Tutorial 15: Naturalness and supersymmetry

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1 Problems with the Standard Model

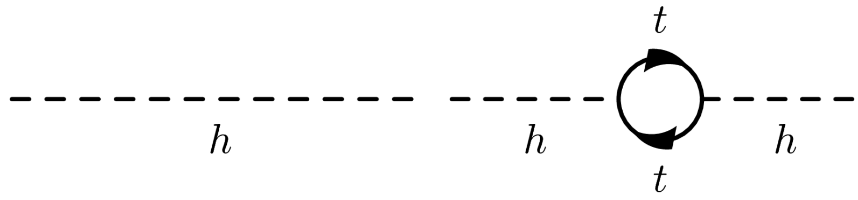
With the Higgs boson discovered, the Standard Model is apparently complete. Assuming that continuing measurements of its properties (and other SM processes) produce no significant deviations from the SM predictions, we have a consistent theory of particle interactions at colliders (and, by implication, nuclear and atomic physics too). Despite this success, the SM is not a final theory of nature, not least because of several observed phenomena that it cannot explain, including:

- Gravitation;
- Neutrino masses;
- Dark matter and dark energy;
- Matter/antimatter asymmetry in the universe;
- Cosmic inflation.

In addition, there are some further issues with the SM, arguably of a more philosophical nature. Most of these concern apparent coincidences in the SM parameters, such as the precisely equal electric charge of the proton and positron, or the fact that there are three generations of both leptons and quarks. These and other features of the SM point to deeper symmetries that may be present in a more complete theory, however there may be no deviation from SM collider phenomenology until energies reach the so-called *Grand Unification*, or GUT, scale.[1] The GUT scale is $\mathcal{O}(10^{16})$ GeV, well beyond our current reach of $\mathcal{O}(10^3)$ GeV. Today, we will instead focus on the *hierarchy problem*,[2] which can be explored at the LHC.

[1] This is the scale at which the three SM interactions may unify into a single gauge interaction described by a larger symmetry group, such as SU(5) or SO(10). The SM fermions in each generation would then be grouped into a single gauge multiplet of this group, with fixed relationships between their quantum numbers.

[2] Often called the "little" hierarchy problem, to distinguish it from other hierarchy problems, e.g. with the observed value of the cosmological constant Λ.



(a) Bare propagator (b) One-loop propagator

Figure 1: Tree-level and one-loop Higgs boson propagators.

2 Why is the SM unnatural?

A "natural" theory of particle physics can be loosely defined as one where the qualitative behaviour of the theory does not depend too closely on the particular values that its free parameters happen to take. In other words, the free parameters are not *finely tuned*. Where parameters are related numerically, it is usually expected that there should be some reason (e.g. a symmetry) "explaining" the relationship, rather than just assigning it to random chance. For example, the proton and neutron have nearly the same mass despite their different quark content, due to the approximate SU(2) isospin symmetry between up and down quarks.[3]

The problem is that the presence of a light ($m \sim \mathcal{O}(100)$ GeV) scalar boson in the SM is highly unnatural, due to the influence of virtual loop corrections to its mass. These are similar to the loop corrections to the Z/γ^* propagator that were discussed in tutorial 6, which led to the numerical values of many observables depending on the interaction scale Q^2. The propagator for a scalar boson, shown in Figure 1(a), is $i/(p^2 - m^2)$, and modifications of this are interpreted as changes in the particle's mass m, under the assumption that p^2 is fixed by external constraints. In the case of the SM Higgs boson, the most important modifications come from loops involving the top quark (**Question:** why?), as illustrated in Figure 1(b).

The first-order loop correction to the Higgs mass m_h is given by

$$\Delta m_h^2 = m_h^2 - m_{h0}^2 = -\frac{|y_t|^2}{8\pi^2}\Lambda^2 + \ldots, \tag{1}$$

where m_{h0} is the leading-order Higgs boson mass parameter and Λ^2 is a *cutoff scale*, where we terminate the integral over the virtual top quark's momentum.[4] Curiously, if there is no high scale where the integral should terminate, there is no problem, as truly infinite terms can be arranged to cancel in the usual procedures for renormalisation. However, we are confident that gravity must ultimately have a quantum-field-theoretic nature,

[3]This, in turn, results from the fact that m_u and m_d are both small compared to the QCD scale of ~ 200 MeV.

