

Testing the Standard Model of Elementary Particle Physics II

3rd lecture

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6th May 2021

4.2 The physics program at the Large Hadron Collider

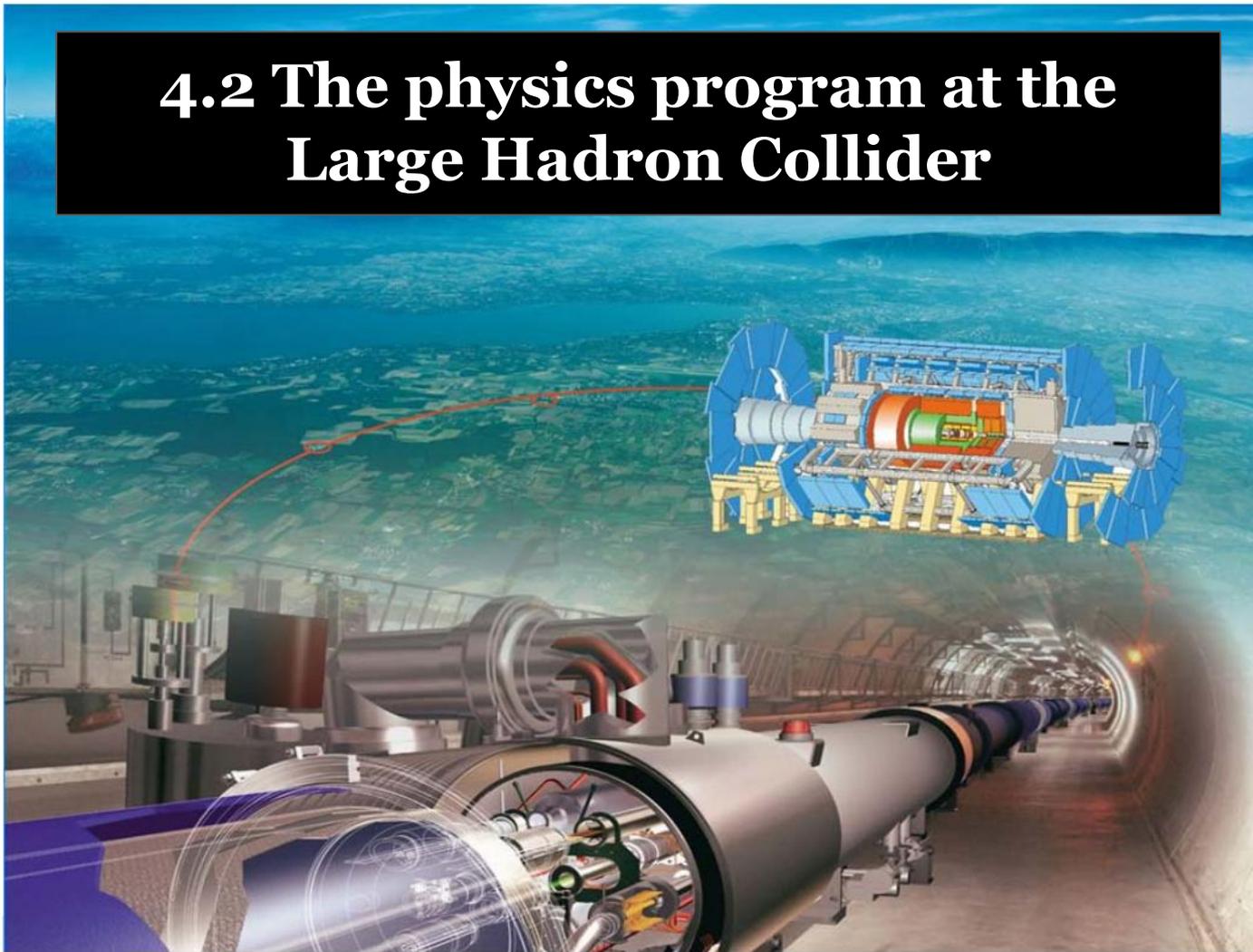


Table of content

- **The physics program at the Large Hadron Collider**
 - A short overview
 - High energy physics (HEP) data analysis
 - Basics of Monte Carlo simulations
 - Statistics for Pedestrians

