Tutorial 7

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"Tests des Standardmodells der Teilchenphysik"

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Neutral Currents – Neutrino-Electron Scattering

The $\nu_\mu e^-$ and $\bar{\nu}_\mu e^-$ elastic scattering processes can only proceed via a neutral current interaction, as illustrated by the Feynman diagram in Figure 1.

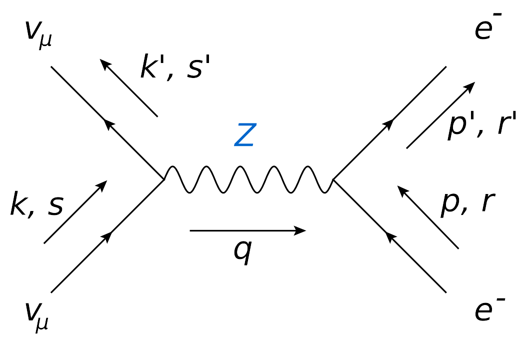

Figure 1: Feynman diagram for the neutral current (NC) $\nu_\mu e^- \to \nu_\mu e^-$ elastic scattering (time flux ↑).

1. We consider the weak process $\nu_\mu e^- \to \nu_\mu e^-$ occurring via *neutral* currents at tree level. Draw the higher-order Feynman diagram of the related *charged* process $\nu_\mu e^- \to \nu_\mu e^-$ and explain why it can be neglected in cross-section calculations.

2. Write down an expression for the matrix element for the electron-neutrino scattering process at tree level assuming low energies, $q^2 \ll m_Z^2$.

3. Sum over the final spin orientations of the outgoing fermions and average over the initial spins of the incident fermions, and show that the squared amplitude takes the following form

$$\langle|\mathcal{M}|^2\rangle = \frac{1}{2}\left(\frac{g_Z}{M_Z}\right)^2 \left[C_1^2(p.k)(p'.k') + C_2^2(p.k')(p'k) - m^2 C_1 C_2 (k.k')\right] \tag{1}$$

   where $m = m_\ell$, $C_1 = c_V^\ell + c_A^\ell$, $C_2 = c_V^\ell - c_A^\ell$, and c_V and c_A are the neutral weak couplings for the involved charged lepton.

4. Work out the kinematics to derive an expression of the differential scattering cross-section $d\sigma/d\Omega$ in the center-of-mass frame and at the high energy limit (i.e. ignoring the electron mass $m_e \to 0$ when $|\mathbf{p}| \simeq E \gg m_e$). Show that the differential scattering cross-section is

$$\frac{d\sigma}{d\Omega} = \frac{2E^2}{\pi^2}\left(\frac{g_Z}{4M_Z}\right)^4 \left(C_1^2 + C_2^2 \cos^4\frac{\theta}{2}\right) \tag{2}$$

   where E is the electron (or neutrino) energy, and θ is the scattering angle in the COM frame.

5. Integrate over angles and show that the total cross-section σ for this process is

$$\sigma = \frac{2E}{3\pi}\left(\frac{g_Z}{2M_Z}\right)^4 \left(c_V^2 + c_A^2 + c_V c_A\right)\ , \tag{3}$$

where c_V and c_A correspond to the neutral vector and axial vector coupling parameters for the electron.

