Tutorial 16: Naturalness and TeV-scale gravity

Dr. M Flowerdew

June 29, 2015

In the last tutorial, we discussed the problem of naturalness in the Standard Model. The fact that the Higgs boson is accessible to the LHC, with a mass close to the electroweak scale, would appear to require some kind of mechanism to suppress large loop corrections to its propagation amplitude. This could be achieved by supersymmetric particles, however so far there is no direct evidence for their existence. This week, we will consider other possible resolutions of the hierarchy problem: TeV-scale gravity and the multiverse landscape.

1 Quantum mechanics in compactified dimensions

The properties of the gravitational interaction arise, according to general relativity, from the space-time metric. Therefore, any modification of the gravitational interaction at the TeV scale requires a change to that metric at small distances. This can be achieved by postulating additional *compactified* dimensions, with a physical size of $\mathcal{O}$(mm) or smaller. This approach, although not any specific choice of scale, is also motivated by string theory. Before turning to the hierarchy problem, we will first consider how a quantum mechanical field would behave with a single compactified dimension.

We will assume that the 5D space-time is flat (i.e. it has no curvature), and that the extra dimension x^5 is finite in size, with a constant length of

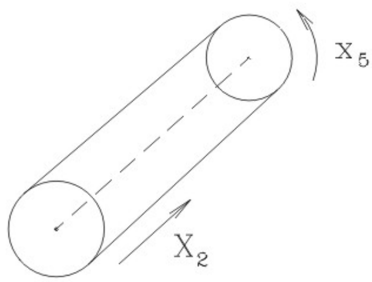


Figure 1: A sketch of a compactified spatial dimension x_5, together with an extended ("normal") dimension x_2.

$2\pi R$. Topologically, the x^5 dimension is described by a loop, as illustrated in Figure 1. As the space is flat, the five-dimensional wave function of a free particle is easy to write down:

$$\psi(x^\mu, x^5) = e^{-ip_\mu x^\mu} e^{-ip_5 x^5}. \tag{1}$$

The enforced periodicity in this particular scenario however means that any wave function must satisfy the following condition:[1]

$$\psi(x^\mu, x^5) = \psi(x^\mu, x^5 + 2\pi R). \tag{2}$$

Substituting this condition into (1), we see that the momentum p_5 must be quantised:

$$e^{2i\pi p_5 R} = 1 \quad \Rightarrow \quad p_5 = \frac{n}{R},\ n = 0, 1, 2, 3, \ldots. \tag{3}$$

We are not in a position to directly observe particles with $p_5 \neq 0$ propagating in this extra dimension. However, this momentum will nevertheless contribute to the overall energy of the particle, via the usual relation:

$$E^2 = p_\mu p^\mu + p_5 p^5 + m^2 = p_\mu p^\mu + \left(m^2 + \frac{n^2}{R^2}\right). \tag{4}$$

From the perspective of our four-dimensional space-time, a non-zero p_5 manifests itself as an additional contribution to the mass. So, instead of a single state with mass m, each quantum field that can propagate in the extra dimension is associated with a *Kaluza-Klein (KK) tower* of states, with mass $\sqrt{m^2 + n^2/R^2}$. In general, the precise mass spectrum will depend on the geometry of the additional spatial dimension(s), however it is a general feature that the first excited state in the tower will have a mass $\mathcal{O}(1/R)$, if we assume that $1/R \gg m$.

Exercise: Why is this a reasonable assumption for most SM particles?

If all SM fields are able to propagate in these extra dimensions, then their KK excitations could be produced and observed in particle colliders as more massive versions of the SM particle. For example, a massive Z' particle with $n = 1$ could decay into two charged leptons and be detected as a resonance in the $\ell^+\ell^-$ invariant mass spectrum. Limits on the masses of new dilepton resonance however set very stringent limits on the allowed minimum value of $1/R \gtrsim$ 2–3 TeV, and therefore such a model could not do much to alleviate the hierarchy problem.[2] If extra dimensions are to help explain the Higgs boson mass, a more complex scenario is required.

[1] In general, any integer multiple of $2\pi R$ could be added to x^5, however considering just this special case is sufficient for our purposes.

[2] We will come back to searches for new dilepton resonances in Section 3.