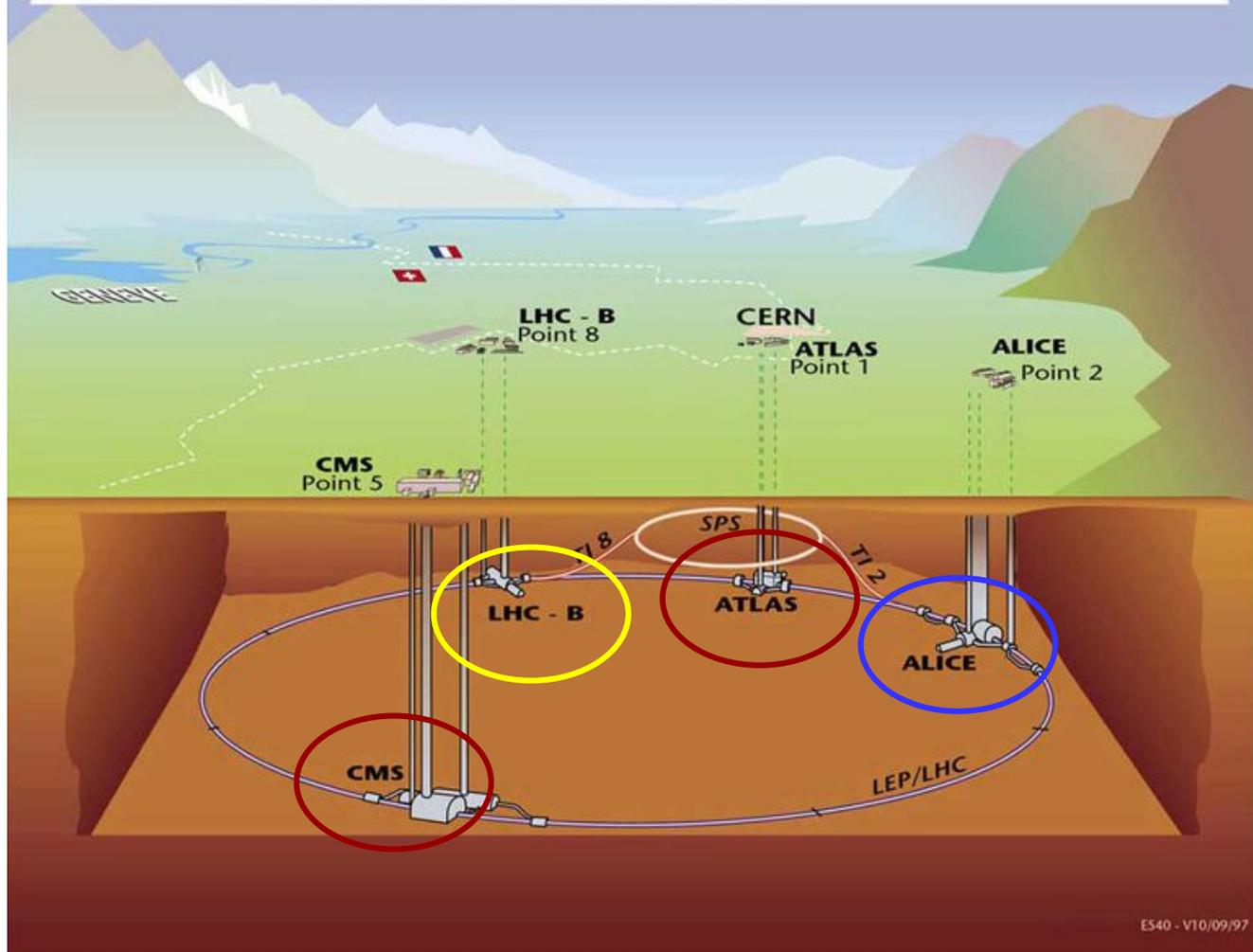
- **Literature**
 - Monte Carlo simulations:
 - [Introduction to parton-shower event generators](#)
 - Statistics:
 - [LHC Statistics for Pedestrians](#)
 - R. J. Barlow: **Statistics**, WILEY, 1997

Overall view of the LHC experiments.