[4]The "…" terms depend only logarithmically on this scale.

with an associated graviton particle. At the Planck scale, $m_{\rm Pl} \sim 10^{19}$ GeV, gravitational interactions become as strong as the other forces, and must be taken into account in the running of the Higgs boson mass. This suggests a value of $\Lambda \sim m_{\rm Pl}$. Thus, in the Standard Model we expect m_h to be of order the Planck mass, independent of the value of m^2_{h0}. The fact that $m_h \approx 125$ GeV, many orders of magnitude below $m_{\rm Pl}$, would seem to require an extremely precise cancellation Δm^2_h and m^2_{h0}, in other words, fine tuning. This is the hierarchy problem.

2.1 Solutions to the hierarchy problem

There are, in principle, three ways to resolve this apparent problem.

1. *Modify the loop diagrams.* If the virtual contributions to the Higgs boson mass are modified by as-yet unobserved particles, the result in Equation (1) is altered, along with the conclusion about naturalness in the SM. There are at least two ways this might be achieved:
   (a) *Composite Higgs,* where the scalar Higgs boson is not fundamental, but made of non-scalar constituents, such that m_h now depends on the coupling strength between those constituents.
   (b) *Loop cancellation,* where additional non-SM particles with $m \ll m_{\rm Pl}$ cancel the loop diagrams of the SM particles, removing the quadratic divergence in $m_{\rm Pl}$.
2. *Reduce the Planck scale.* It could be that the structure of space-time at small distances naturally creates a hierarchy between the Planck and electroweak scales.
3. *The universe is finely-tuned.* A finely-tuned universe could arise naturally in a multiverse "landscape", if fine-tuning is necessary for the development of a cosmologically stable universe with atoms and stars.

In all except the last solution, new phenomena are expected (often dubbed "new physics"), associated with some energy scale. If the hierarchy problem is to be solved, that scale should be close to the electroweak scale ($v \approx$ 246 GeV), and probably not much larger than a TeV. As the details depend stongly on the particular theory, and the amount of "allowed" fine-tuning is subjective, this constraint is regarded as somewhat soft – scales of up to 2–3 TeV are often accepted – but usually still within range of the LHC or one of its planned upgrades. In this tutorial, we will explore the most popular solution from the list above: cancellation of the loop corrections via the introduction of supersymmetry.

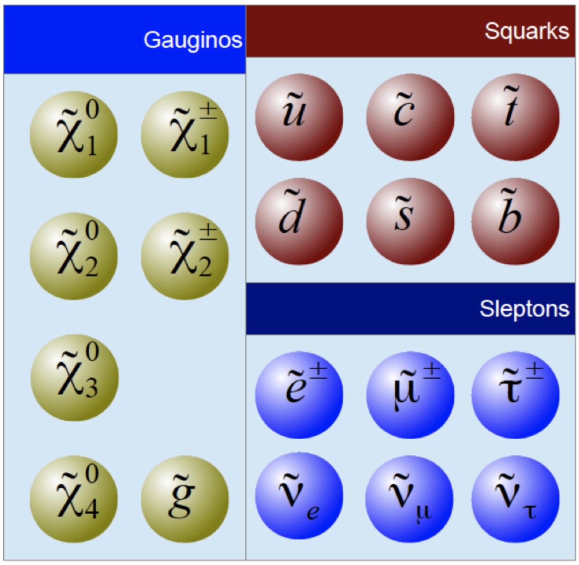


Figure 2: The SUSY particle content of the MSSM.

