Tutorial 14: Neutrino masses and mixings

Dr. M Flowerdew

June 8, 2015

Neutrinos hold a unique place in the Standard Model. They are the only fermions which do not interact via either the strong or the electromagnetic force, a fact which makes them notoriously difficult to detect and measure. In addition, their masses are known to be exceedingly small, far less than for the other SM particles. Furthermore, it is not yet known whether the right-handed neutrino ν_R even exists, as it would be neutral under all of the SM gauge groups and could be very massive. We will explore the current knowledge and theory of neutrinos in this tutorial, with a focus on their masses and mixings.

1 Describing neutrino masses

In the Standard Model, neutrinos are massless. Certainly, within the measurement precision of the time this was a valid approximation.[1] With the discovery of neutrino oscillations (see Section 3), we now know that at least two of the three neutrino species have non-zero masses.

There are, in fact, many possible mechanisms to allow the neutrinos to have non-zero masses, all of which extend the field content of the SM. One of the conceptually simplest approaches is to introduce right-handed neutrino fields ν_R, with appropriate couplings to the Higgs field that allow mass generation in the same way as for other fermions. This possibility was already noted in tutorial 3. After electroweak symmetry breaking, the neutrino mass terms for one generation would be

$$\mathcal{L}_{m_\nu} = -m_\nu \left(\overline{\nu_\mathrm{L}} \nu_\mathrm{R} + \overline{\nu_\mathrm{R}} \nu_\mathrm{L} \right) . \tag{1}$$

These terms have exactly the same structure as the mass terms for the charged leptons and the quarks. Similarly to the quarks, a more complex structure can emerge when this procedure is extended to three generations, where the interaction basis (defined by the charged lepton mass states) does not necessarily correspond to the mass basis (defined by the extension of Equation (1)). The leptonic equivalent of the CKM matrix is the PMNS

[1] If we consider only kinematic measurements, e.g. of β decays, this is still true.

matrix, named after Pontecorvo, Maki, Nakagawa and Sakata. It is a 3×3 unitary matrix U_{PMNS}, defined as follows:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U_{\text{PMNS}} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}, \tag{2}$$

where ν_1, ν_2 and ν_3 are the three mass eigenstates.

As with the CKM matrix, the PMNS matrix can be parameterised by three angles (θ_{12}, θ_{23} and θ_{13}) and a complex phase, δ. Measurements of oscillations (see Section 3) indicate that the mixing between the second and third mass states is nearly maximal, i.e. $\theta_{23} \sim 45°$, and that mixing between the first and third states is small ($\theta_{13} \approx 8.5°$). This allows us to write the PMNS matrix in the following approximate form:

$$U_{\text{PMNS}} \approx \begin{pmatrix} c_{12} & s_{12} & s_{13}e^{-i\delta} \\ -\frac{1}{\sqrt{2}}s_{12} & \frac{1}{\sqrt{2}}c_{12} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}}s_{12} & -\frac{1}{\sqrt{2}}c_{12} & \frac{1}{\sqrt{2}} \end{pmatrix}, \tag{3}$$

where $s_{12} = \sin\theta 12$ and $c_{12} = \cos\theta_{12}$. Note the lack of hierarchical structure – unlike the CKM matrix, only one element, U_{e3}, is small.

With this simple theory, one can accommodate our current knowledge of neutrino masses. However, one may perhaps wonder why the neutrino mass parameters are so small with respect to the other fermion masses. One attraction of the above theory is that it is compatible with the *seesaw mechanism*, which could explain this apparent anomaly.[2]

1.1 The seesaw mechanism and Majorana masses

Equation (1) is not the most general Lagrangian that can be written for neutrino masses involving ν_{R}. A more complete expression includes an additional pair of terms (again returning to the case of just one generation):[3]

$$\mathcal{L}_{m_\nu} = -M_{\text{D}}\left(\overline{\nu_{\text{L}}}\nu_{\text{R}} + \overline{\nu_{\text{R}}}\nu_{\text{L}}\right) - \frac{1}{2}M_{\text{R}}\left(\overline{\nu_{\text{R}}^c}\nu_{\text{R}} + \overline{\nu_{\text{R}}}\nu_{\text{R}}^c\right). \tag{4}$$

The last terms are possible because ν_{R} is a gauge singlet. In other words, it only interacts with the Higgs field, and it does not couple to the photon, W, Z or gluon. Therefore, explicit mass terms do not violate any gauge symmetries of the SM, and can be included even without the BEH mechanism, unless it is forbidden by some (as yet unknown) high-scale symmetry. In addition, it would be natural for M_{R} to have a high scale, $\gg 1$ TeV.