B-hadron physics

Heavy ions

Multi-purpose



Physics program at ATLAS

**B-Physics and
light states**

**Standard
Model physics**

**Top quark
physics**

**Higgs boson
physics**

**Heavy Ion
Physics**

**Exotics
Searches**

**Extended
Higgs sector
searches**

**Supersymmetry
searches**



Physics program at CMS

**B-Physics and
Quarkonia**

**Standard
Model physics**

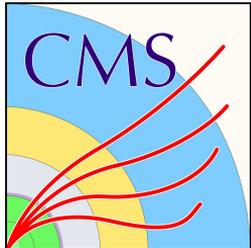
**Top quark
physics**

**Higgs boson
physics**

**Heavy Ion
Physics**

**Exotics
Searches**

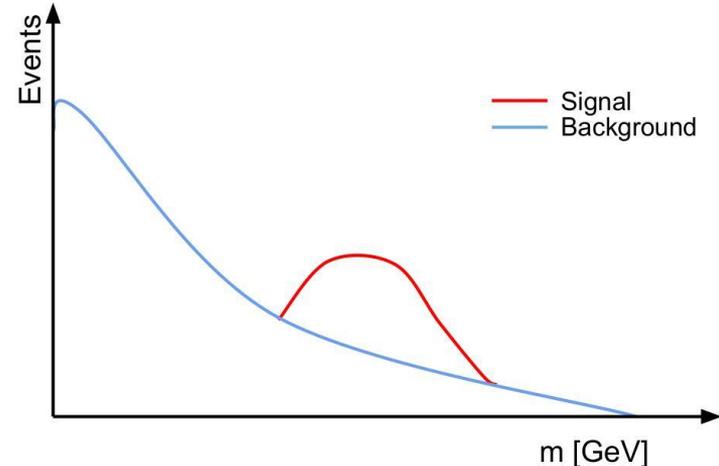
**Supersymmetry
searches**



Public web page listing all recent results: <http://cms-results.web.cern.ch/cms-results/public-results/publications/>

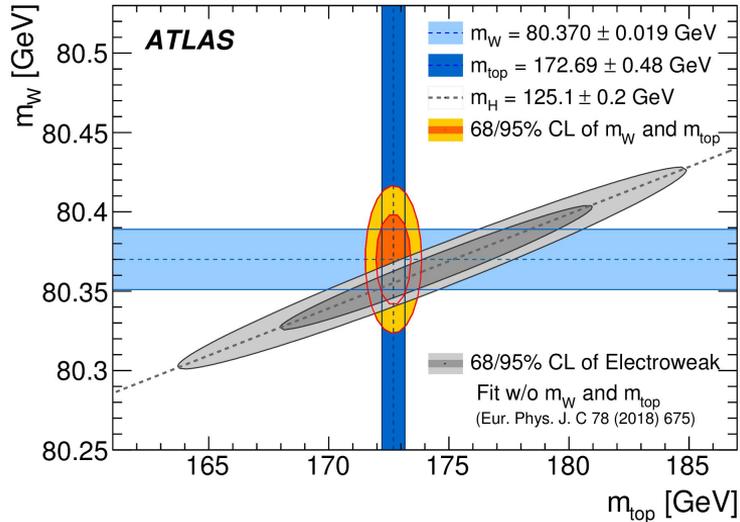
Measurements vs. Searches

- **Measurements:** Study of properties of already know particles
 - Long-term efforts involving in-depth examinations of all relevant systematic uncertainties
- **Searches:** Probe for excess (or peak from resonant production) in data due to contribution from new particle
 - Often performed in “extreme” phase space regions:
 - At high p_T
 - Large (b-)jet multiplicities
 - Large amount of missing transverse momentum
 - Multi-lepton final states