3 Supersymmetry

Supersymmetry (also known as SUSY) is a space-time symmetry, and is in fact the only way that the Poincaré symmetry of special relativity can be extended, whilst still respecting the requirements of quantum field theory. SUSY postulates a complete symmetry between boson and fermion degrees of freedom, such that the Lagrangian density is unchanged if the two are swapped. Phenomenologically, this requires the introduction of a SUSY partner (*sparticle*) for each SM particle: a SUSY fermion for each each SM boson, and a SUSY boson for each SM fermion. The SUSY theory with the fewest new particles is called the minimal supersymmetric Standard Model (MSSM), and it is shown in Figure 2. The SUSY partners of the SM fermions are scalar quarks and leptons, called squarks ($\tilde{q}$) and sleptons ($\tilde{\ell}$), respectively. The gauge and Higgs bosons all gain spin-$\frac{1}{2}$ partners: the gluino ($\tilde{g}$), four neutralinos ($\tilde{\chi}_i^0$) and two charginos ($\tilde{\chi}_i^{\pm}$).[5]

Apart from their spin, each sparticle experiences the same interactions as the corresponding SM particle. This means that they also contribute to the evolution of m_h with energy. According to the Feynman rules, boson and fermion loops have amplitudes with opposite signs, and therefore the sparticle loops nearly cancel those from the SM particles. The non-cancellation arises from the inequality between SM and SUSY particle masses, meaning m_h ultimately depends on the sparticle masses (especially $m_{\tilde{t}}$), and not m_{Pl}.

[5]As noted in tutorial 12, SUSY is a two-Higgs doublet model, which explains why there are more charginos and neutralinos than one would expect from the SM boson count alone.

Exercise: If realised in nature, SUSY is a broken symmetry (compare $\mathrm{SU(2)_L} \times \mathrm{U(1)}_Y$ in the SM). Unbroken SUSY would imply that the SUSY particles also have the same mass as their SM partners. What would be some of the consequences of unbroken SUSY?

To see what other interactions exist, consider a sfermion gauge interaction vertex (e.g. $g - \tilde{q} - \tilde{q}$). This is related to the SM $g - q - q$ vertex, except that two spin-$\frac{1}{2}$ quarks have been replaced by two spin-0 squarks. Similarly, other sparticle vertices can be constructed from SM vertices by replacing two particles with their SUSY partners. For example:

- $q - \tilde{q} - \tilde{g}$ (again from $g - q - q$);
- $Z - \tilde{\ell} - \tilde{\ell}$ (from $Z - \ell - \ell$);
- $f - \tilde{f} - \tilde{\chi}_i^0$ (from $f - f - Z/\gamma/h/H/A$);

and so on.

By itself, supersymmetry also allows other interactions, unrelated to gauge symmetries, that would violate lepton and baryon number. This would in general lead to extremely rapid proton decay, which is usually avoided by insisting that another quantum number called *R-parity* is conserved. R-parity is defined such that all sparticles have odd R-parity and SM particles have even R-parity. This forbids the additional non-gauge interactions, and protects the proton from decay. It also means that the least massive SUSY particle (the *LSP*) is stable, as in any decay diagram the initial state is R-parity-odd, while the final state with only SM particles would be R-parity-even. Thus, the LSP is also potentially a dark matter particle, usually assumed to be the lightest neutralino ($\tilde{\chi}_1^0$, a linear combination of bino $\tilde{B}$, wino $\tilde{W}^0$ and higgsino $\tilde{H}$ fields). R-parity conservation is not the only way to prevent proton decay in SUSY models, however we will not explore other options here for lack of time.

3.1 Searches for SUSY at the LHC

If they exist, sparticles with $m \sim 1$ TeV can be directly produced at the LHC, and their decay products can be detected. If R-parity conservation is assumed, the sparticles must be produced in pairs, and their decay products will always include the LSP. The LSP, being electrically neutral and weakly interacting, will escape the detector like a neutrino, making large $E_\mathrm{T}^\mathrm{miss}$ one of the distinguishing features of sparticle production events. The primary difficulty in experimental searches is our ignorance of the rest of the SUSY particle mass spectrum, and therefore which visible particles should accompany this $E_\mathrm{T}^\mathrm{miss}$. For example, the detection of charged leptons is a powerful experimental technique to reduce the background from SM processes, however the rate at which they are produced in sparticle decays depends critically upon the mass(es) of the sleptons.