2 The ADD model

One such model is the ADD model.[3] In this model, a number N of compactified extra dimensions are introduced, with a radius R. The primary difference with respect to the simple Kaluza-Klein theory is that, while gravity propagates in the full $(3+N)$-dimensional space, the SM fields are restricted to a subspace (*membrane*, or *brane*) containing the three non-compactified dimensions. This helps to avoid the constraints on KK excitations mentioned in the previous section, and the compactified dimensions can be very much larger than $R \sim 1/\text{TeV}$.

To illustrate how this might help address the hierarchy problem, it is helpful to consider the classical properties of such a space. The classical gravitational potential between two masses m_1 and m_2 can, for small radii r, be written on dimensional grounds as

$$V(r) \propto \frac{m_1 m_2}{{m^*_{\text{Pl}}}^{N+2}} \cdot \frac{1}{r^{N+1}} \qquad (r \lesssim R). \tag{5}$$

Here, m^*_{Pl} is the true Planck scale that sets the strength of gravitational interactions.

Exercise: Verify that Equation (5) has the correct units.

For large radii $r \gg R$, the limiting behaviour of the potential must be that of Newtonian gravity:

$$V(r) \propto \frac{m_1 m_2}{{m_{\text{Pl}}}^2} \cdot \frac{1}{r} \qquad (r \gg R). \tag{6}$$

We can find the observable Planck scale, m_{Pl}, by noting that the distance between the two masses in each of the N extra dimensions will be $\mathcal{O}(R)$. Replacing this into Equation (5), we obtain

$$V(r) \propto \frac{m_1 m_2}{{m^*_{\text{Pl}}}^{N+2} R^N} \frac{1}{r} \qquad (r \gg R). \tag{7}$$

Comparing this with Equation (6), we find that

$$m^2_{\text{Pl}} \sim {m^*_{\text{Pl}}}^{N+2} R^N, \tag{8}$$

to within a factor of order 1.

If we suppose that m^*_{Pl} is $\mathcal{O}(\text{TeV})$, then we can write R as a function of N. Very approximately, $R(N) \sim 10^{32/N-19}$ m. The case of $N = 1$ would imply $R \sim 10^{13}$ m, comparable to the size of the solar system and clearly impossible. In contrast, $N = 2$ would imply $r \sim 1$ mm, which could give rise to deviations from Newtonian gravity in high-precision measurements of gravity at short distances. For larger N, the associated radius is too small to be detectable in this manner, but the fact that $m^*_{\text{Pl}} \sim 1$ TeV means that the model can be probed at the LHC.

[3] ADD stands for the proposers of the model: Arkani-Hamed, Dimopoulos and Dvali.

2.1 LHC signatures of ADD-like extra dimensions

The primary feature of TeV-scale physics in the ADD model is that gravitational interactions become observable in high-energy collisions, i.e. gravitons can be produced at detectable rates. This is possible because the large physical size of the N extra dimensions means that the KK excitations of the graviton are very closely spaced in energy. The large number of possible states means that even though the coupling to any *one* mode is small, the collective coupling to *all* modes can be significant. Produced individually, these gravitons will propagate into the bulk space and likely not interact further with the SM brane, due to their vanishingly small coupling to normal matter. From a phenomenological point of view, they behave like neutrinos, and can only be inferred via an excess of events with substantial $E_{\mathrm{T}}^{\mathrm{miss}}$. In order to have any substantial transverse momentum, the graviton needs to recoil against other SM particles involved in the collision, most likely a quark or a gluon. The experimental signature is therefore one high-p_{T} jet that is directed away from a substantial amount of missing transverse momentum.[4]

Exercise: What other non-SM phenomena would give the same signature? How does it differ from the signature for squark and gluino production considered last week?

Both ATLAS and CMS have performed searches in this channel. After a veto on events with isolated electrons and muons (to reject $W \to \ell\nu$ production), the dominant background arises from $Z \to \nu\nu$ decays where the Z boson recoils from a jet. This background is modeled using the data in $Z \to \ell\ell$ and $W \to \ell\nu$ control samples, to avoid possible inaccuracies in the simulation of quark and gluon radiation. Figure 2 shows the results of the ATLAS search, as a function of the true Planck scale. The constraints are strongest if there are only two extra dimensions, in which case $m_{\mathrm{Pl}}^* > 5$ TeV. The limits get weaker as N increases, but even for $N = 6$ (equivalent to a 10-dimensional space-time), the true Planck scale must be at least 3 TeV.

