Tutorial 4

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"Tests des Standardmodells der Teilchenphysik"

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Part 2: Time Evolution of a Neutral Kaon State

Suppose that at time $t = 0$ we have the state

$$\psi(0) = |K_S^0\rangle \equiv |K_S^0(t = 0)\rangle \tag{1}$$

and we want to know how does this state evolve in time. We should have at time t,

$$\psi(t) = e^{-iHt}|K_S^0\rangle. \tag{2}$$

For a free particle, the energy is

$$\omega_S = \sqrt{p^2 + m_S^2}, \tag{3}$$

where m_S is the mass of the K_S^0 flavor state. However, if we just use this state for H, we won't have a particle which decays in time. We know that, if we start with a particle at $t = 0$ the probability to find it undecayed at a later time t if it has a lifetime $\tau_S = 1/\Gamma_S$ is given by

$$P(t) = e^{-\Gamma_S\, t}. \tag{4}$$

Thus, the amplitude should have an $\exp(-\Gamma_S\, t/2)$ time dependence, in addition to the phase variation

$$\psi(t) = e^{-i\omega_S t - \Gamma_S t/2}|K_S^0\rangle. \tag{5}$$

Letting $\omega_L = \sqrt{p^2 + m_L^2}$, where m_L is the mass of the K_L^0, and $\Gamma_L = 1/\tau_L$, we similarly have for an initial K_L^0 state ($\psi(0) = |K_L^0\rangle$):

$$\psi(t) = e^{-i\omega_L t - \Gamma_L t/2}|K_L^0\rangle. \tag{6}$$

In the $\{|K_S^0\rangle, |K_L^0\rangle\}$ basis, the Hamiltonian operator is:

$$H = \begin{pmatrix} \omega_S - i\Gamma_S/2 & 0 \\ 0 & \omega_L - i\Gamma_L/2 \end{pmatrix}. \tag{7}$$

How did we know that H is diagonal in this basis and not, perhaps, in the $|K^0\rangle, |\overline{K}^0\rangle$ basis? The answer is that we are assuming that CP is conserved and hence, $[H, CP] = 0$. The Hamiltonian cannot mix states of differing CP quantum numbers, so there are no off-diagonal terms in H in the $|K_S^0\rangle, |K_L^0\rangle$ basis.

The second point is that we have allowed the possibility that the masses of the two CP eigenstates are not the same, having already noted that the lifetimes are different. This might be a bit worrisome since the C operation does not change mass. [1] However, the $|K_S^0\rangle$ and $|\overline{K}_L^0\rangle$ are not antiparticles of one another, so there is no constraint that their masses must be equal. Therefore, we allow the possibility that they may be different.

Now suppose that at time $t = 0$ we have a pure $\overline{K}^0$ state

$$\psi(0) = |\overline{K}^0\rangle. \tag{8}$$

[1] It is a fundamental theorem in relativistic quantum mechanics that particle and anti-particle have the same mass (as well as the same total lifetime).

Experimentally, this is a reasonable proposition, since we may produce such states via the strong interaction. For example, if we collide two particles with no initial strangeness (perhaps a proton and an anti-proton), we make strange particles in "associated production", that is in the production of $s\bar{s}$ pairs. Thus, we might have the reaction $\bar{p}p \to n\overline{\Lambda}\overline{K}^0$. The presence of the $\overline{\Lambda}$, which contains the $\bar{s}$ quark, suggests that the kaon produced is a $\overline{K}^0$, since it contains the s quark.

So, one could realistically imagine producing a $\overline{K}^0$ at $t = 0$. But the time-evolution to later times is governed by the Hamiltonian, which is not diagonal in the $|K^0\rangle, |\overline{K}^0\rangle$ basis. Thus, we might expect that at some later time we may observe a K^0.

We now are interested in estimating the probability, $P_{K^0}(t)$ that a K^0 meson is observed at time t, given a pure $\overline{K}^0$ state at $t = 0$. The way to proceed it by first noting that $\psi(0) = |\overline{K}^0\rangle = \left(|K_S^0\rangle - |K_L^0\rangle\right)/\sqrt{2}$, and thus

$$\begin{aligned} P_{K^0}(t) &= |\langle K^0|\psi(t)\rangle|^2 & (9)\\ &= \frac{1}{2}|\langle K^0|K_S^0\rangle e^{-i\omega_S t-\Gamma_S t/2} - \langle K^0|K_L^0\rangle e^{-i\omega_L t-\Gamma_L t/2}|^2 \\ &= \frac{1}{4}\left\{e^{-\Gamma_S t} + e^{-\Gamma_L t} - 2e^{-\frac{\Gamma_S+\Gamma_L}{2}t}\cos\left[(\omega_S - \omega_L)t\right]\right\}. & (10)\end{aligned}$$

By measuring the frequency of the oscillation in the last term, we may measure the mass difference between the K_S^0 and the K_L^0 when the momentum is small, $\omega_S - \omega_L \approx m_S - m_L$. Because this difference is very small, it is experimentally intractable to attempt this with direct kinematic measurements. Measurements of the oscillation frequency yield a mass difference of

$$\begin{aligned} |m_S - m_L| &= 0.5\times 10^{10}\ \mathrm{s}^{-1} & (11)\\ &= \frac{0.5\times 10^{10}\ \mathrm{s}^{-1}}{3\times 10^{23}\ \mathrm{fm/s}} 200\times 10^6\ \mathrm{eV\text{-}fm} \\ &= 3\ \mu\mathrm{eV}, & (12)\end{aligned}$$

a difference comparable to the energy of a microwave photon. Since the mass of the kaon is approximately 500 MeV, this is a fractional difference of order one part in 10^{14}!