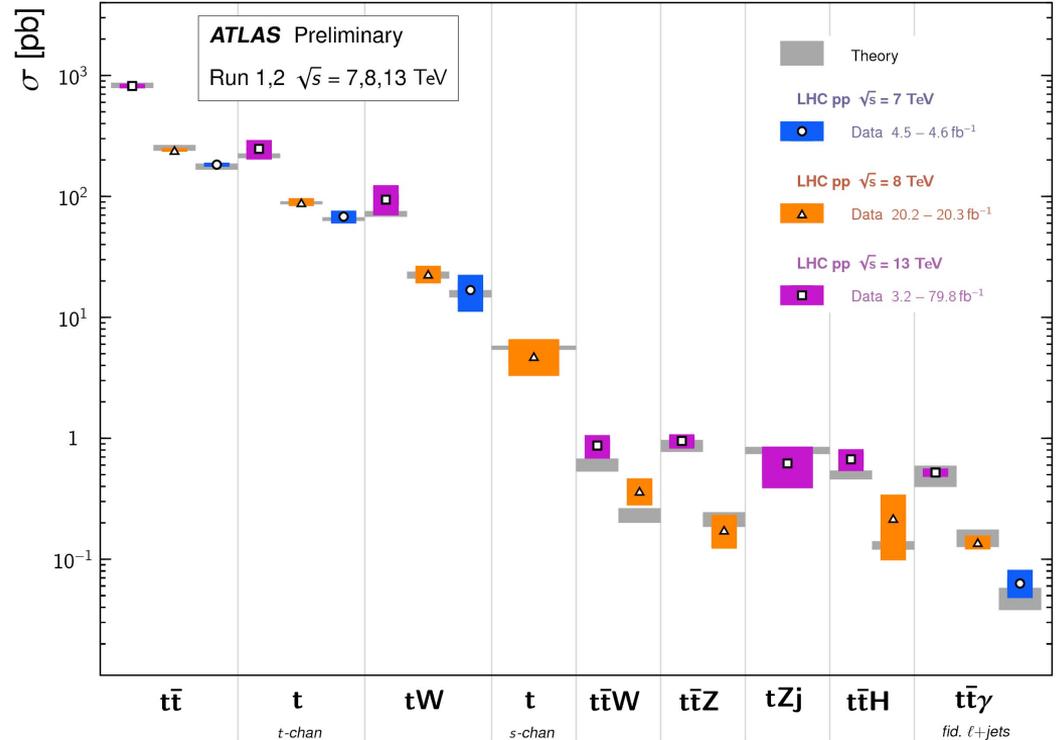
Measurements

- Aim to determine:
 - Cross sections
 - Couplings
 - Masses
 - Spin/CP
 - Polarisation

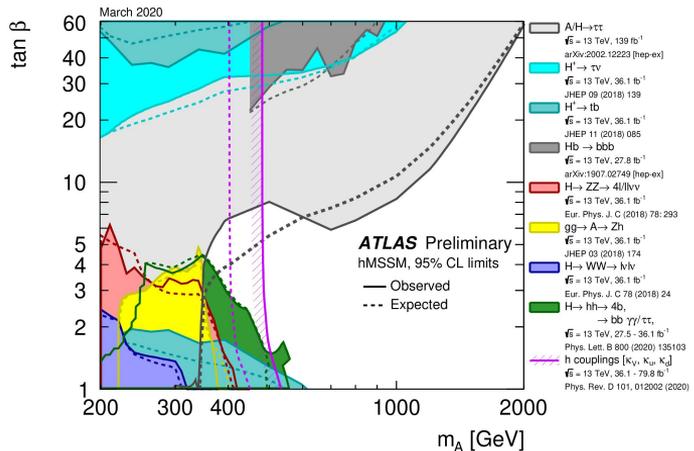
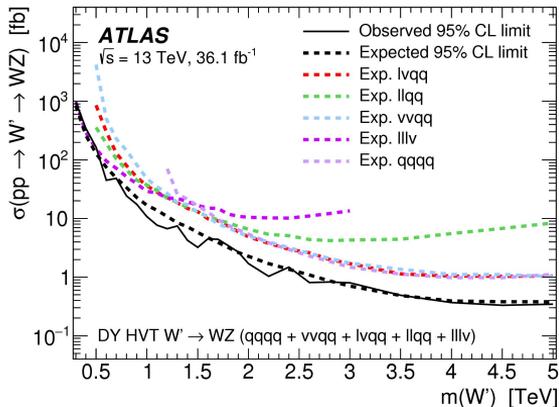


