. To see how these are computed, consider the mass term for a fermion f in a Type I model (or up-type quarks in any model):

$$\mathcal{L}_{m_f} = -\frac{y_f v_2}{\sqrt{2}} \bar{f} f = -m_f \bar{f} f. \tag{14}$$

In terms of the SM Higgs sector parameters, $m_f = y_f^{\mathrm{SM}} v/\sqrt{2}$. Therefore, $y_f = y_f^{\mathrm{SM}} v/v_2 = y_f^{\mathrm{SM}}/\sin\beta$. Now, the coupling of the fermion to the neutral, scalar Higgs sector is given by

$$\mathcal{L}_{m_f} = -\frac{y_f}{\sqrt{2}} H_2^0 \bar{f} f. \tag{15}$$

Using Equation (10) to express H_2^0 in terms of mass eigenstates, it is evident then that the couping to the lighter field h is proportional to $\cos\alpha/\sin\beta$, and the coupling to the heavier field H is proportional to $\sin\alpha/\sin\beta$. The fermion couplings in the other models (and to A) arise in a similar way, via modifications of the Yukawa couplings from the SM values and the mixing between the different Higgs fields. Note that in the case of a Type I model, it is possible for the h (or H) to be fermiphobic, i.e. to have zero or negligible couplings to fermions. This is strongly disfavoured by the LHC observations, as the gluon fusion production channel would be absent.

Exercise: For what value(s) of α and β is h indistinguishable from the SM Higgs boson? The answer may be different to what you expect. Compute also the couplings of H and A to the SM particles in this case.

Table 1: Couplings to the neutral Higgs boson fields in 2HDM models that forbid FCNCs at tree level, relative to the SM. The couplings for gauge bosons and up-type quarks are independent of the model variety, but are included for completeness.

<table>
<tr><th></th><th></th><th>u</th><th>d</th><th>ℓ</th><th>W/Z</th></tr>
<tr><td rowspan="3">Type I</td><td>h</td><td colspan="3">$\cos\alpha/\sin\beta$</td><td>$\sin(\beta-\alpha)$</td></tr>
<tr><td>H</td><td colspan="3">$\sin\alpha/\sin\beta$</td><td>$\cos(\beta-\alpha)$</td></tr>
<tr><td>A</td><td>$\cot\beta$</td><td colspan="2">$-\cot\beta$</td><td>0</td></tr>
<tr><td rowspan="3">Type II</td><td>h</td><td>$\cos\alpha/\sin\beta$</td><td colspan="2">$-\sin\alpha/\cos\beta$</td><td>$\sin(\beta-\alpha)$</td></tr>
<tr><td>H</td><td>$\sin\alpha/\sin\beta$</td><td colspan="2">$\cos\alpha/\cos\beta$</td><td>$\cos(\beta-\alpha)$</td></tr>
<tr><td>A</td><td>$\cot\beta$</td><td colspan="2">$\tan\beta$</td><td>0</td></tr>
<tr><td rowspan="3">Type III(X)</td><td>h</td><td colspan="2">$\cos\alpha/\sin\beta$</td><td>$-\sin\alpha/\cos\beta$</td><td>$\sin(\beta-\alpha)$</td></tr>
<tr><td>H</td><td colspan="2">$\sin\alpha/\sin\beta$</td><td>$\cos\alpha/\cos\beta$</td><td>$\cos(\beta-\alpha)$</td></tr>
<tr><td>A</td><td>$\cot\beta$</td><td>$-\cot\beta$</td><td>$\tan\beta$</td><td>0</td></tr>
<tr><td rowspan="3">Type IV(Y)</td><td>h</td><td>$\cos\alpha/\sin\beta$</td><td>$-\sin\alpha/\cos\beta$</td><td>$\cos\alpha/\sin\beta$</td><td>$\sin(\beta-\alpha)$</td></tr>
<tr><td>H</td><td>$\sin\alpha/\sin\beta$</td><td>$\cos\alpha/\cos\beta$</td><td>$\sin\alpha/\sin\beta$</td><td>$\cos(\beta-\alpha)$</td></tr>
<tr><td>A</td><td>$\cot\beta$</td><td>$\tan\beta$</td><td>$-\cot\beta$</td><td>0</td></tr>
</table>

2 LHC constraints on 2HDMs

When considering 2HDM models at the LHC, it is usually assumed that the recently discovered scalar boson is h. If this is correct, then these models can be probed in two main ways:

- Precision measurements of the couplings of h.

- Searches for the direct or indirect production of the other Higgs bosons (H, A and $H^{\pm}$).

We will now briefly consider each in turn.