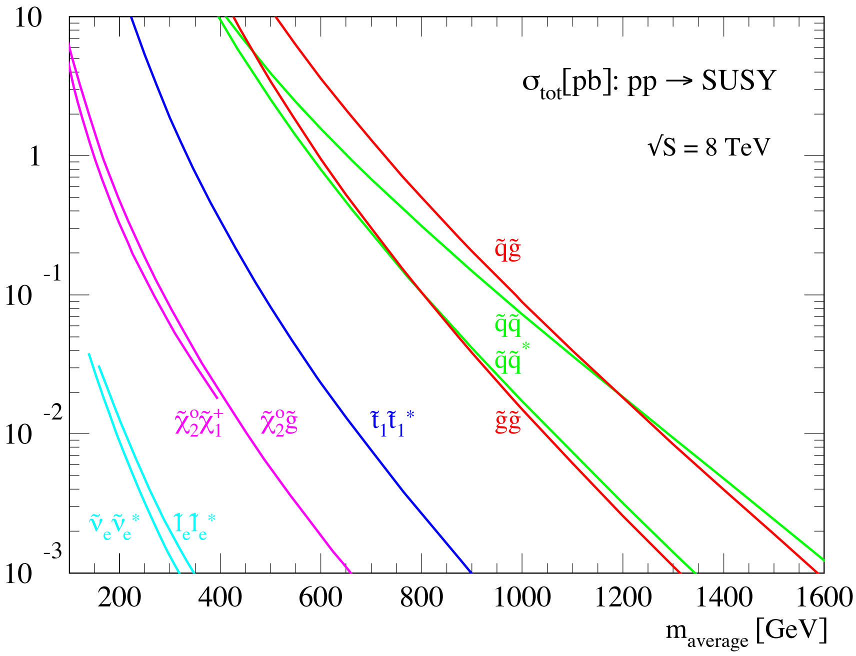


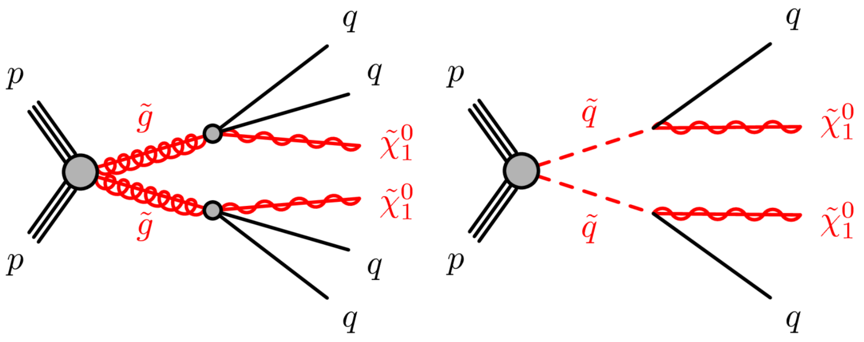
Figure 3: Example sparticle production cross-sections at the LHC with $\sqrt{s} =$ 8 TeV. The x-axis is the average mass of the two sparticles being produced.

Exercise: Why? Consider both sleptons produced as decay products of more massive sparticles and the direct production process $pp \to Z^*/\gamma^* \to \tilde{l}^+\tilde{l}^-$.

Thus, multiple search strategies are required, each optimised for some particular assumption of the true sparticle mass hierarchy. We will look at just a few examples.

If all sparticle masses are of the same order of magnitude, squarks and gluinos will be produced the most often at the LHC, as they interact via the strong nuclear force (see Figure 3). Figure 4 shows example diagrams of gluino and squark pair production in pp collisions. As is common in experimentally-oriented studies, antiparticles are not explicitly indicated in the figure, because most of the time it is irrelevant for the detector-level phenomenology.

Exercise: Draw explicit tree-level Feynman diagrams for the process in Figure 4(a), i.e. expanding the "blobs". At the gluino production vertex, consider the following exchange processes: (a) s-channel gluon, (b) t-channel squark, (c) t-channel gluino. For the decay vertex, consider the decay via a virtual squark. The relevant SUSY vertices are $q - \tilde{g} - \tilde{q}$, $g - \tilde{q} - \tilde{q}$, $g - \tilde{g} - \tilde{g}$ and $q - \tilde{q} - \tilde{\chi}_1^0$.