[2]More precisely, it allows a Type I seesaw mechanism.

[3]ν_{R}^c refers to the antiparticle of the ν_{R}, which has opposite chirality, lepton number etc. It is necessary to use this in order to ensure that all terms in Equation (4) are Lorentz scalars, by coupling a right-handed field to the adjoint of a left-handed field.

The ν_R field in Equation (4) is a *Majorana* field, as the M_R term mediates transitions between ν_R and its antiparticle, ν_R^c. This means that the right-handed neutrino and anti-neutrino share the same relationship as (say) the right- and left-handed electron, i.e. in a sense they are “the same” particle. This motivates the construction of explicitly Majorana neutrino states:

$$L = \frac{1}{\sqrt{2}}\left(\nu_\mathrm{L} + \nu_\mathrm{L}^c\right); \qquad\qquad R = \frac{1}{\sqrt{2}}\left(\nu_\mathrm{R} + \nu_\mathrm{R}^c\right). \tag{5}$$

Using these states, Equation (4) can be re written in a matrix form:

$$\mathcal{L}_{m_\nu} = -\begin{pmatrix}\overline{L} & \overline{R}\end{pmatrix}\begin{pmatrix}0 & M_\mathrm{D}\\ M_\mathrm{D} & M_\mathrm{R}\end{pmatrix}\begin{pmatrix}L\\ R\end{pmatrix}, \tag{6}$$

Exercise: Derive Equation (6). You will need to take special care of the order of operations (i.e. adjoint vs. charge conjugation), and recall that $\nu_\mathrm{L} = P_\mathrm{L}\nu$ and $\nu_\mathrm{R} = P_\mathrm{R}\nu$.

Exercise: Diagonalise the mass matrix in Equation (6), and find the corresponding mass eigenstates in terms of ν_L and ν_R to leading order in $M_\mathrm{D}/M_\mathrm{R}$.

The mass matrix has eigenvalues of magnitude M_R and $M_\mathrm{D}^2/M_\mathrm{R}$, assuming $M_\mathrm{R} \gg M_\mathrm{D}$. The first eigenvalue corresponds to a mostly ν_R state, which remains massive, while the mass of the ν_L state varies inversely as M_R. This is an attractive feature, as it could explain the smallness of the (left-handed) neutrino masses in a natural way. For example, if we assume that M_D is of the same order as a top mass ($\sim$ 200 GeV), and that M_R is of order the GUT scale ($\sim 10^{15}$ GeV), the left-handed neutrino mass comes out as $\sim$ 40 meV, which, as we will see, is of the correct order of magnitude to describe the oscillation data.

In this version of the seesaw mechanism, neutrinos are necessarily Majorana particles, due to the influence of the right-handed component. One consequence of this is that the PMNS matrix obtains two additional complex phases. Another is that $\nu \leftrightarrow \bar{\nu}$ transitions would be allowed. There are hopes that this could be observed in rare variations of double beta decays like ${}^{76}\mathrm{Ge} \to {}^{76}\mathrm{Se} + 2e^- + 2\bar{\nu}_e$. If one of the antineutrinos undergoes a helicity flip (the amplitude of this is proportional to its mass), then it could be absorbed by the second beta decay, resulting in ${}^{76}\mathrm{Ge} \to {}^{76}\mathrm{Se} + 2e^-$, a *neutrinoless double beta decay.* The rate of this process depends upon a combination of neutrino masses and PMNS matrix elements:

$$\Gamma \propto \langle m_{\beta\beta}\rangle^2 = \left|\sum_i U_{ei}^2 m_i\right|^2. \tag{7}$$