2.1 Precision measurements of the $m = 125$ GeV scalar boson

From Table 1, we see that in 2HDM models it is possible for the couplings of h to differ substantially from those of the SM Higgs boson. This means that the measurements of the boson's couplings, discussed in the previous tutorial, can set very stringent constraints on the parameters α and β. Figure 1 show a selection of constraints from the ATLAS experiment on the 2HDM parameter space, for each of the four model types. In each case, the x axis is chosen to be $\cos(\beta - \alpha)$, which is directly constrained by measurements of the Higgs boson's coupling to the W and Z bosons. The y-axis, chosen to be $\tan\beta$, varies the fermionic couplings for fixed values of $\cos(\beta - \alpha)$, in a way that is different for the four model types. Current measurements are all in agreement with the Standard Model predictions, and in particular values of $|\cos(\beta - \alpha)|$ greater than about 0.5 are almost entirely ruled out.

Exercise: In most panels of Figure 1, $\cos(\beta - \alpha)$ is less well constrained when $\tan\beta \sim 1$. Why do you think this might be? (Refer to Table 1 for help).

Exercise: Following on from the previous question, why are Type I models less constrained than the other types for $\tan\beta >> 1$?

2.2 Searches for additional Higgs bosons

Like the SM, 2HDM models do not explicitly predict the masses of the associated Higgs bosons. However, it seems natural that the additional bosons H, A and $H^{\pm}$ should have masses not too far from those of the lightest state h. Many searches for these additional bosons have been performed at the LHC and other colliders, including in the low mass range $m < 125$ GeV. Here, we will focus on the high-mass range, where the additional bosons are expected to be nearly degenerate in mass.

Exercise: Consider the neutral A and H bosons, assuming $m_{A/H} \gtrsim 200 GeV$. Assuming that $\cos(\beta - \alpha) \sim 0$ (see Figure 1), what are the likely decay modes? Consider the couplings listed in Table 1 and also couplings to other Higgs bosons.

Exercise: In a similar way, what decay modes might be available to the charged Higgs boson $H^{\pm}$?