(a) $pp \to \tilde{g}\tilde{g} \to 4q2\tilde{\chi}_1^0$ (b) $pp \to \tilde{q}\tilde{q} \to 2q2\tilde{\chi}_1^0$

Figure 4: Example diagrams for gluino and squark pair-production and decay in pp collisions. The "blobs" represent effective interactions via off-shell propagators, and spectator quarks emerging from the production vertex are not shown. Also, particle/antiparticle indicators are not explicitly shown.

Exercise: What diagrams are possible for squark production (Figure 4(b))? Consider both $pp \to \tilde{q}\tilde{q}$ and $pp \to \tilde{q}\bar{\tilde{q}}$.

These production and decay modes are predicted to occur in many SUSY models. The event signature consists of several (2–4) hadronic jets in addition to the $E_{\mathrm{T}}^{\mathrm{miss}}$ from the two $\tilde{\chi}_1^0$ particles.

Exercise: What SM background processes could yield the same event signature? For each, try to think of ways that it might be reduced.

Example constraints on squark and gluino masses from the first LHC run, assuming only the decay modes from Figure 4, are shown in Figure 5. The lower limits on the squark and gluino masses are consistently above 1.5 TeV, even if the LSP is very massive ($\sim$ 700 GeV).

This result, while strong, relies on many assumptions and it also does not mean that *all* SUSY particles must lie above this limit. For this reason, searches are performed in many other channels, targeting different sparticle production and decay modes. Of the other sparticles, the third generation squarks (stop and sbottom) are of particular interest, due to their key role in solving the hierarchy problem.[6]

Top squarks with $m \lesssim 1$ TeV could be produced in pairs at the LHC just like any other squark. The process analogous to Figure 4(b) is the following:

$$pp \to \tilde{t}\bar{\tilde{t}} \to \left(t\tilde{\chi}_1^0\right)\left(\bar{t}\tilde{\chi}_1^0\right). \tag{2}$$

Note that in this case, the antiparticles have been explicitly indicated.

[6] Also, many supersymmetric theories generically predict either the $\tilde{t}$ or the $\tilde{b}$ to be the least massive squark, and therefore the most experimentally accessible.

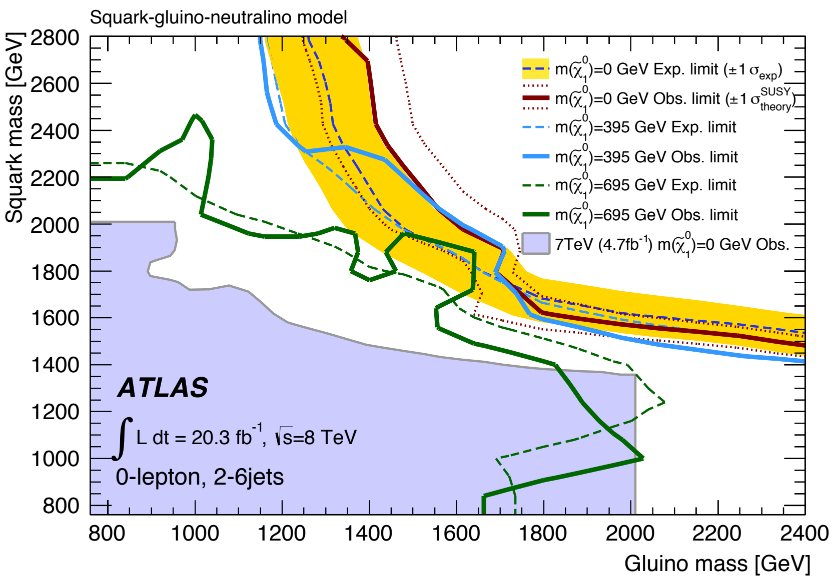


Figure 5: 95% CL exclusion limits on squark and gluino masses from ATLAS using LHC Run-1 data, assuming they decay as shown in Figure 4 with 100% branching ratio. The thick, solid lines show the excluded mass ranges for three different assumed $\tilde{\chi}_1^0$ masses.

Exercise: Why would the process $pp \to \tilde{t}\tilde{t}$ be suppressed?