Another example of a gravitational phenomenon at the LHC would be the creation of microscopic black holes. Recall that the Schwarzschild radius, which essentially sets a limit on the non-black-hole matter density, is dependent on the Planck scale.[5] Therefore, as the true Planck scale m_{Pl}^* becomes apparent for TeV-scale processes, it becomes possible to exceed the density limit in parton-parton collisions and to form a black hole. These micro black holes must have a mass of at least m_{Pl}^*, however a complete theory of quantum gravity would be needed to predict their behaviour close to this bound. Therefore, searches for black holes at the LHC only consider black holes produced with a minimum threshold mass $m_{\mathrm{th}} \gtrsim 2m_{\mathrm{Pl}}^*$. These

[4]Due to final-state QCD radiation, it is also possible to produce two or more jets rather than just one. This is taken into account in the search methodology.

[5]This is usually expressed in terms of the gravitational coupling G, as $r_s = 2GM/c^2$.

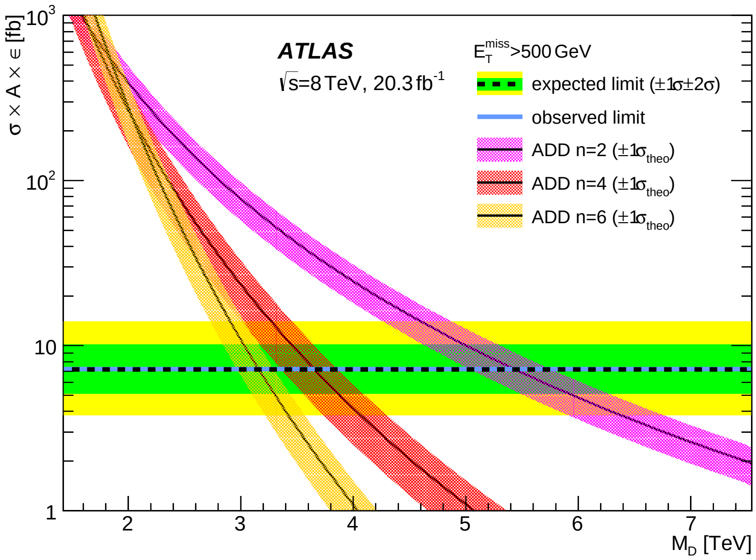


Figure 2: Production cross-section (corrected for analysis acceptance and efficiency) for ADD graviton production at the LHC as a function of the true Planck scale $M_D = m^*_{\mathrm{Pl}}$, for different numbers of extra dimensions. These are compared to the limit from the ATLAS monojet search obtained from events with $E_{\mathrm{T}}^{\mathrm{miss}} > 500$ GeV, shown as a horizontal line.

black holes evaporate rapidly via Hawking radiation, which can be modeled relatively well. Gravity couples equally to all particle types, however from statistical considerations alone, quarks or gluons are the most likely particles to be radiated. Therefore, microscopic black holes are expected to produce events with large numbers of jets. Searches for such events have placed limits on the m_{th} in the range of ~ 4.5–6 TeV, with some dependence on m^*_{Pl}, the black hole production mechanisms and the decay of the black hole remnant (i.e. what is left once its mass decreases to m^*_{Pl}).

3 The Randall-Sundrum model

Similar to the ADD model, the Randall-Sundrum (RS) model supposes that gravitons are able to propagate into spatial dimensions inaccessible to the SM particles. In this case, the apparent weakness of gravity is explained by the *warped* metric of these extra dimensions, rather than their physical size. As a simple example, consider the following metric with one extra dimension y in addition to our conventional 4D space-time x^μ:

$$\mathrm{d}s^2 = \mathrm{d}y^2 + e^{-2ky}\eta_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu. \tag{9}$$