Thus, searches for double beta decay are sensitive to the neutrino mass scale, but cancellations between the PMNS elements could reduce $\langle m_{\beta\beta}\rangle$ to

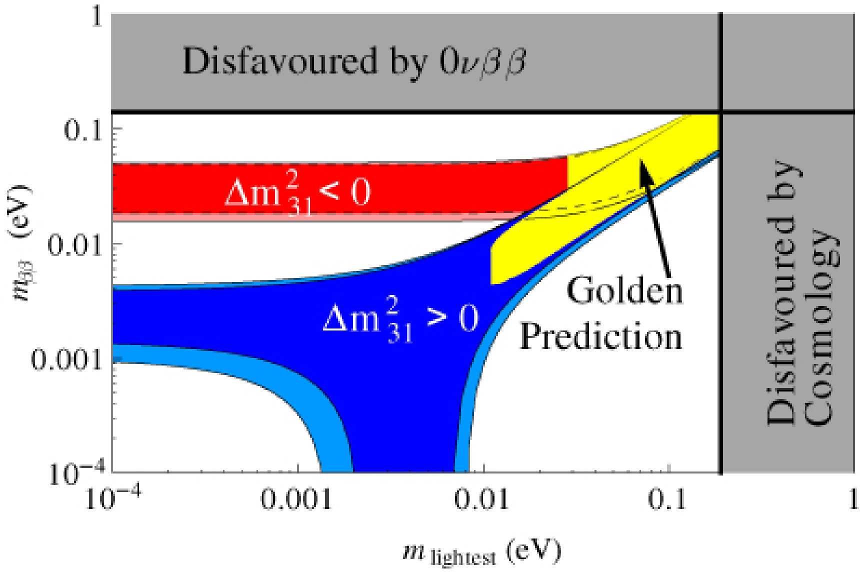

Figure 1: Allowed neutrino masses, expressed in terms of the smallest neutrino mass and $m_{\beta\beta}$.

arbitrarily small values. With the currently measured values of the PMNS matrix, this kind of cancellation is only possible if the lightest neutrino mass lies between approximately 2 meV and 10 meV, as shown in Figure 1. In all other cases $\langle m_{\beta\beta}\rangle$ is bounded from below ($\gtrsim$ 1 meV), meaning that neutrinoless double beta decay should be observable. Current experimental limits disfavour values of $\langle m_{\beta\beta}\rangle$ above about 200 meV.

2 Neutrino oscillations

As with quarks, the non-trivial nature of the PMNS matrix allows for flavour-changing interactions involving neutrinos. Unlike quarks, the different neutrino mass eigenstates are not immediately distinguishable, and so we cannot simply measure, say, the relative rates of ν_1 and ν_2 produced from a source of ν_e. Instead, as we can only produce and measure flavour eigenstates, we must measure transition rates between the flavours, which occur via flavour oscillations. There are some similarities here with the neutral meson oscillations discussed in the previous tutorial, the main difference being that we now work in an ultra-relativisitc regime, and this changes the dependence of the oscillation frequency with mass.

We consider a simple model in one dimension, with just two neutrino mass eigenstates $|1\rangle$ and $|2\rangle$. These states have masses m_1 and m_2, respectively. The two flavour eigenstates, $|a\rangle$ and $|b\rangle$, are related to the mass

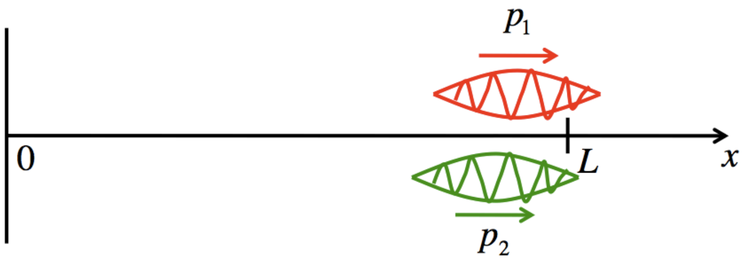

Figure 2: Sketch of two wave-packets travelling along x with momenta p_1 and p_2.

eigenstates by a rotation angle θ:

$$\begin{aligned} |a\rangle &= \quad \cos\theta|1\rangle + \sin\theta|2\rangle, \\ |b\rangle &= -\sin\theta|1\rangle + \cos\theta|2\rangle. \end{aligned} \tag{8}$$

We suppose that a neutrino $|a\rangle$ is created at $t = x = 0$, with energy E, travelling along the positive x axis. The two mass eigenstates propagate independently in space, with dfferent momenta p_1 and p_2, due to their different masses, as sketched in Figure 2.