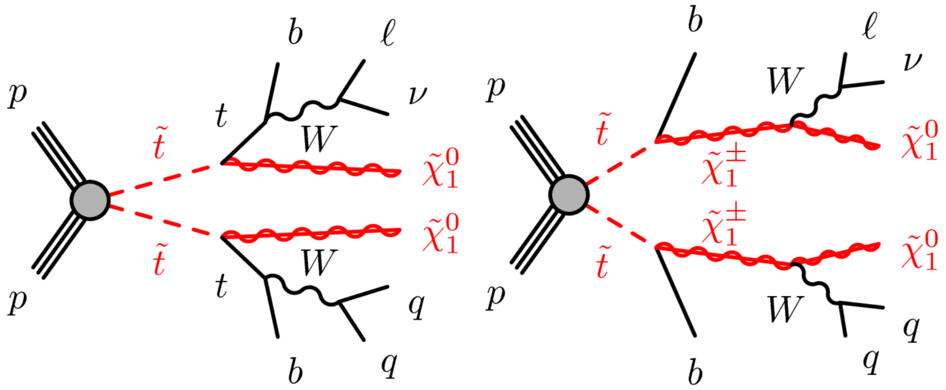
If the branching ratio for $\tilde{t} \to t\tilde{\chi}_1^0$ is large, then the experimental signature of stop pair production is the same as for $t\bar{t}$ production, with additional $E_{\mathrm{T}}^{\mathrm{miss}}$ from the two LSPs. This means that searches for this signature are divided into fully leptonic, semileptonic and fully hadronic channels, in a similar way to SM studies of the top quark (see tutorial 9). An example of the semileptonic channel is shown in Figure 6(a). It may be, however, that the $\tilde{\chi}_1^0$ is too massive to allow the decay of Equation (2) to proceed with on-shell particles. In this case, the following decays may be possible:

$$\tilde{t} \to W^+ b\tilde{\chi}_1^0 \qquad (m_{\tilde{\chi}_1^0} + m_t > m_{\tilde{t}} > m_{\tilde{\chi}_1^0} + m_b + m_W), \tag{3}$$

$$\text{or } \tilde{t} \to f\bar{f}' b\tilde{\chi}_1^0 \qquad (m_{\tilde{\chi}_1^0} + m_b + m_W > m_{\tilde{t}}), \tag{4}$$

where in both cases the top quark is off-shell, and in the latter the W boson is also off-shell. If it is assumed that the $\tilde{t}$ decays *only* via these modes, then the resulting theory has just two parameters: $m_{\tilde{t}}$ and $m_{\tilde{\chi}_1^0}$. Figure 7(a) shows the current exclusion limits on these scenarios as a function of both of these masses. Limits of up to $m_{\tilde{t}} \sim 670$ GeV are obtained, although this weakens substantially if $m_{\tilde{\chi}_1^0} \gtrsim 100$ GeV or the stop mass is near one of the thresholds between different decay modes.

Depending on the other SUSY particle masses, it is of course possible for the stop to decay in other ways. One of the most studied is where



(a) Semileptonic $pp \to \tilde{t}\tilde{t} \to 2t2\tilde{\chi}_1^0$ (b) Semileptonic $pp \to \tilde{t}\tilde{t} \to 2b2\tilde{\chi}_1^\pm$

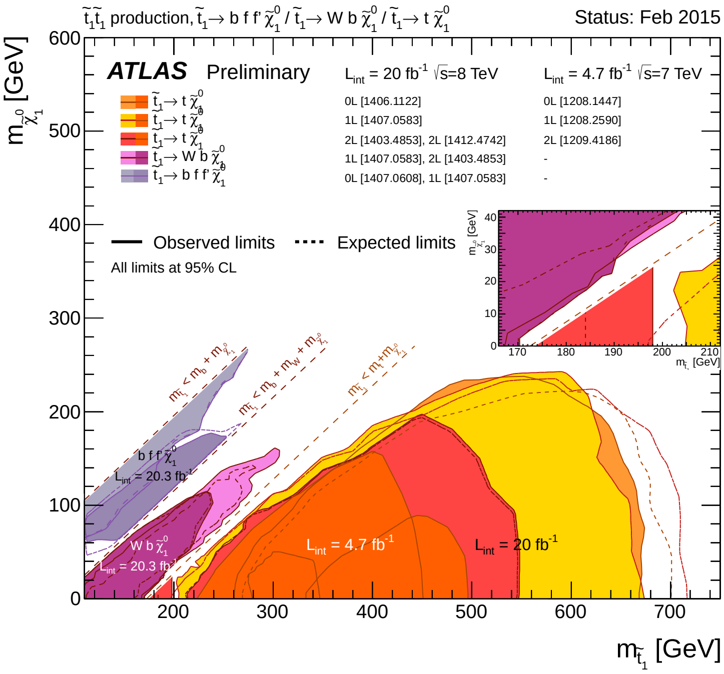
Figure 6: Example diagrams for top squark pair-production and decay in pp collisions. The image style is the same as Figure 4.

