Tutorial 1

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“Tests des Standardmodells der Teilchenphysik”

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Higgs Boson Decays Into Fermions

Consider the Higgs boson mass m_h as a free parameter. The coupling of the Higgs boson h^0 to a fermion f, either lepton or quark, is given by

$$\mathcal{L}_f = -\frac{1}{\upsilon} m_f h^0 \bar{f} f \tag{1}$$

and hence at tree level the matrix element is

$$i\mathcal{M} = -i\frac{1}{\upsilon} m_f \; \bar{u}^s(p) v^r(k) \; . \tag{2}$$

Therefore, the $h^0 \longrightarrow \bar{f}f$ decay involves a single Yukawa vertex only of strength m_h/υ. The corresponding Feynman diagram is shown in Figure 1. Calculate the squared amplitude and simply sum over the fermion

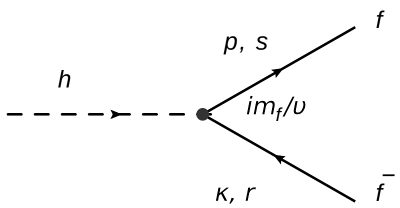


Figure 1: Higgs decay into charged fermions.

spins

$$\langle |\mathcal{M}|^2 \rangle = \sum_{\text{spins}} (i\mathcal{M})(i\mathcal{M})^\dagger \, . \tag{3}$$

since the Higgs boson has only one polarization state.

The differential decay width for a two-body decay is given by

$$\frac{d\Gamma}{d\Omega} = \frac{1}{m_h} \; \langle |\mathcal{M}|^2 \rangle \; \frac{1}{32\pi^2} \; \frac{2|p_f|}{m_h} \tag{4}$$

Integrating over $d\Omega = d\phi \; d\cos\theta$ (and since there is no angular dependence, $\int d\Omega = 4\pi$), one obtains the partial decay width. Show, therefore, that the decay width of the Higgs boson to fermions is

$$\Gamma\left(h^0 \longrightarrow \bar{f}f\right) = N_c^f \; \frac{G_F}{4\pi\sqrt{2}} m_h m_f^2 \left[1 - \left(\frac{2m_f}{m_h}\right)^2\right]^{3/2} \; , \quad \text{where} \quad N_c^f = \begin{cases} 1 & \text{for leptons} \\ 3 & \text{for quarks} \end{cases} , \tag{5}$$

or equivalently,

$$\Gamma\left(h^0 \longrightarrow \bar{f}f\right) = N_c^f \; \frac{m_f^2}{\upsilon^2} \frac{m_h}{8\pi} \beta_f^3 \tag{6}$$

where β_f is the velocity of the outgoing fermions

$$\beta_f = 2\frac{|p_f|}{m_h} = \sqrt{1 - 4\frac{m_f^2}{m_h^2}} \tag{7}$$

Therefore, the Higgs boson decays dominantly into the heaviest fermions and width $\propto m_h$.

Higgs Boson Decays Into Massive Gauge Bosons

The Yukawa couplings of the Higgs bosons to the massive gauge bosons $V = W^\pm$, Z^0 are

$$\mathcal{L}_V = \frac{1}{\upsilon}\left(2M_W^2\; h^0 W_\mu^+ W^{-\mu} + \; M_Z^2\; h^0 Z_\mu^0 Z^{0\mu}\right) \tag{8}$$

and thus the amplitude for the $h^0 \to W^+W^-$ process is

$$i\mathcal{M} = \frac{1}{2}\upsilon g^2 g^{\mu\nu}\epsilon_\mu^*(p,\, \lambda)\epsilon_\nu^*(k,\, \rho)\; . \tag{9}$$

The corresponding Feynman diagram is shown in Figure 2.

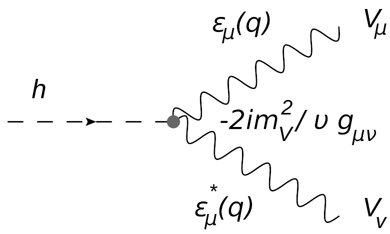


Figure 2: Higgs decay into charged bosons.

1. Calculate the amplitude of this decay process, $\langle|\mathcal{M}|^2\rangle$. Apply the rule for gauge bosons

$$\sum_{\text{pol}} \epsilon_\mu(q)\epsilon_\nu^*(q) = -\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{M_V^2}\right)\; . \tag{10}$$

2. The differential decay width into massive bosons is given by

$$\frac{d\Gamma}{d\Omega} = \frac{1}{m_h}\;\langle|\mathcal{M}|^2\rangle\;\frac{1}{32\pi^2}\;\frac{2|p_V|}{m_h} \times S_V \tag{11}$$

with $S_V = \frac{1}{2}$ for two identical final Z^0 bosons and 1 otherwise. Show that in the on-shell approximation the decay width is finally ($\int d\Omega = 4\pi$)

$$\Gamma(h^0 \longrightarrow VV) = S_V \frac{G_F}{8\pi\sqrt{2}} m_h^3 (1 - 4\lambda_V)^{\frac{1}{2}} \left(1 - 4\lambda_V + 12\lambda_V^2\right)\; , \tag{12}$$

where $\lambda_V = (M_V/m_h)^2$.