To proceed further, we should use a full wave-packet description of the propagation, however it turns out that a much simpler plane-wave approach suffices to obtain the main results. Consider the state at a later time t, after which the neutrino has travelled a distance $x = L \approx ct$:

$$|\psi(t)\rangle = \cos\theta e^{-i(Et-p_1L)}|1\rangle + \sin\theta e^{-i(Et-p_2L)}|2\rangle. \tag{9}$$

The probability of detecting a neutrino in state $|b\rangle$ (i.e. that an oscillation has occurred) is proportional to the square of the corresponding amplitude:

$$\begin{aligned} |\langle b|\psi(t)\rangle|^2 &= |-\sin\theta\cos\theta e^{-i(Et-p_1L)} + \cos\theta\sin\theta e^{-i(Et-p_2L)}|^2 \\ &= 2\sin^2\theta\cos^2\theta\,[1-\cos(p_1-p_2)L]\,. \end{aligned} \tag{10}$$

Thus, the oscillation probability depends upon the difference between the momenta of the two mass eigenstates. If we assume that both masses are small compared to E, we can expand the momenta in powers of m_i^2/E^2:

$$\begin{aligned} p_1 - p_2 &= \sqrt{E^2 - m_1^2} - \sqrt{E^2 - m_2^2} \\ &\approx E\left(1 - \frac{m_1^2}{2E^2} - 1 + \frac{m_2^2}{2E^2}\right) \\ &= \frac{\Delta m^2}{2E}. \end{aligned} \tag{11}$$

With a little rearrangement of the trigonometric terms, we find that:

$$|\langle b|\psi(t)\rangle|^2 = \sin^2 2\theta \sin^2\left(\frac{\Delta m^2 L}{4E}\right). \tag{12}$$

From Equation (12), we observe that:

- Oscillations occur with a period of $4\pi E/\Delta m^2$, so states with higher Δm^2 oscillate more rapidly. Equivalently, for a given neutrino energy, detectors placed at different distances probe different values of Δm^2.
- Measurements must be made at multiple values of E and/or L to accurately measure both θ and Δm^2.
- The oscillations do not depend on the sign of Δm.
- The amplitude of the oscillations varies as $\sin^2 2\theta$, and vanish as θ tends to 0 or π.

In reality, the oscillation amplitude also diminishes with distance. This is not evident in Equation (12) because we did not consider the uncertainty on the neutrino's energy. In reality, if the typical spread in energy is δE, only about $E/\delta E$ oscillations can be observed before interference between different energy eigenstates washes the oscillations out, leaving just the long-term average transition probability of $\frac{1}{2}\sin^2 2\theta$.

3 Measurements of neutrino oscillations

The detection of neutrinos can be achieved in several ways. Some early measurements used the reactions of neutrinos with nuclei, for example ${}^{37}\mathrm{Cl} + \nu_e \rightarrow {}^{37}\mathrm{Ar} + e^-$ in the Homestake experiment in the 1960s. The electron neutrino flux was inferred from the rate of production of argon in the detector, however this method is only sensitive to electron neutrinos and provides no information about the direction or the energy of incoming neutrinos, only the number that are above the reaction threshold.

More modern detectors aim to at least partially reconstruct the neutrino interactions, recording the direction and energy of at least one outgoing particle. A good example of this is the SNO experiment,[4] which first confirmed that oscillations occurred for solar neutrinos. SNO was a heavy water (${}^{2}\mathrm{H}$) detector, where high-energy electrons could be detected by the Čerenkov radiation that they produce. It was able to attribute the missing neutrinos to other flavours through the use of a three-channel detection system:

[4] Sudbury Neutrino Observatory.