the stop decays to a chargino instead of a neutralino. In many cases, this gives a very similar event signature to $\tilde{t} \to t\tilde{\chi}_1^0$, although the kinematic properties of the observable leptons and jets may be very different. An example semileptonic decay in this channel is shown in Figure 6(b). If possible, this decay channel will compete with that from Figure 6(a), and in general the branching ratio of each is unknown.[7] Figure 7(b) shows how the limits for on-shell semileptonic stop decays vary as a function of the branching ratio $\tilde{t} \to t\tilde{\chi}_1^0$, assuming that the only other active decay channel is $\tilde{t} \to b\tilde{\chi}_1^\pm$. In general, the limits weaken as $\mathrm{BR}(\tilde{t} \to b\tilde{\chi}_1^\pm)$ increases.

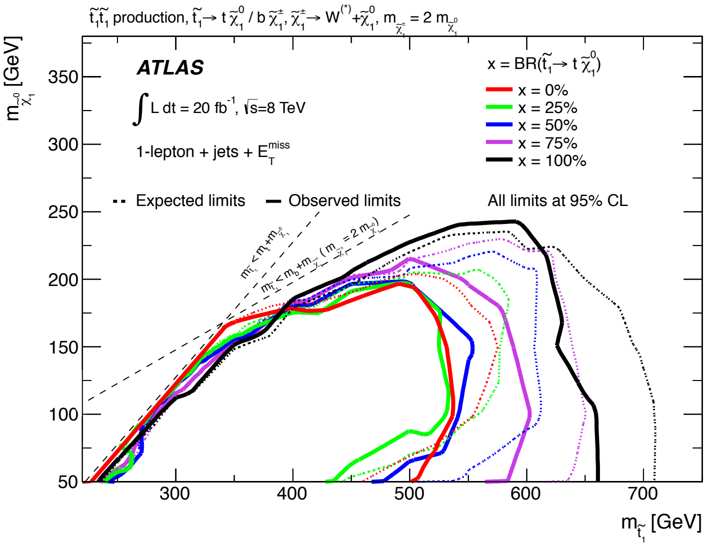
Exercise: What specific feature of the decay $\tilde{t} \to t\tilde{\chi}_1^0$ might explain the higher sensitivity to this decay mode?

In addition to the searches described here, many more have been performed looking for longer cascade decays, electroweak sparticle production, and non-standard (e.g. R-parity-violating) decays. No evidence for the existence of TeV-scale SUSY has yet been seen. We will continue next week with a discussion of non-SUSY solutions to the hierarchy problem, and the possible implications of continued non-observation in the future.

[7]This is due to the fact that all of the models described here are *simplified models*. In a specific, complete, SUSY theory the stop decay modes are of course calculable. However we do not know which, if any, complete SUSY theory is correct, and so the statement that the branching ratios are "unknown" is still valid.



(a)



(b)

Figure 7: 95% CL exclusion limits on top squark production from ATLAS using LHC Run-1 data in the $m_{\tilde{t}}$–$m_{\tilde{\chi}_1^0}$ plane. (a) Summary of results from many searches assuming 100% branching ratio for $\tilde{t} \to t^{(*)}\tilde{\chi}_1^0$. (b) Constraints from the 1-lepton search channel as the branching ratios for $\tilde{t} \to t\tilde{\chi}_1^0$ and $\tilde{t} \to b\tilde{\chi}_1^\pm$ are varied, assuming that $m_{\tilde{\chi}_1^\pm} = 2m_{\tilde{\chi}_1^0}$.