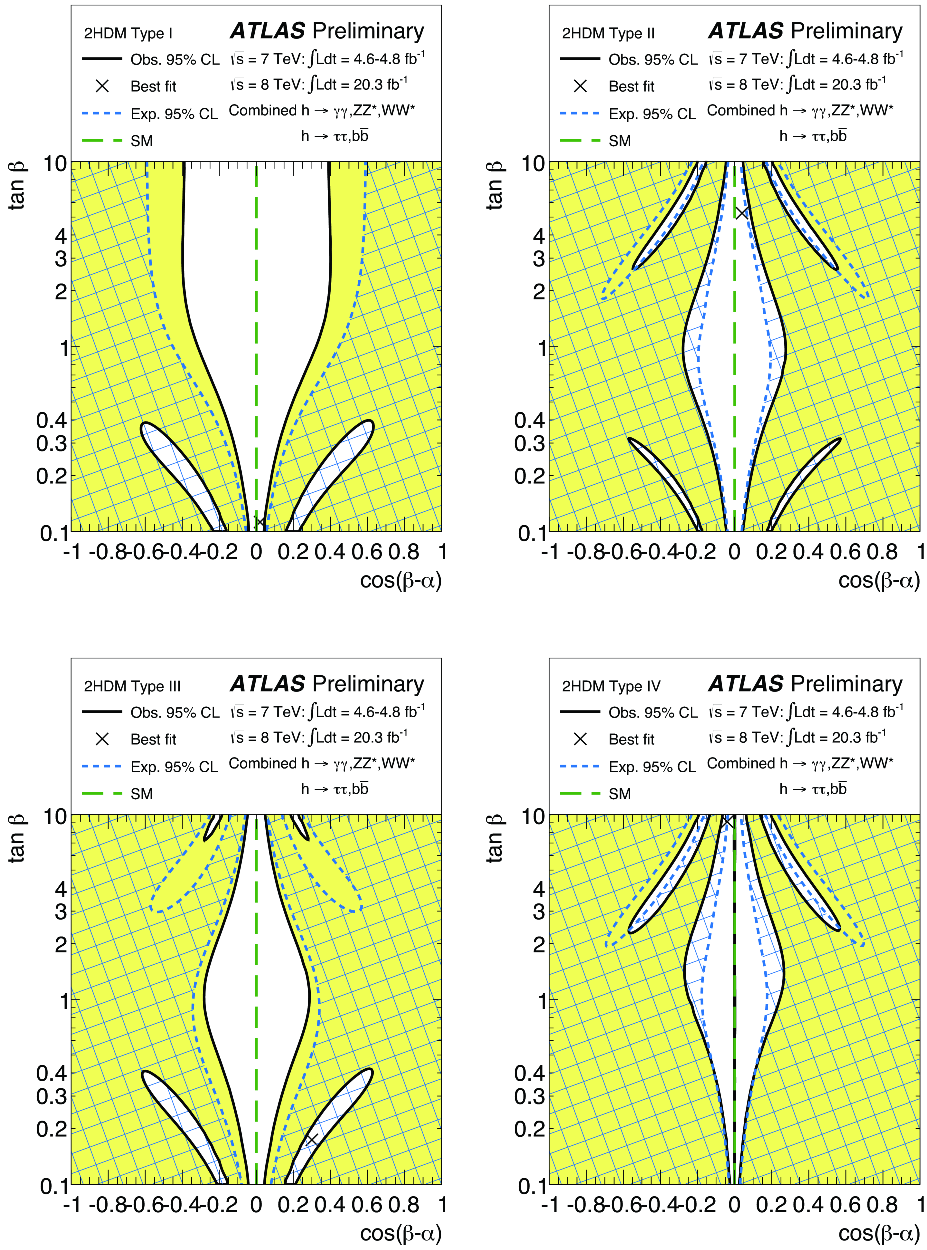

Figure 1: Constraints on 2HDMs from measurements of the Higgs boson coupling strengths by ATLAS. A value of $\cos(\beta - \alpha) = 0$ corresponds to the decoupling limit, where the coupling of h to bosons is the same as in the SM.

The pseudoscalar A is CP-odd, and therefore the symmetric decays like $A \to ZZ$ and $A \to hh$ are suppressed (Why?). Therefore the decay channel $A \to Zh$ is a promising one for discovery, as long as the A is not so massive as to allow $A \to t\bar{t}$.[4] In the Zh channel, events are easily triggered using the decays $Z \to e^+e^-$ and $Z \to \mu^+\mu^-$, allowing sensitivity to the experimentally challenging $h \to b\bar{b}$ and $h \to \tau^+\tau^-$ decays. These decay channels were used to obtain the constraints shown in Figure 2, assuming a mass $m_A = 300$ GeV. For low values of $\tan\beta$, the constraints are much improved over those of Figure 1. For $\tan\beta \gtrsim 5$, the results become highly dependent on the type of 2HDM model considered, with stronger constraints for Type II and Type IV models.

Exercise: Using this fact, what can you deduce about the relative strengths of the $h \to \tau^+\tau^-$ and $h \to b\bar{b}$ search channels?

The final search examples focus on a particular form of Type II 2HDM model based on the ideas of supersymmetry, which we will return to later in the course. In this case, the neutral boson H and A are both expected to have substantial branching fractions to the $\tau^+\tau^-$ final state. If their masses are much larger than m_h, then the difference between them cannot be experimentally resolved, and the search is performed assuming that both are produced. Results for this channel are shown in Figure 3, where the mass scale of the additional Higgs bosons is parametrised by m_A.[5] This model also predicts m_h, contours of this parameter can be seen in the diagram for comparison with the measured value. The constraints are complementary to those from LEP, such that $m_A < 130$ GeV is nearly completely ruled out (in this particular model).

Exercise: Why is the constraint in this channel best for high values of $\tan\beta$? (Again, use Table 1 to help.)

It is also possible to search for the charged Higgs state $H^\pm$ at the LHC. Like the neutral Higgs bosons, it couples to mass, but its production and decay modes differ substantially from the other Higgs bosons due to its non-zero electric charge.

Exercise: Try to come up with at least one way that a $H^\pm$ could be produced at the LHC, via the $H^\pm - t - b$ vertex. *Hint:* Try working backwards from the $H^\pm$ boson. In all of the most likely cases, at least one other particle (quark) is produced along with the charged Higgs boson.

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[4] Do not be confused by the "zero" coupling between A and the gauge bosons in Table 1. This is referring to the $A - Z - Z$ and $A - W - W$ vertices, while here we consider the $A - Z - h$ vertex. If you are interested, the coupling can be deduced from Equation (11).

[5] There is no need to consider $\cos(\beta - \alpha)$ in this case, as it is constrained by other assumptions of the theory.