Identities

$$\begin{aligned}
\gamma^5 &= \frac{i}{4!}\varepsilon_{\mu\nu\alpha\beta}\gamma^\mu\gamma^\nu\gamma^\alpha\gamma^\beta && (4)\\
(\gamma^5)^\dagger &= \gamma^5 && (5)\\
(\gamma^5)^2 &= 1 && (6)\\
\{\gamma^5, \gamma^\mu\} &= \gamma^5\gamma^\mu + \gamma^\mu\gamma^5 = 0 && (7)\\
\mathrm{tr}(\gamma^\mu) &= 0 && (8)\\
\mathrm{tr}(\gamma^\mu\gamma^\nu) &= 4g^{\mu\nu} && (9)\\
\mathrm{tr}(\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma) &= 4(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\rho}) && (10)\\
\mathrm{tr}(\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma^5) &= -4i\epsilon^{\mu\nu\rho\sigma} && (11)\\
\mathrm{tr}(\gamma^5\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma) &= +4i\epsilon^{\mu\nu\rho\sigma} && (12)\\
\mathrm{tr}(\gamma^5) &= \mathrm{tr}(\gamma^\mu\gamma^\nu\gamma^5) = 0 && (13)\\
\mathrm{tr}(\text{odd number of }\gamma\text{'s}) &= 0 && (14)\\
\varepsilon_{ijkl} &= \begin{cases} +1 & \text{if } (i,j,k,l) \text{ is an even permutation of } (1,2,3,4)\\ -1 & \text{if } (i,j,k,l) \text{ is an odd permutation of } (1,2,3,4)\\ 0 & \text{otherwise}\end{cases} && (15)\\
\varepsilon^{ijkl} &= \varepsilon_{ijkl} && (16)\\
\varepsilon_{\mu\nu\rho\sigma}\varepsilon^{\mu\nu\kappa\lambda} &= -2\left(\delta_\sigma{}^\kappa\delta_\rho{}^\lambda - \delta_\rho{}^\kappa\delta_\sigma{}^\lambda\right) && (17)\\
& && (18)
\end{aligned}$$

Data

f	Q_f	c_A^f	c_V^f
ν_e, ν_μ, ν_τ	0	$+1/2$	$+\frac{1}{2}$
e^-, μ^-, τ^-	-1	$-1/2$	$-\frac{1}{2} + 2\sin^2\theta_W \simeq -0.03$
u, c, t	$+2/3$	$+1/2$	$+\frac{1}{2} - \frac{4}{3}\sin^2\theta_W \simeq +0.19$
d, d, b	$-1/3$	$-1/2$	$-\frac{1}{2} + \frac{2}{3}\sin^2\theta_W \simeq -0.34$

Table 1: Neutral vector and axial-vector $Z^0 f\bar{f}$ couplings in the GWS model with $\sin^2\theta_W = 0.234$.

Formulæ

$$Z^0 f f \text{ vertex factor} = -i g_z \gamma^\mu \frac{1}{2}\left(c_V^f - c_A^f \gamma^5\right), \; f = \text{any fermion} \tag{19a}$$

$$W^\pm \ell^- \bar{\nu}_\ell \text{ vertex factor} = -i \frac{g_w}{\sqrt{2}} \gamma^\mu \frac{1}{2}\left(1-\gamma^5\right) \tag{19b}$$

$$W^\pm q_i q_j \text{ vertex factor} = -i \frac{g_w}{\sqrt{2}} \gamma^\mu \frac{1}{2}\left(1-\gamma^5\right) U_{ij}, \; i = (u,\, c,\, t),\; j = (d,\, s,\, b) \tag{19c}$$

$$Z^0,\, W^\pm \text{ propagator} = -i \frac{g_{\mu\nu} - \frac{q_\mu q_\nu}{M^2_{Z/W}}}{q^2 - M^2_{Z/W}} \tag{19d}$$

$$g_W = \sqrt{4\pi\alpha_W} = \text{weak coupling constant for CC} \tag{19e}$$

$$\alpha_W = \frac{g_W^2}{4\pi} = \frac{1}{29} = \text{weak fine structure constant} \tag{19f}$$

$$g_W = \frac{g_e}{\sin\theta_W} \tag{19g}$$

$$g_Z = \frac{g_e}{\sin\theta_w \cos\theta_W} = \text{weak coupling constant for NC} \tag{19h}$$