Charged current: $\nu_e + n \to e^- + p$, which produces an energetic electron. Solar neutrinos are not energetic enough to produce muons or taus, hence this measures only the ν_e rate, but with higher energy sources muon and tau production can also be observed.

Neutral current: $\nu + {}^2\text{H} \to \nu + p + n$. The neutron can be captured by another deuterium atom, releasing a gamma ray. This gamma ray scatters off one or more electrons in the heavy water, which in turn emit Čerenkov radiation. All neutrino flavours contribute equally to this interaction.

Electron scattering: $\nu + e^- \to \nu + e^-$. Here the scattered electron is detected directly. All neutrino flavours contribute to this interaction via the neutral current, but electron neutrinos can also exchange a W boson. Overall, about 75% of the rate comes from ν_e, the remainder from ν_μ and ν_τ.

Similar multi-channel detection techniques can be achieved through the use of scintillators and photographic emulsion. The technology choice ultimately depends upon the required detector size, energy threshold, and cost.

The kind of measurement made in a neutrino experiment depends on the capabilities of the detector and the nature of the neutrino source. Oscillation experiments can be separated into *appearance* experiments, where the detected neutrino type (e.g. ν_e) is different from the flavour produced by the source (e.g. ν_μ), and *disappearance* experiments, where the produced and detected flavours are the same.

3.1 Measurements of atmospheric neutrinos

Cosmic rays striking the upper atmosphere produce muon and electron neutrinos in well-predicted ratios, through the production and subsequent decays of charged pions. The energies of these neutrinos can be very high ($\sim$ TeV), allowing them to be detected and identified by large underground detectors. Detectors capable of measuring the direction of the neutrino (such as Super Kamiokande) can differentiate between upward- and downward-going neutrinos, allowing the baseline L to be varied from several kilometres to the diameter of the Earth. These observations demonstrate that muon neutrinos disappear at a rate consistent with oscillations corresponding to $\Delta m^2_{\text{atm.}} \sim 2.4 \times 10^{-3}$ eV^2 and $\tan^2\theta_{\text{atm.}} \sim 1.3$. These results are quoted in terms of effective parameters, which are experimentally accessible but may be combinations of the underlying PMNS matrix parameters. As the loss of muon neutrinos does not correspond to a gain in electron neutrinos, it is assumed that they oscillate to tau neutrinos, therefore the atmospheric results are associated (by convention) with mixing between mass eigenstates 2 and 3.

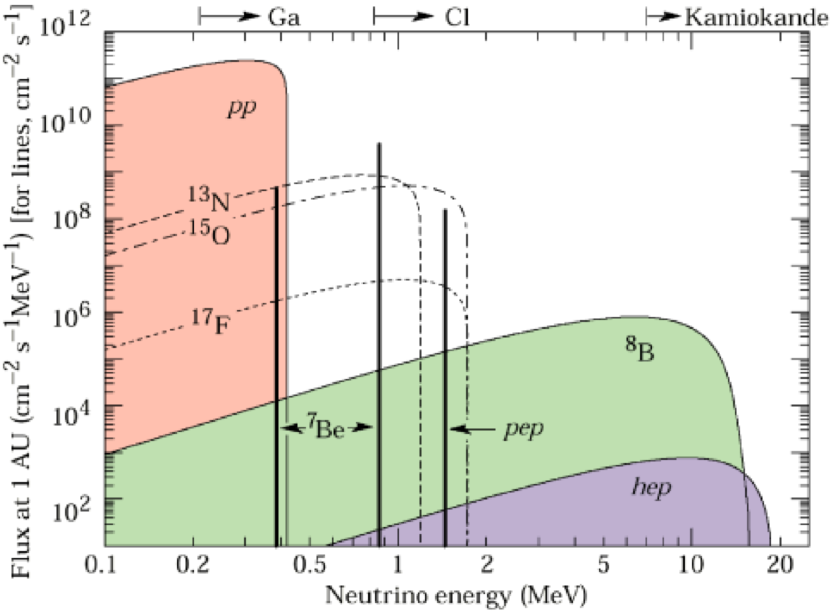

Figure 3: The solar neutrino spectrum at a distance of 1 AU. The thresholds for different detection technologies are also shown.