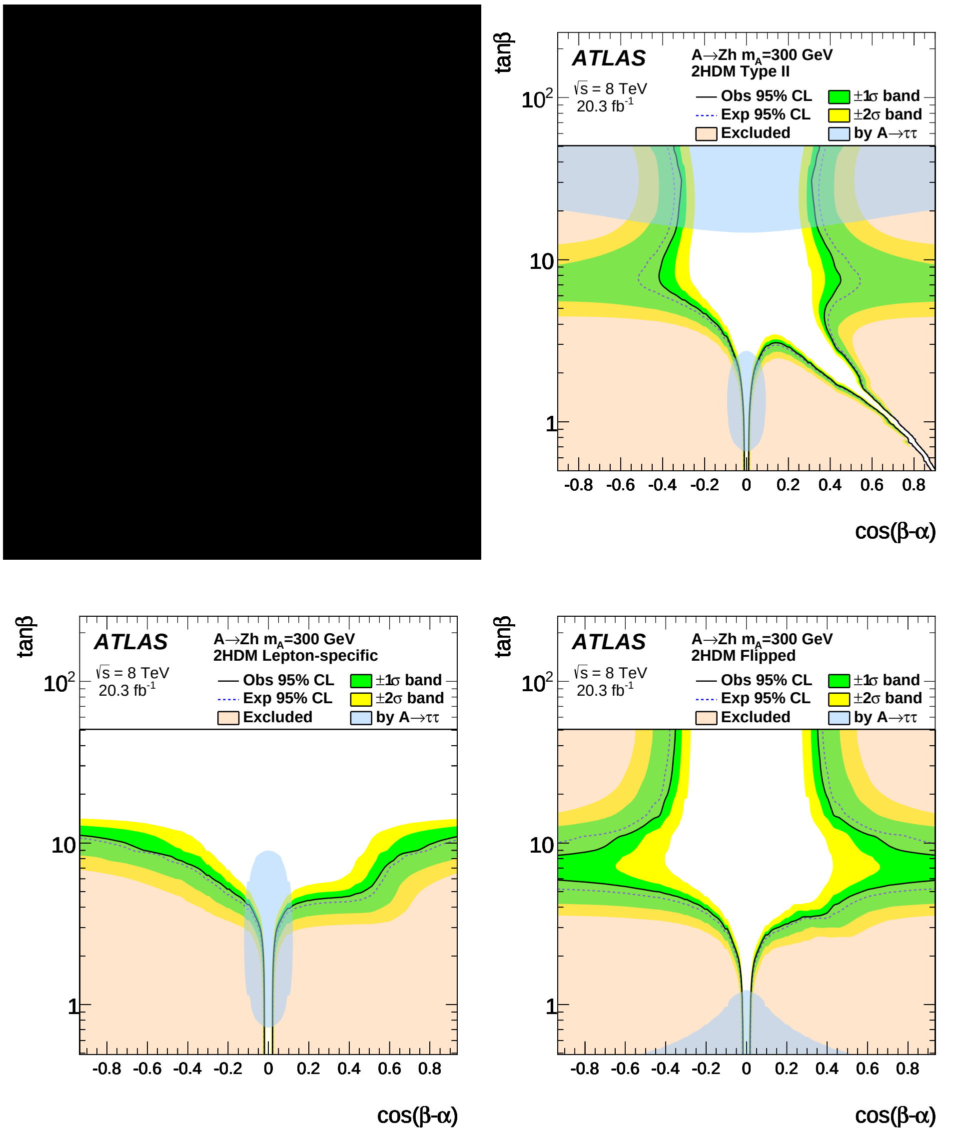

Figure 2: Constraints on 2HDMs from a search for $A \to Zh$, assuming $m_A = 300$ GeV.

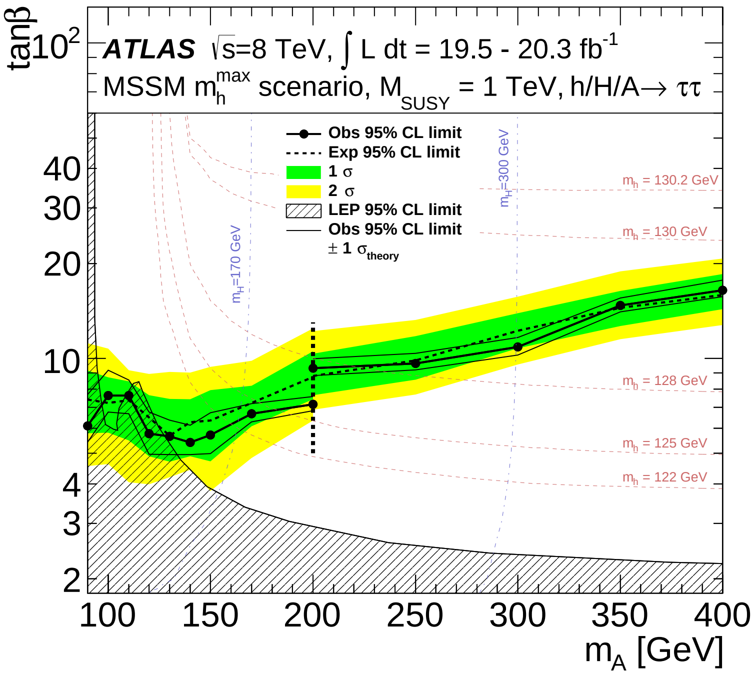

Figure 3: 95% CL limits on additional Higgs bosons in the MSSM, from the $H/A \to \tau^+\tau^-$ channel.

