# Tutorial 17: Problem set

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Several of these problems are copied or adapted from Alan Barr's notes on nuclear and particle physcis, `http://www-pnp.physics.ox.ac.uk/~barra/teaching/subatomic.pdf`.

1. The $\Omega^-$, $\Delta^{++}$ and $\Delta^-$ baryons are members of the $J^P = \frac{3}{2}^+$ baryon decouplet. Explain why there are no baryons with equivalent quark content in the $J^P = \frac{1}{2}^+$ baryon octet.

2. Write down the Klein-Gordan equation. Show that

$$\phi = \frac{e^{-\mu r}}{r} \tag{1}$$

$$\text{and } \phi = Ae^{-ip_\mu x^\mu} \tag{2}$$

are both valid solutions. What are the physical interpretations of these solutions?

3. Draw the lowest-order Feynman diagram(s) for the following processes. Where given, $X$ refers to unspecified hadrons.
   (a) $e^+e^- \to e^+e^-$
   (b) $e^-e^- \to e^-e^-$
   (c) $\gamma \to e^+e^-$ (in the presence of matter)
   (d) $\gamma\gamma \to \gamma\gamma$
   (e) $\nu p \to e^+X$
   (f) $p\bar{p} \to ZX$ (choose the most likely parton-parton combination)
   (g) $pp \to W^+X$ (choose the most likely parton-parton combination)
   (h) $pp \to W^+W^-X$
   (i) $pp \to W^+W^+X$

4. Why does the ratio

$$\frac{\sigma\left(e^+e^- \to \mu^+\mu^-\right)}{\sigma\left(e^+e^- \to \tau^+\tau^-\right)} \tag{3}$$

tend to unity at high energies? What values would you expect for the following two ratios?

$$\frac{\sigma\left(e^+e^- \to \mu^+\mu^-\right)}{\sigma\left(e^+e^- \to e^+e^-\right)} \tag{4}$$

$$\frac{\sigma\left(e^+e^- \to q\bar{q}\right)}{\sigma\left(e^+e^- \to \mu^+\mu^-\right)} \tag{5}$$

5. Write down the propagator factors for photon and $Z$ boson exchange in the reaction $e^+e^- \to \mu^+\mu^-$. Neglecting any differences in the vertex factors, at what beam energy does the contribution from these two diagrams become equal? What contribution comes from interference between the two diagrams at this energy?

6. The $J/\psi$ has a mass of 3097 MeV, a width of 87 keV, and equal branching ratios of 6% to $e^+e^-$ and $\mu^+\mu^-$. What would you expect for these branching ratios if the $J/\psi$ only decayed electromagnetically? Discuss the result. For comparison, the $\psi''$ has a mass of 3770 MeV, a width of 24 MeV, and a branching ratio to $e^+e^-$ of about $10^{-5}$.

7. Draw diagrams for the decays $D^0 \to K^-\pi^+$ and $D^0 \to K^-e^+\nu_e$. Estimate the relative rates of these decays, disregarding the difference in phase-space factors.

8. What decay modes are available to a $W$ boson produced on-shell? What fraction of $W^+$ decays would you expect to produce positrons?

9. Justify the following expression for the four-momentum of a lepton in a $pp$ collisions at the LHC:

$$p^\ell = \begin{pmatrix} E^\ell \\ p_x^\ell \\ p_y^\ell \\ p_z^\ell \end{pmatrix} = p_\mathrm{T}^\ell \begin{pmatrix} \cosh\eta^\ell \\ \cos\phi^\ell \\ \sin\phi^\ell \\ \sinh\eta^\ell \end{pmatrix}. \tag{6}$$

In the process $pp \to W \to \ell\nu$, how can the transverse momentum of the neutrino ($\boldsymbol{p}_\mathrm{T}^\nu$) be determined? What prevents direct calculation of its longitudinal momentum ($p_z^\nu$, equivalently $\eta^\nu$)? By applying the constraint

$$\left|p^\ell + p^\nu\right|^2 = m_W^2, \tag{7}$$

where $p^\ell$ and $p^\nu$ are the four-momenta of the muon and neutrino, respectively, find an expression for $\cosh\left(\eta^\nu - \eta^\ell\right)$. This equation may have two solutions, one solution, or none – provide a physical interpretation for each case.

*Extension:* Show how the above calculation relates to the *transverse mass*

$$m_\mathrm{T} = \sqrt{2p_\mathrm{T}^\ell E_\mathrm{T}^\mathrm{miss}\left(1 - \cos\Delta\phi[\boldsymbol{p}^\ell, \boldsymbol{p}_\mathrm{T}^\mathrm{miss}]\right)} \tag{8}$$

and why this might be a useful quantity to identify $W$ boson production at hadron colliders.

10. (a) The LHC beam pipe is evacuated to reduce loss of beam from collisions with gas molecules. If less than 5% of the beam protons are to be lost from collisions with gas nuclei over a ten hour run, estimate the maximum persmissible number density of gas atoms in the beam pipe. Assume that the collision cross-section for protons with gas atoms is 50 mb.
    (b) Assuming that each bunch has a circular cross-section with radius 17 $\mu$m, how many protons are required per bunch to have an average of ten interactions per bunch crossing?
    (c) What is the total kinetic energy of each bunch?
    (d) If such bunches collide every 25 ns, what instantaneous luminosity (in $\mathrm{cm^{-2}s^{-1}}$) is achieved?
    (e) If the cross-section for producing a Higgs boson is 50 pb, how many will be made each second?