Top Quark Production Cross Section Measurements

Status: November 2018



Searches



ATLAS Exotics Searches* - 95% CL Upper Exclusion Limits

Status: May 2019

ATLAS Preliminary

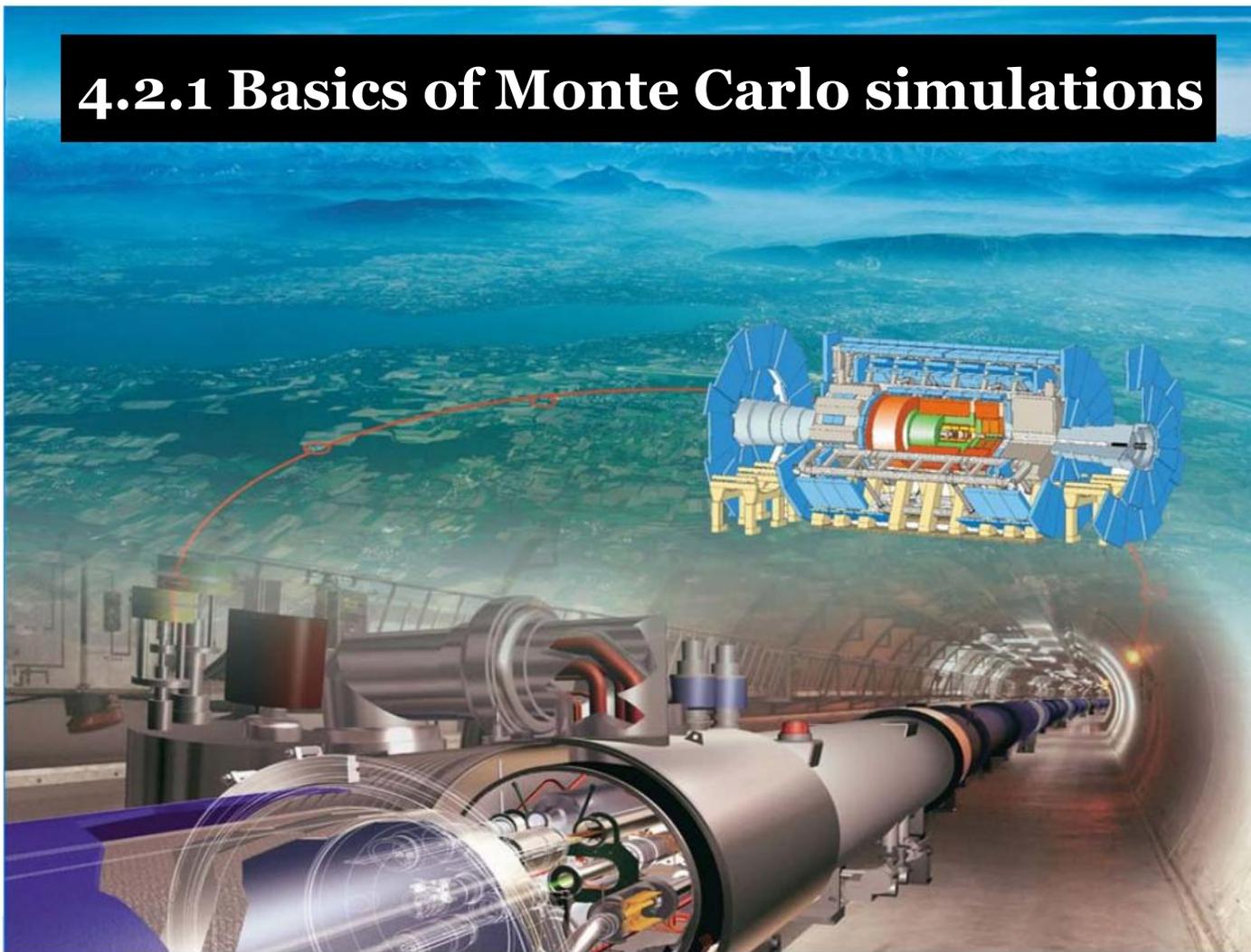
$\int \mathcal{L} dt = (3.2 - 139) \text{ fb}^{-1}$ $\sqrt{s} = 8, 13 \text{ TeV}$