Now, let us see examples in more detail.

1. Find the neutral kaon Hamiltonian in the $|K^0\rangle$, $|\overline{K}^0\rangle$ basis. Is the symmetry of this result consistent with the notion that the masses of particles and antiparticles are the same? Same question for the decay rates.

2. Repeat the derivation of Eq. 10, but work in the density matrix formalism. We did not consider the possibility of decay when we developed this formalism, so we should be careful; we may find that we need to modify some of our discussion. Show that in the end

$$\begin{aligned} P_{K^0}(t) &= \mathrm{tr}\left[\rho(t)|K^0\rangle\langle K^0|\right] \\ &= \cdots & (13)\\ &= \frac{1}{4}\left\{e^{-\Gamma_S t} + e^{-\Gamma_L t} - 2e^{-\left(\frac{\Gamma_S+\Gamma_L}{2}\right)t}\cos\left[(\omega_S - \omega_L)t\right]\right\}. & (14)\end{aligned}$$

3. We discussed the neutral kaon (K) meson, and in particular the phenomenon of $K^0 - \bar{K}^0$ mixing. Let us think about this system a bit further. The K^0 and $\bar{K}^0$ mesons interact in matter, dominantly via the strong interaction. Approximately, the cross section for an interaction with a deuteron (the nucleus of deuterium ^2H) is:

$$\begin{aligned} \sigma(K^0 d) &= 36\,\mathrm{millibarns} & (15)\\ \sigma(\bar{K}^0 d) &= 59\,\mathrm{millibarns}, & (16)\end{aligned}$$

at a kaon momentum of, say, 1.5 GeV. Note that a "barn" is a unit of area equal to $10^{-24}\,\mathrm{cm}^2$.

   (a) Consider a beam of kaons (momentum 1.5 GeV) incident on a target of liquid deuterium. Let λ be the K^0 "interaction length", that is the average distance that a K^0 will travel in the deuterium before it interacts according to the above cross section. Similarly, let $\bar{\lambda}$ be the $\bar{K}^0$ interaction length. To a good enough approximation for our purposes, you may treat the deuterium as a collection of deuterons. The density of liquid deuterium is approximately $\rho = 0.17\,\mathrm{g/cm}^3$. What are λ and $\bar{\lambda}$, in centimeters?

(b) Suppose we have prepared a beam of K_L^0 mesons, for example by first creating a K^0 beam and waiting long enough for the K_S^0 component to decay away. If we let this K_L^0 beam traverse a deuterium target, the K^0 and $\bar{K}^0$ components will interact differently, and we may end up with some K_S^0 mesons exiting the target. Let us make an estimate for the size of this effect.

Since the kaon is relativistic, we need to be a little careful compared with our previous discussion. In the K_L^0 rest frame, the amplitude depends on time t^* according to:

$$\exp(-im_L t^* - \Gamma_L t^*/2), \tag{17}$$

where $\Gamma_L = 1/\tau_L$ is the K_L^0 decay rate. In the laboratory frame, where the kaon is moving with speed v ($c = 1$), and $\gamma = 1/\sqrt{1-v^2}$, $t^* \to t/\gamma$, where t is the time as measured in the laboratory frame. In the lab frame, we have $t/\gamma = x/\gamma v$, and we may write the amplitude as for the K_L^0 as:

$$\exp(-im_L x/\gamma v - \Gamma_L x/2\gamma v), \tag{18}$$

Let us consider a deuterium target, of thickness w, along the beam direction. At a distance x into the target, an interaction may occur, resulting in a final state:

$$\frac{1}{\sqrt{2}}(f|K^0\rangle - \bar{f}|\bar{K}^0\rangle), \tag{19}$$

where, for example, the amplitude f for the K^0 component traversing distance dx is just:

$$f = e^{-dx/2\lambda} \approx 1 - \frac{dx}{2\lambda}. \tag{20}$$

Put all this together and find an expression for the probability to observe a K_S^0 to emerge from the deuterium, for a K_L^0 incident. Assume that $w \ll \lambda$. You may wish to use $\Delta m \equiv m_L - m_S$, $\Gamma_{S,L} \equiv 1/\tau_{S,L}$, and $\Delta\Gamma \equiv \Gamma_L - \Gamma_S \approx -\Gamma_S$

(c) Suppose $w = 10\,\text{cm}$ and $\gamma v = 3$. What is the probability to observe a K_S^0 emerging from the target? What is the probability to observe a K_L^0? You may use:

$$\Gamma_S = 1.1 \times 10^{10}\,\text{s}^{-1}, \tag{21}$$
$$\Delta m = 0.5 \times 10^{10}\,\text{s}^{-1}. \tag{22}$$

References

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