Muon neutrinos can also be produced with reasonably high purity by particle beams. In these, low energy pions are allowed to decay into muons and (anti-)neutrinos, with relatively small contamination from muon decays. The characteristics of the beam can be precisely controlled, giving a high-intensity source of $\sim$ GeV neutrinos. The disappearance of muon neutrinos in particle beams have been confirmed to be consistent with atmospheric neutrino oscillations.

3.2 Measurements of solar and reactor neutrinos

Electron neutrinos are produced copiously in the Sun via the reaction $p \to n + e^+ + \nu_e$. Mostly this occurs in the conversion of hydrogen to helium, producing neutrinos with a typical energy of ~ 0.3 MeV, but energies of up to ~ 10 MeV can be produced in the decays of ^{8}B (see Figure 3). Measurements of the solar neutrino flux by SNO confirm that the electron neutrinos disappear via oscillations to other flavours, with effective parameters of $\Delta m^2_{\text{solar}} \sim 8 \times 10^{-5}$ eV^2, and $\tan^2\theta_{\text{solar}} \sim 0.5$.

Exercise: Why can SNO not tell which neutrino flavour(s) the ν_e oscillate to? *Hint:* Consider the detection mechanisms outlined at the start of the section, and the neutrino energies involved.

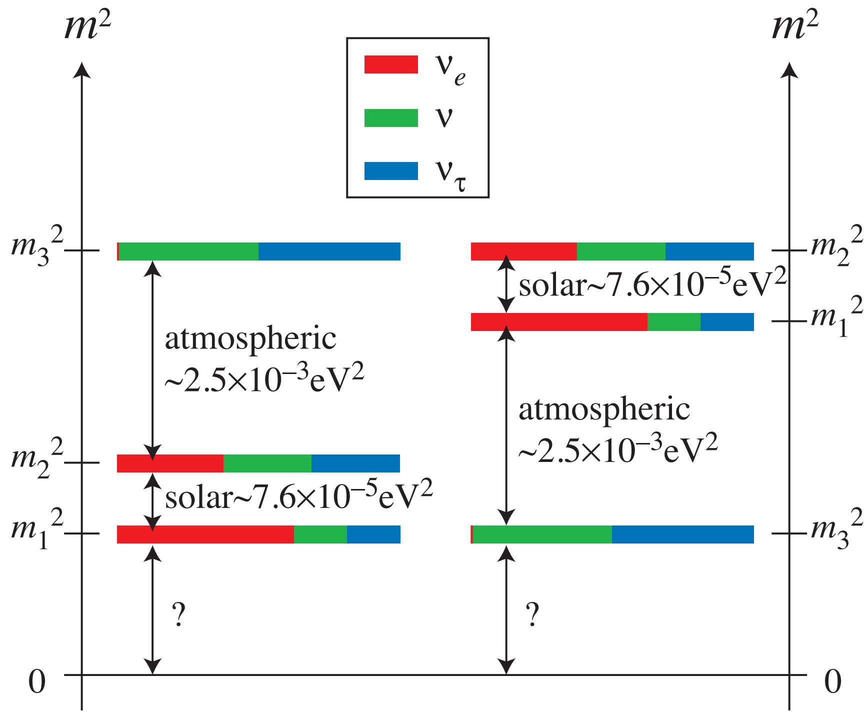

Figure 4: Illustration of the two potential hierarchies of neutrino masses, together with their flavour compositions. The normal hierarchy is shown on the left, and the inverted hierarchy on the right.

There is a large difference (about a factor of 30) between the values of Δm^2 governing atmospheric and solar neutrino oscillations. This tells us that solar oscillations must occur between two nearly degenerate mass eigenstates (conventionally labeled 1 and 2), separated from the third state, which is involved in atmospheric oscillations. It is not known, however, whether m_3 is greater or less than $m_{1,2}$, due to the ambiguity in the sign of $\Delta m^2_{\text{atm.}}$.[5] This is illustrated in Figure 4, which shows both the *normal hierarchy* with $m_3 > m_{1,2}$ and the *inverted hierarchy* with $m_3 < m_{1,2}$.