Model	l, γ	Jets [†]	$E_{T, \text{miss}}$	$\int \mathcal{L} dt [\text{fb}^{-1}]$	Limit	Reference	
Extra dimensions	ADD $G_{KK} + g/g$	$0 e, \mu$	1-4 J	Yes	36.1	M_0 7.7 TeV $n = 2$	1711.03301
	ADD non-resonant $\gamma\gamma$	2γ	-	-	36.7	M_5 8.6 TeV $n = 3 \text{ HLZNL0}$	1707.04147
	ADD QBH	-	$\geq 2 j$	-	37.0	M_{th} 8.9 TeV $n = 6$	1703.09127
	ADD BH high Σp_T	$\geq 1 e, \mu$	$\geq 2 j$	-	3.2	M_{th} 8.2 TeV $n = 6, M_{\text{pl}} = 3 \text{ TeV, rol BH}$	1606.02665
	ADD BH multijet	-	$\geq 3 j$	-	3.6	M_{th} 9.55 TeV $n = 6, M_{\text{pl}} = 3 \text{ TeV, rol BH}$	1512.02586
	RSI $G_{KK} \rightarrow \gamma\gamma$	2γ	-	-	36.7	$G_{KK} \text{ mass}$ 4.1 TeV $k/\bar{M}_{\text{pl}} = 0.1$	1707.04147
	Bulk RS $G_{KK} \rightarrow WW/ZZ$	multi-channel	-	-	36.1	$G_{KK} \text{ mass}$ 2.3 TeV $k/\bar{M}_{\text{pl}} = 1.0$	1808.02380
	Bulk RS $G_{KK} \rightarrow WW \rightarrow qqqq$	$0 e, \mu$	2 J	-	139	$G_{KK} \text{ mass}$ 1.6 TeV $\Gamma/m = 15\%$	ATLAS-CONF-2019-003
	Bulk RS $g_{KK} \rightarrow tt$	$1 e, \mu$	$\geq 1 b, \geq 1 J/2$	Yes	36.1	$g_{KK} \text{ mass}$ 3.8 TeV $\Gamma/m = 15\%$	1804.10823
	ZUED/RPP	$1 e, \mu$	$\geq 2 b, \geq 3 j$	Yes	36.1	$KK \text{ mass}$ 1.8 TeV Tier (1,1), $\mathcal{R}(A^{(1,1)} \rightarrow tt) = 1$	1803.09678
Gauge bosons	SSM $Z' \rightarrow ll$	$2 e, \mu$	-	-	139	$Z' \text{ mass}$ 5.1 TeV $\Gamma/m = 1\%$	1903.06248
	SSM $Z' \rightarrow \tau\tau$	2τ	-	-	36.1	$Z' \text{ mass}$ 2.42 TeV	1709.07242
	Leptophobic $Z' \rightarrow bb$	-	2 b	-	36.1	$Z' \text{ mass}$ 2.1 TeV	1805.09299
	Leptophobic $Z' \rightarrow tt$	$1 e, \mu$	$\geq 1 b, \geq 1 J/2$	Yes	36.1	$Z' \text{ mass}$ 3.0 TeV	1804.10823
	SSM $W' \rightarrow ll$	$1 e, \mu$	-	Yes	139	$W' \text{ mass}$ 6.0 TeV	1801.06992
	SSM $W' \rightarrow \tau\nu$	1τ	-	Yes	139	$W' \text{ mass}$ 3.7 TeV	CERN-EP-2019-100
	HVT $V' \rightarrow WZ \rightarrow qqqq$ model B	$0 e, \mu$	2 J	-	139	$V' \text{ mass}$ 3.6 TeV $g_V = 3$	ATLAS-CONF-2019-003
	HVT $V' \rightarrow WH/ZH$ model B	multi-channel	-	-	36.1	$V' \text{ mass}$ 2.93 TeV $g_V = 3$	1712.06518
	LRSM $W_g \rightarrow tb$	multi-channel	-	-	36.1	$W_g \text{ mass}$ 3.25 TeV $m(N_g) = 0.5 \text{ TeV, } g_t = g_b$	1807.10473
	LRSM $W_g \rightarrow \mu N_k$	2μ	1 J	-	80	$W_g \text{ mass}$ 5.0 TeV	1904.12679
CI	CI $q\bar{q}q\bar{q}$	-	2 j	-	37.0	A 21.8 TeV η_{LL}	1703.09127
	CI $l\bar{l}q\bar{q}$	$2 e, \mu$	-	-	36.1	A 40.0 TeV η_{LL}	1707.02424
	CI $t\bar{t}t\bar{t}$	$\geq 1 e, \mu$	$\geq 1 b, \geq 1 j$	Yes	36.1	A 2.57 TeV $ C_{tt} = 4\epsilon$	1811.02305
DM	Axial-vector mediator (Dirac DM)	$0 e, \mu$	1-4 J	Yes	36.1	m_{med} 1.55 TeV $g_e = 0.25, g_b = 1.0, m(\chi) = 1 \text{ GeV}$	1711.03301
	Colored scalar mediator (Dirac DM)	$0 e, \mu$	1-4 J	Yes	36.1	m_{med} 1.67 TeV $g = 1.0, m(\chi) = 1 \text{ GeV}$	1711.03301