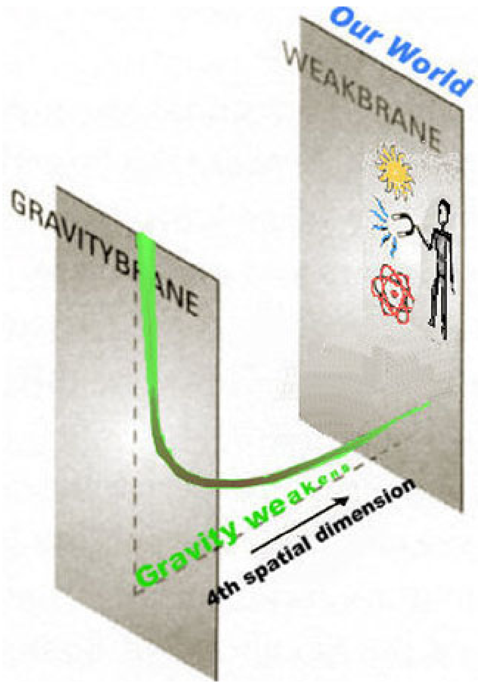


Figure 3: A sketch of the Randall-Sundrum model, showing the separation of the weak brane and the gravity (or Planck) brane.

Here, k is assumed to be $\mathcal{O}(m_{\rm Pl})$. As long as y is constant, this is identical to the usual 4D space-time metric, scaled by a *warp factor* e^{-ky}. The so-called *Planck brane*, where gravitational interactions are strong, is located at $y = 0$, while the SM fields are restricted to the *weak brane*, where $y = y_{\rm SM}$.[6] This arrangement of branes is sketched in Figure 3.

To see how this might address the hierarchy problem, consider a displacement $\mathrm{d}x^1 \sim 1/m_{\rm Pl}$ on the Planck brane. This corresponds to $\mathrm{d}s = 1/m_{\rm Pl}$, as $y = 0$. The same displacement $\mathrm{d}s$ near the weak brane corresponds to a physical distance of $\mathrm{d}x^1 = e^{ky_{\rm SM}}/m_{\rm Pl}$. If we suppose that $ky_{\rm SM} \approx 37$, this is a distance of $\sim 1/\mathrm{TeV}$, equivalent to a much lower energy scale. In fact, all particle masses are scaled by the warp factor, while the effective Planck scale for gravitational interactions only varies weakly as $\sqrt{1 - e^{-2ky_{\rm SM}}}$ because gravitons probe the whole 5D space-time. Thus, both $m_{\rm Pl}$ and the electroweak scale can be explained by a single mass scale (and other $\mathcal{O}(1)$ parameters), and the hierarchy problem is avoided.

[6]Formally, this requires cyclic boundary conditions to be imposed on Equation (9), to allow for the finite allowed range of y.

3.1 LHC signatures of RS gravitons

The production of (excited) gravitons is again the key to searches at the LHC for RS-like models. There are however two important differences in the behaviour of RS gravitons compared to those in ADD-like models:

- The KK excitations of gravitons are widely spaced in the RS model, with ∼TeV masses.
- These excited states couple strongly to matter, with a strength $\sim E/\text{TeV}$.

This means that individual excited graviton states (G^*) produced at the LHC would rapidly decay and could be reconstructed as distinct resonances, much like the Z boson. Full reconstruction of the decay would allow the properties of the resonance to be studied, for example to confirm its spin-2 nature. As with the Higgs boson, the most promising channels for discovery are not necessarily those with the greatest branching ratio; the particle identification efficiency, amount of background from SM processes and the energy/momentum resolution of the detector all play an important role.

Exercise: Why is a good energy/momentum resolution so important for discovery of an excited graviton?

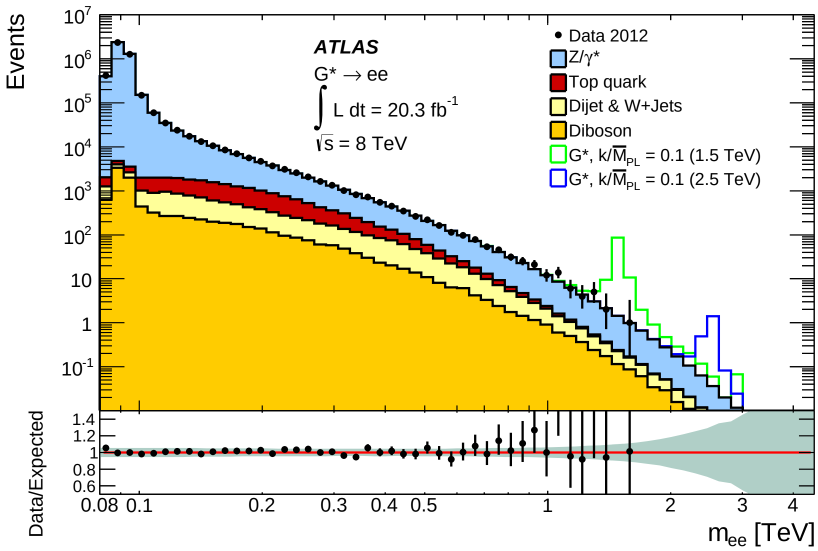
Exercise: Consider the experimental advantages and disadvantages of searching for an excited graviton in the following channels:

$$\begin{array}{lll} G^* \to gg & G^* \to q\bar{q} & \\ G^* \to e^+e^- & G^* \to \mu^+\mu^- & G^* \to \tau^+\tau^- \\ G^* \to W^+W^- & G^* \to ZZ & G^* \to \gamma\gamma \end{array}$$

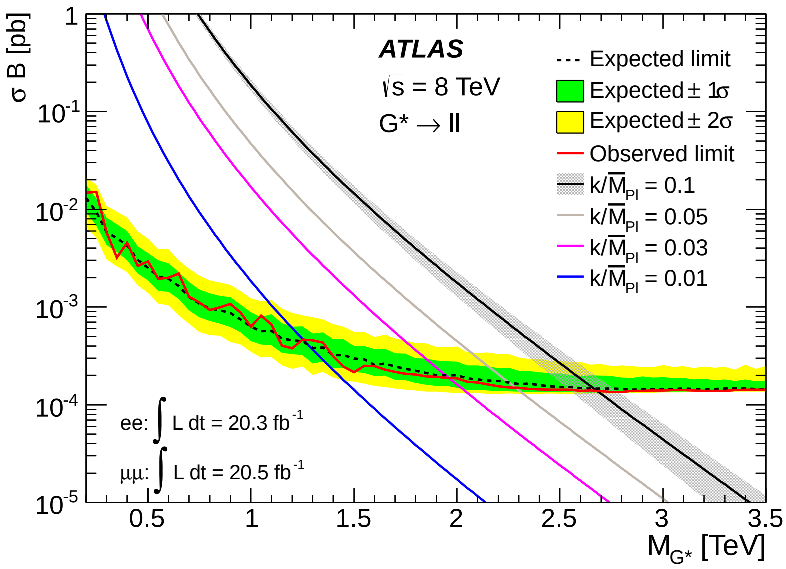
Which decay mode(s) would you expect to have the best intrinsic sensitivity, and why? Do not forget to consider different decay modes for channels with unstable SM particles, but you may ignore the (here unspecified) branching ratio of the graviton to each decay mode.

Exercise: Why are only two-body decays considered in the above exercise?

An example of a search for RS gravitons in the e^+e^- decay channel is shown in Figure 4(a). In this case, the irreducible $Z/\gamma^* \to e^+e^-$ process is the dominant SM background. The absence of any significant deviation at high mass allows limits to be placed on the production cross-section for excited RS gravitons (Figure 4(b)), which constrain the excited graviton mass to be $\gtrsim 1$ TeV unless k is very small. Under certain modifications of the basic RS model (designed to suppress contributions to flavour-changing neutral currents and other electroweak observables), these constraints do not apply and limits must instead be obtained from the decays of the graviton to W and Z bosons. In this case, weaker constraints of $G^* \gtrsim 500$–700 GeV are obtained, again depending on the value of k.



(a)



(b)

Figure 4: (a) Distribution of the e^+e^- invariant mass in the ATLAS search for $X \to e^+e^-$. The expectations from two RS graviton models are overlaid. (b) Constraints on the dilepton ($e^+e^- + \mu^+\mu^-$) cross-section from the same analysis, compared to predictions for the RS model with various values of k. $\bar{M}_{\rm Pl}$ is the reduced Planck mass, equal to $m_{\rm Pl}/\sqrt{8\pi}$.