The decay width grows like m_h^3, i.e. is very large for $m_h \gg m_V$. For small m_h, one (two) V bosons can be off-shell, the width is

$$\Gamma = \frac{\Gamma_0}{\pi^2}\int_0^{m_h^2} \frac{dq_1^2\; M_V\Gamma_V}{(q_1^2 - M_V^2)^2 + M_V^2\Gamma_V^2}\int_0^{m_h^2 - q_1^2} \frac{dq_2^2\; M_V\Gamma_V}{(q_2^2 - M_V^2)^2 + M_V^2\Gamma_V^2} \tag{13}$$

with

$$\Gamma_0 = S_V \frac{m_h^3}{8\pi\upsilon^2}\Lambda^{1/2}\left(\Lambda - 12\frac{q_1^2 q_2^2}{m_h^4}\right),\; \Lambda = \left(1 - \frac{q_1^2}{m_h^2} - \frac{q_2^2}{m_h^2}\right)^2 - 4\frac{q_1^2 q_2^2}{m_h^4}\; . \tag{14}$$

3. Calculate the ratios

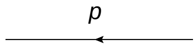
$$\frac{\Gamma(h^0 \to b\bar{b})}{\Gamma(h^0 \to c\bar{c})},\quad \frac{\Gamma(h^0 \to b\bar{b})}{\Gamma(h^0 \to \tau^+\tau^-)},\quad \frac{\Gamma(h^0 \to b\bar{b})}{\Gamma(h^0 \to \mu^+\mu^-)} \quad \text{and} \tag{15}$$

using the data provided below to provide arithmetic results. Make a plot of the ratio

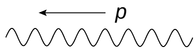
$$\frac{\Gamma(h^0 \to W^+W^-)}{\Gamma(h^0 \to Z^0 Z^0)} \tag{16}$$

as a function of the Higgs mass, m_h.

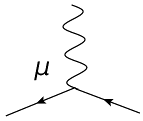
Feynman Rules for Quantum Electrodynamics

Dirac propagator: 

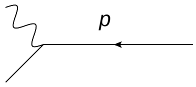
$$= \frac{i(\not{p} + m)}{p^2 - m^2 + i\epsilon} \qquad (17)$$

Photon propagator: 

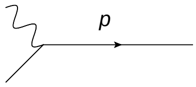
$$= \frac{-ig_{\mu\nu}}{p^2 + i\epsilon} \qquad (18)$$

QED vertex: 

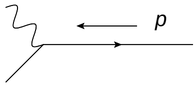
$$= iQe\gamma^\mu \quad (Q = -1 \text{ for an electron}) \qquad (19)$$

External fermions: 

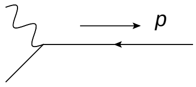
$$= u^s(p) \quad \text{(initial)} \qquad (20)$$



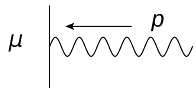
$$= \bar{u}^s(p) \quad \text{(final)} \qquad (21)$$

External anti-fermions: 

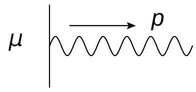
$$= \bar{v}^s(p) \quad \text{(initial)} \qquad (22)$$



$$= v^s(p) \quad \text{(final)} \qquad (23)$$

External photons: 

$$= \epsilon_\mu(p) \quad \text{(initial)} \qquad (24)$$



$$= \epsilon^*_\mu(p) \quad \text{(final)} \qquad (25)$$

Data

$m_h = 125.09$ GeV	$m_\tau = 1.77$ GeV	$m_b = 4.18$ GeV	$m_c = 1.27$ GeV
$m_\mu = 0.106$ GeV	$m_Z = 91.19$ GeV	$m_W = 80.2$ GeV	$\sin^2\theta_W = 0.23$
$v = \frac{1}{\sqrt{\sqrt{2}G_F}} = 246$ GeV	$(1\text{ GeV})^{-2} = 0.389$ mb		

Mathematical structure of the γ matrices

The defining property for the gamma matrices to generate a Clifford algebra is the anticommutation relation

$$\{\gamma^\mu, \gamma^\nu\} = \gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2g^{\mu\nu} I_4 \qquad (26)$$

where $\{,\}$ is the anticommutator, $\eta^{\mu\nu}$ is the Minkowski metric with signature $(+ - --)$ and I_4 is the 4×4 identity matrix. This defining property is more fundamental than the numerical values used in the specific representation of the gamma matrices. Covariant gamma matrices are defined by

$$\gamma_\mu = g_{\mu\nu}\gamma^\nu = \left\{\gamma^0, -\gamma^1, -\gamma^2, -\gamma^3\right\}, \qquad (27)$$

and Einstein notation is assumed.

Numerator algebra

Trace identities Three of the main properties of the trace operator are:

$$\begin{aligned} \operatorname{tr}(A+B) &= \operatorname{tr}(A) + \operatorname{tr}(B) && (28) \\ \operatorname{tr}(rA) &= r\operatorname{tr}(A) && (29) \\ \operatorname{tr}(ABC) &= \operatorname{tr}(CAB) = \operatorname{tr}(BCA) && (30) \end{aligned}$$

Trace properties involving γ matrices Traces of γ matrices can be evaluated as follows:

$$\mathrm{tr}(\mathbf{1}) = 4 \tag{31}$$
$$\mathrm{tr}(\text{any odd number of } \gamma\text{'s}) = 0 \tag{32}$$
$$\mathrm{tr}(\gamma^\mu \gamma^\nu) = 4g^{\mu\nu} \tag{33}$$
$$\mathrm{tr}(\gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma) = 4(g^{\mu\nu} g^{\rho\sigma} - g^{\mu\rho} g^{\nu\sigma} + g^{\mu\sigma} g^{\nu\rho}) \tag{34}$$
$$\tag{35}$$

Another identity allows one to reverse the order of γ matrices inside a trace:

$$\mathrm{tr}(\gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma \cdots) = \mathrm{tr}(\cdots \gamma^\sigma \gamma^\rho \gamma^\nu \gamma^\mu) \tag{36}$$

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