	VV_{1X} EFT (Dirac DM)	$0 e, \mu$	1 J, $\leq 1 j$	Yes	3.2	M, $m_{\text{pl}}(\chi) = 150 \text{ GeV}$	1608.02372
Scalar reson. $\phi \rightarrow t\bar{t}$ (Dirac DM)	$0-1 e, \mu$	1 b, 0-1 J	Yes	36.1	m_{pl} 3.4 TeV $y = 0.4, \lambda = 0.2, m(\chi) = 10 \text{ GeV}$	1812.09743	
LQ	Scalar LQ 1 st gen	$1, 2 e$	$\geq 2 j$	Yes	36.1	$LQ \text{ mass}$ 1.4 TeV $\beta = 1$	1902.00377
	Scalar LQ 2 nd gen	$1, 2 \mu$	$\geq 2 j$	Yes	36.1	$LQ \text{ mass}$ 1.56 TeV $\beta = 1$	1902.00377
	Scalar LQ 3 rd gen	2τ	2 b	Yes	36.1	$LQ \text{ mass}$ 1.03 TeV $\mathcal{R}(LQ \rightarrow b\tau) = 1$	1902.08103
	Scalar LQ 3 rd gen	$0-1 e, \mu$	2 b	Yes	36.1	$LQ \text{ mass}$ 970 GeV $\mathcal{R}(LQ \rightarrow \tau\nu) = 0$	1902.08103
	Heavy quarks	VLO $TT \rightarrow H_t/Z_t/W_b + X$	multi-channel	-	-	36.1	T mass 1.37 TeV SU(2) doublet
VLO $BB \rightarrow W_t/Z_b + X$		multi-channel	-	-	36.1	B mass 1.34 TeV SU(2) doublet	1808.02343
VLO $T_{3/2} T_{3/2} \rightarrow Wt + X$		$2(SS) \geq 3 e, \mu$	$\geq 1 b, \geq 1 j$	Yes	36.1	$T_{3/2} \text{ mass}$ 1.64 TeV $\mathcal{R}(T_{3/2} \rightarrow Wt) = 1, c(T_{3/2} Wt) = 1$	1807.11863
VLO $Y \rightarrow Wb + X$		$1 e, \mu$	$\geq 1 b, \geq 1 j$	Yes	36.1	Y mass 1.85 TeV $\mathcal{R}(Y \rightarrow Wb) = 1, c_Y(Wb) = 1$	1812.07343
VLO $B \rightarrow Hb + X$		$0 e, \mu, 2 \gamma$	$\geq 1 b, \geq 1 j$	Yes	79.8	B mass 1.21 TeV $\kappa_B = 0.5$	ATLAS-CONF-2018-024
VLO $QQ \rightarrow WbWq$	$1 e, \mu$	$\geq 4 j$	Yes	20.3	Q mass 690 GeV	1509.04261	
Excited fermions	Excited quark $q^* \rightarrow qg$	-	2 j	-	139	$q^* \text{ mass}$ 6.7 TeV	ATLAS-CONF-2019-007
	Excited quark $q^* \rightarrow q\gamma$	1γ	1 j	-	36.7	$q^* \text{ mass}$ 5.3 TeV	1709.10440
	Excited quark $b^* \rightarrow b\gamma$	-	1 b, 1 j	-	36.1	$b^* \text{ mass}$ 2.6 TeV	1805.09299
	Excited lepton l^*	$3 e, \mu$	-	-	20.3	$l^* \text{ mass}$ 3.0 TeV $\Lambda = 3.0 \text{ TeV}$	1411.2921
	Excited lepton ν^*	$3 e, \mu, \tau$	-	-	20.3	$\nu^* \text{ mass}$ 1.6 TeV $\Lambda = 1.6 \text{ TeV}$	1411.2921
Other	Type III Seesaw	$1 e, \mu$	$\geq 2 j$	Yes	79.8	$N^0 \text{ mass}$ 560 GeV $m(W_2) = 4.1 \text{ TeV, } g_t = g_b$	ATLAS-CONF-2018-020
	LRSM Majorana ν	2μ	2 j	-	36.1	$N_{\mu} \text{ mass}$ 3.2 TeV	1809.11105
	Higgs triplet $H^{\pm\pm} \rightarrow ll$	$2, 3, 4 e, \mu$ (SS)	-	-	36.1	$H^{\pm\pm} \text{ mass}$ 870 GeV DY production	1710.09748
	Higgs triplet $H^{\pm\pm} \rightarrow \tau\tau$	$3 e, \mu, \tau$	-	-	20.3	$H^{\pm\pm} \text{ mass}$ 400 GeV DY production, $\mathcal{R}(H^{\pm\pm} \rightarrow \tau\tau) = 1$	1411.2921
	Multi-charged particles	-	-	-	36.1	multi-charged particle mass 1.22 TeV DY production, $ q = 5e$	1812.03673
Magnetic monopoles	-	-	-	34.4	monopole mass 2.37 TeV DY production, $ g = 1g_D, \text{spin } 1/2$	1905.10130	