These results can be cross-checked by using nuclear reactors as a source of electron anti-neutrinos (via $n \to p + e^- + \bar{\nu}_e$). The energies of these neutrinos are similar to those from the Sun (i.e. a few MeV), and the baselines are typically $\sim 10 - 10^5$ m. Reactor experiments initially confirmed oscillations corresponding to the solar oscillation parameters. More recently, oscillations have been observed at distances corresponding to $\Delta m^2_{\text{atm.}}$, the first evidence for a non-zero value of θ_{13}, now measured by the Daya Bay experiment to be $\sin^2 2\theta_{13} = 0.084 \pm 0.005$.

A summary of neutrino measurement results is shown in Figure 5.

[5]The sign of $\Delta m^2_{\text{solar}}$ is known, due to flavour-specific modifications to neutrino mixing caused by ν_e interactions in the Sun's core.

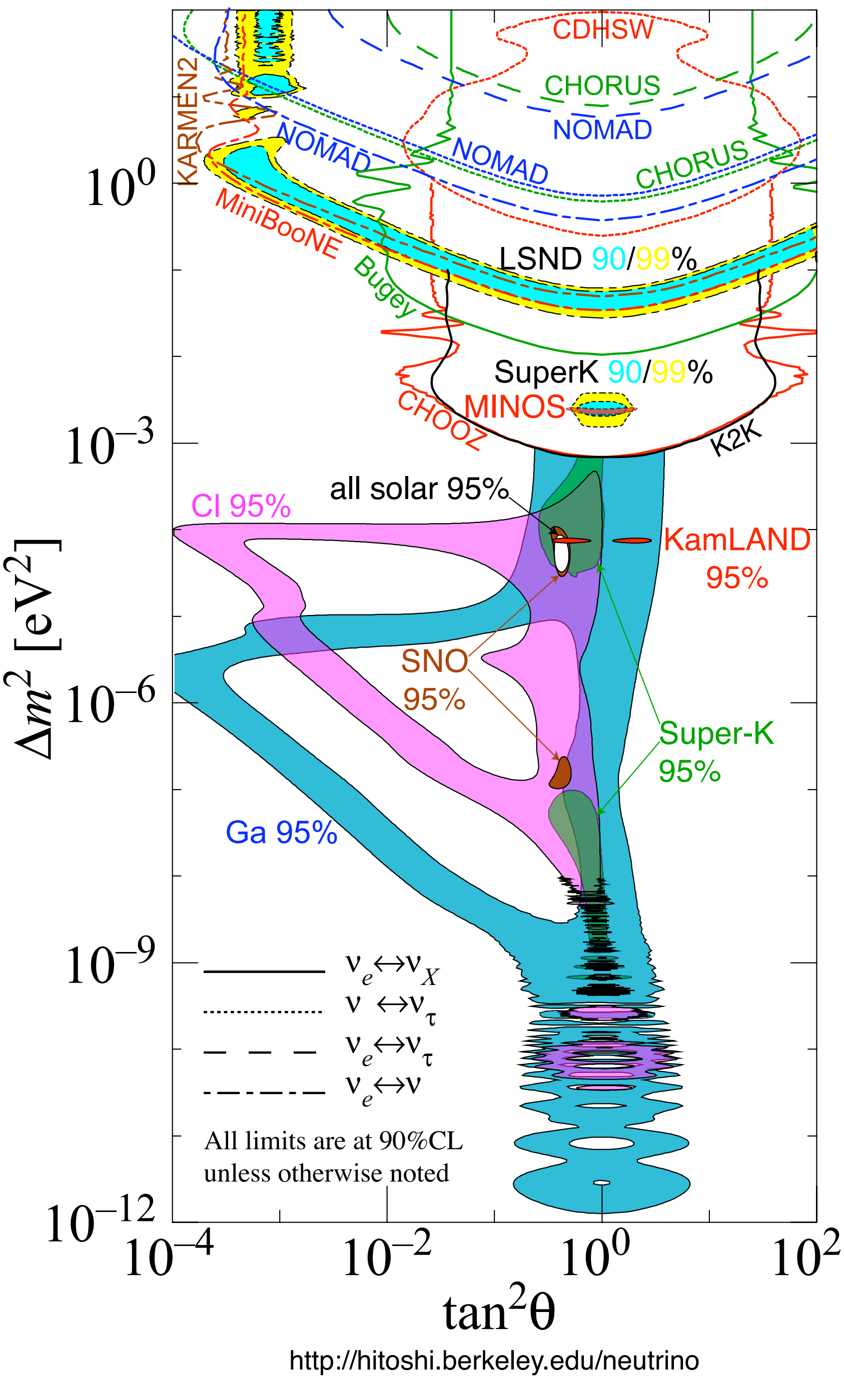

Figure 5: Summary of current and historical constraints on $\tan^2\theta$ and Δm^2 for different neutrino mixing experiments, with the exception of the recent θ_{13} results. Shaded areas denote measurements (disputed in the case of LSND), while unshaded areas denote exclusions. The two confirmed mixings are the atmospheric mixing (marked "SuperK" and "MINOS") and the solar mixing (best combination marked "all solar 95%").

4 Outstanding questions in neutrino physics

The results presented so far are consistent with the simplest form of the PMNS matrix given in Equation (2), however there are many outstanding questions relating to the neutrino sector. These include:

Internal consistency. The PMNS matrix has 4 parameters. As with the CKM matrix, these can in principle be overconstrained by experimental measurements, and to some extent they already are. As new processes are measured and higher precision is attained, the ability to be able to describe all experimental data with just four parameters will be an important test of the PMNS approach.

CP violation. One of the PMNS parameters is a CP-violating phase δ. This can be measured by comparing the interactions of neutrinos with anti-neutrinos. At the moment, there is some evidence that $\delta \sim \pi$, which would indicate that there is no CP violation, but the current sensitivity is poor.