In the MSSM, decays to taus are preferred, as for the A and H, however in this case the decay is $H^\pm \to \tau^\pm \nu_\tau$. Two different signatures are considered, depending on whether the charged Higgs boson mass is less than or greater than m_t:

- $pp \to t\bar{t} \to (H^+ b)(\bar{b}W^-)$ for $m_{H^\pm} < m_t$,
- $pp \to \bar{t}(b)H^+$ for $m_{H^\pm} > m_t$.

In both cases, the charge-conjugate processes are also allowed. The event signature therefore consists of one hadronically-decaying tau lepton, a number of jets (including b-jets) and non-zero $E_{\mathrm{T}}^{\mathrm{miss}}$. This is challenging to separate from sources of SM background (in particular $t\bar{t}$ production), however extremely strong results can be obtained in the low-mass case. This is shown in Figure 4, which displays the results for the same model as Figure 3 in the $m_{H_\pm} - \tan\beta$ plane. Charged Higgs boson masses below about 155 GeV are completely ruled out in this model.

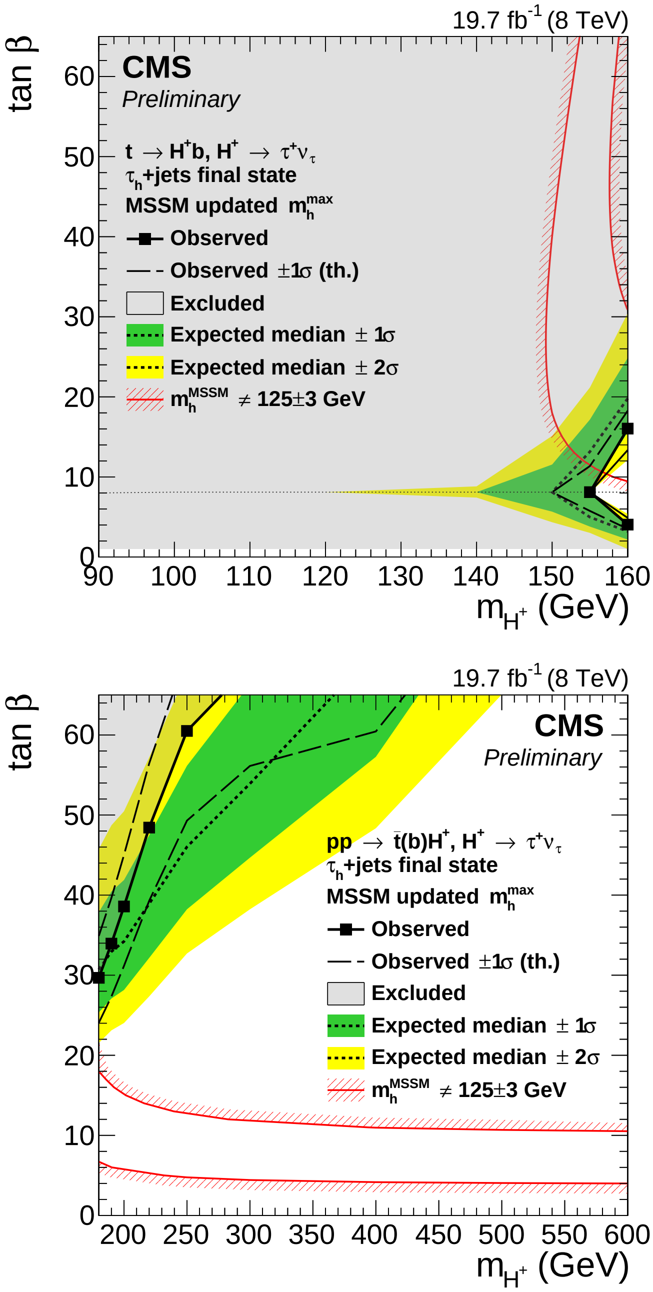

Figure 4: 95% CL limits on charged Higgs bosons in the MSSM, from the low- and high-mass $H^{\pm} \to \tau\nu$ channels.