$$\frac{g_Z}{M_Z} = \frac{g_W}{M_W} \tag{19i}$$

$$g_e = \sqrt{4\pi\alpha} = \text{electromagnetic coupling constant} \tag{19j}$$

$$\alpha = \frac{e^2}{4\pi} = \text{fine structure constant} \tag{19k}$$

$$\frac{G_F}{\sqrt{2}} = \frac{g_W^2}{8 m_W^2} = \text{Fermi's coupling constant} \tag{19l}$$

$$m_W = m_Z \cos\theta_W \tag{19m}$$

$$\sin^2\theta_W = 0.23120 \pm 0.00015 = \text{weak mixing angle} \tag{19n}$$

$$c_V^f = T_f^3 - 2\sin^2\theta_W . Q_f = \text{vector weak coupling} \tag{19o}$$

$$c_A^f = T_f^3 = \text{axial-vector weak coupling} \tag{19p}$$

$$T_f^3 = \text{3rd component of the weak isospin of fermion } f \tag{19q}$$

$$Q_f = \text{charge of interacting fermion } f \tag{19r}$$

$$\tag{19s}$$

It is customary to present the results of the neutrino-electron cross-section measurements as an ellipse of all possible values of the c_A and c_V parameters, in the plane c_A vs c_V.

The experimental ellipses in Figure 2 intersect to give two possible solutions:

$$c_V^e = -0.52 \pm 0.06 \tag{20a}$$

$$c_A^e = 0.06 \pm 0.08 \tag{20b}$$

in reasonable agreement with the GWS model and $\sin^2\theta_W \simeq \frac{1}{4}$.

References

- *Collider Physics (Frontiers in Physics)*, Vernon D. Barger, Roger J.N. Phillips, 1996 **Chapter 2 & 8**

- *Introduction to Elementary Particles*, D. J. Griffiths, Wiley-VCH, Second/Revised edition, 2013 **Chapter 9**

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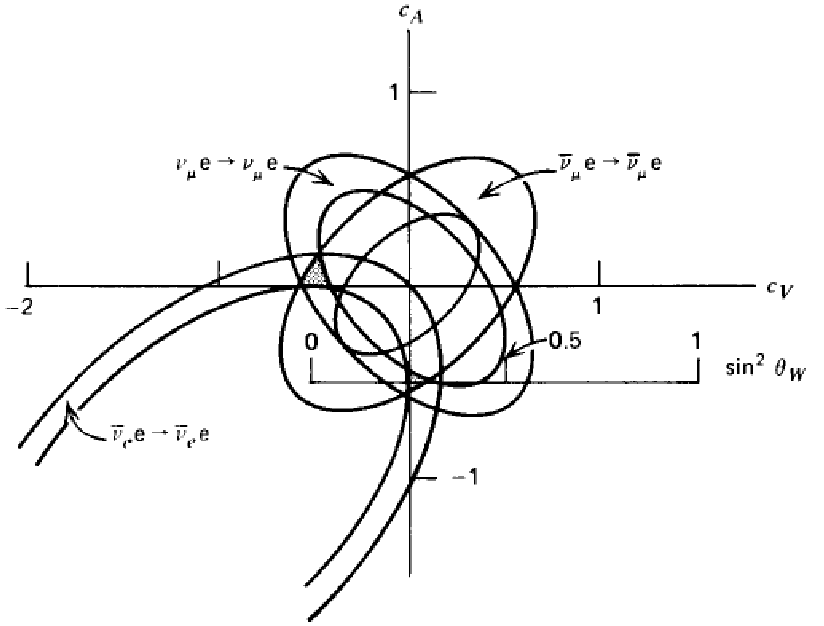

Figure 2: Determination of the parameters c_V and c_A by only neutrino-electron data, as of 1981 before the direct observation of the weak bosons W and Z ("The Structure of Neutral Currents",P Q Hung, and J J Sakurai, Annual Review of Nuclear and Particle Science, Vol. 31: 375-438,December 1981, DOI: 10.1146/annurev.ns.31.120181.002111). The absence of ν_e data is because reactor beams only provide $\bar{\nu}_e$.