Which mass hierarchy? Now that θ_{13} is known to be non-zero, it will be possible for future experiments to determine the mass hierarchy by measuring the detailed structure of this oscillation in the reactor $\bar{\nu}_e$ disappearance rate. As Δm^2_{13} and Δm^2_{23} are nearly equal, their oscillations will interfere, with a phase that depends on which of them is larger – if Δm^2_{13} is larger, the hierarchy is normal, else it is inverted (see Figure 4).

Majorana vs. Dirac mass. The detection of neutrinoless double beta decay would be an unambiguous sign that neutrinos are Majorana particles. In the case of an inverted hierarchy, $\langle m_{\beta\beta} \rangle$ is constrained to be above 0.01 eV, and could be reached by future experiments. In the case of a normal hierarchy, there is no lower limit to $\langle m_{\beta\beta} \rangle$, and so lack of observation does not necessarily mean that neutrinos are purely Dirac particles.

What is the absolute neutrino mass scale? If neutrinoless double beta decay is observed, the neutrino mass scale could be deduced from that. In addition, the KATRIN experiment is undertaking to measure the (effective) electron neutrino mass in nuclear beta decays. The experiment is expected to have sensitivity down to masses of approximately 0.2 eV, substantially better than the current limit of about 2.2 eV. Cosmological observations require the lightest neutrino to have $m \lesssim 0.1$ eV.

Are there sterile neutrinos? Recent results from reactor experiments suggest that, even when accounting for three neutrino flavours, fewer neutrinos are detected than are produced. If attributed to neutrino oscillations, this disappearence would correspond to a very large mass splitting of $\Delta m^2 \sim 0.1$ eV^2, incompatible with the three-flavour PMNS model based on solar and atmospheric oscillations. This has led some to suggest that the $\bar{\nu}_e$ are oscillating into a fourth neutrino flavour over very short ($\sim$ m) distances, where this neutrino does not interact with other SM fields (i.e. it is *sterile*). New results from Daya Bay indicate that, for a fourth neutrino with a squared mass difference in that range, $\sin^2 2\theta_{14} \lesssim 0.05$, corresponding to $\theta_{14} \lesssim 6.5°$.