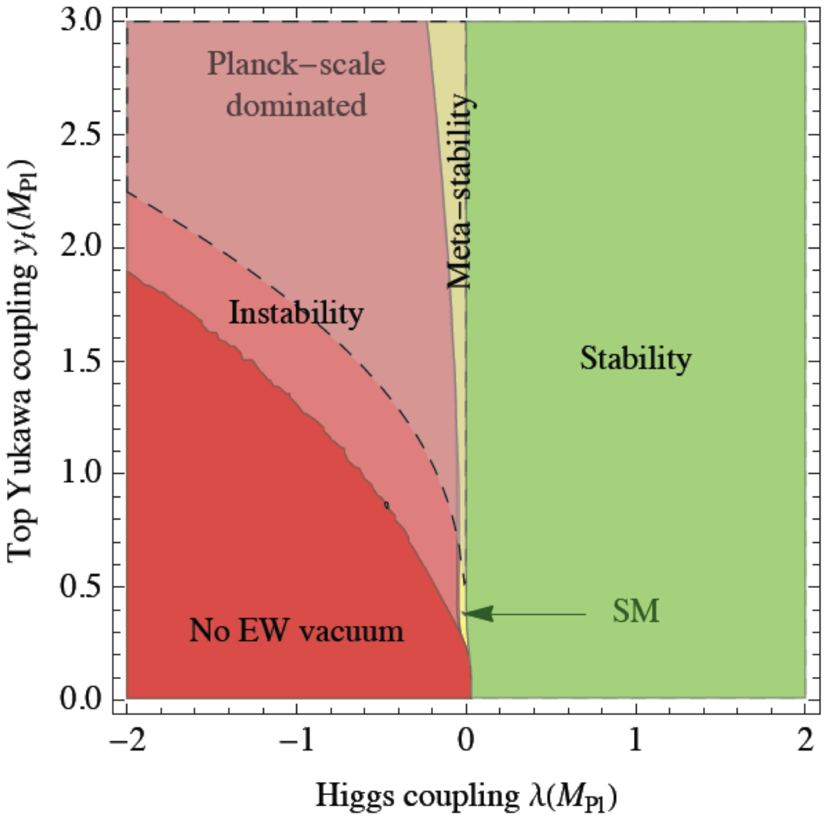


Figure 5: SM phase diagram as a function of the Higgs field coupling λ and the top quark yukawa coupling, both calculated at the Planck scale. The measured values lie within the yellow band of metastability.

4 The finely-tuned SM

The lack of any new TeV-scale phenomena observed at the LHC is beginning to put severe pressure on theories that attempt to explain the Higgs boson mass in a natural way, such as supersymmetry and TeV-scale gravity. While it is possible to relieve the tension between theory and experiment by considering more complex variations of these ideas, most would still require *some* observable phenomena at the LHC. If search results continue to be negative, then the consequences of a finely-tuned SM must be considered. This situation could be allowed, for instance, if our universe is one of many, and the values of the SM parameters[7] vary from universe to universe. Under these conditions, extreme values of certain parameters (such as m_h) could arise in our universe through a kind of selection pressure.

This proposal is also motivated by the apparently near-critical status of

[7]or even the structure of the gauge groups, etc.

the SM parameters. Figure 5 shows the regions for stability and instability of the SM Higgs potential, which depends primarily on the sign of the Higgs quartic coupling λ at the Planck scale. If λ is negative, then the Higgs potential is unbounded from below for the true vacuum, although an unstable false vacuum with a bounded potential may also exist. For values of $-0.04 \lesssim \lambda < 0$, the false vacuum is metastable, with a lifetime longer than the current age of the universe. The measured values of m_t and m_h imply that our universe lies within this metastable region, assuming that the SM is valid up to $m_{\rm Pl}$. If the probability distribution of λ within the multiverse is biased towards negative values, then one would naturally expect a universe with a viable electroweak vacuum to lie in or near this metastable region, from selection pressure alone. However, there is no argument yet as to why negative values of λ should be preferred.

Similar observations can be made regarding the other SM parameters, although in any argument of this type it is notoriously difficult to develop testable predictions. For this reason, the rejection of naturalness remains a controversial idea. In Run-2 of the LHC, starting now, the proton collision energy is $\sqrt{s} = 13$ TeV, nearly double what it was in Run-1. This will give greater sensitivity to new particles with $m > 1$ TeV, and will allow certain loopholes in existing searches to be closed. In this way, it will be possible to further test our ideas of naturalness. What is clear is that the results of these searches, positive or negative, will have significant implications for our understanding of the universe we live in.