*Only a selection of the available mass limits on new states or phenomena is shown.

[†]Small-radius (large-radius) jets are denoted by the letter j (J).

4.2.1 Basics of Monte Carlo simulations



Introduction

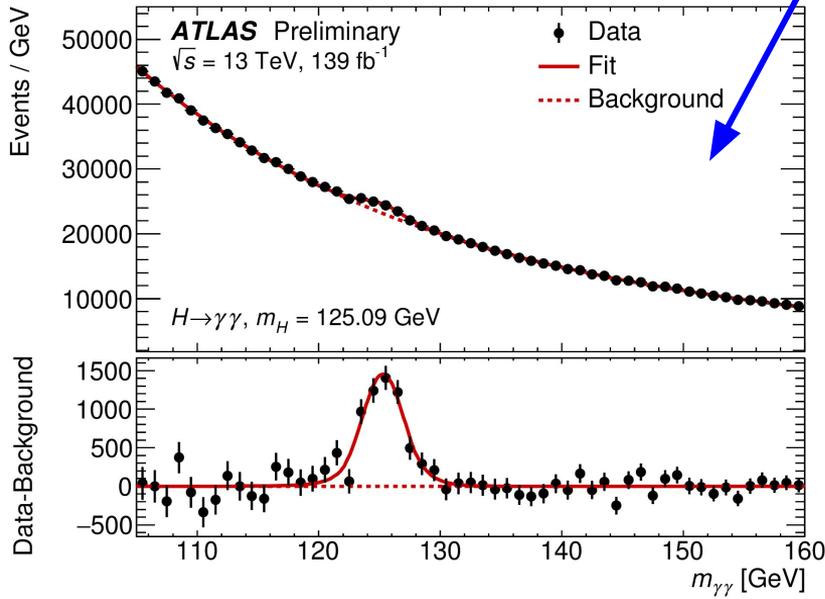
- Hadron colliders are discovery machines.
 - **Advantage:**
 - Proton beams can be accelerated to higher kinetic energies than electron beams
 - **Disadvantage:**
 - The event structure at hadron colliders is significantly more complex than at lepton colliders (due to the composite nature of the beam particles)
- **The description of full final states (from parton collisions) necessitates involved multi-particle calculations**
 - The high-dimensional phase space leaves Monte-Carlo integration as the only viable option.

→ **Monte Carlo Method**

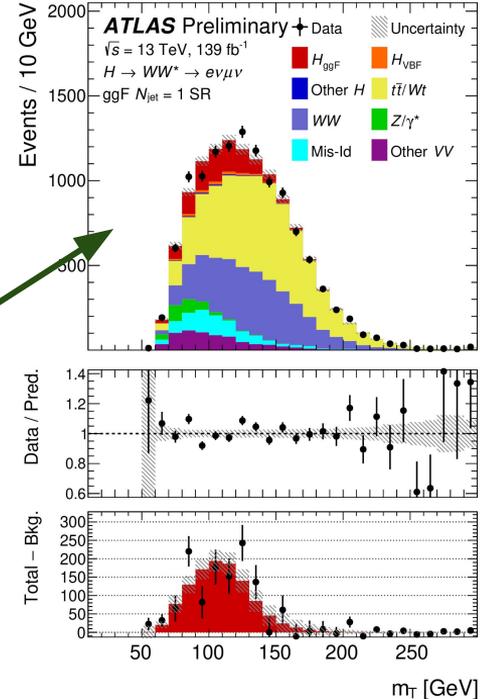
Monte Carlo simulations

- Almost all data analyses at the LHC experiments rely (at least to some extent) on Monte Carlo (MC) simulations

Validation of signal and/or background estimations



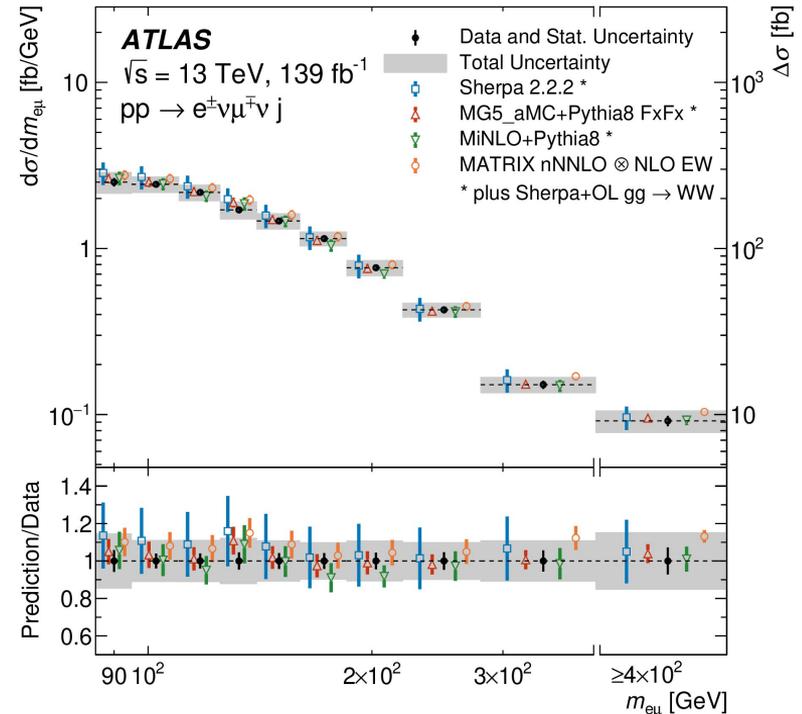
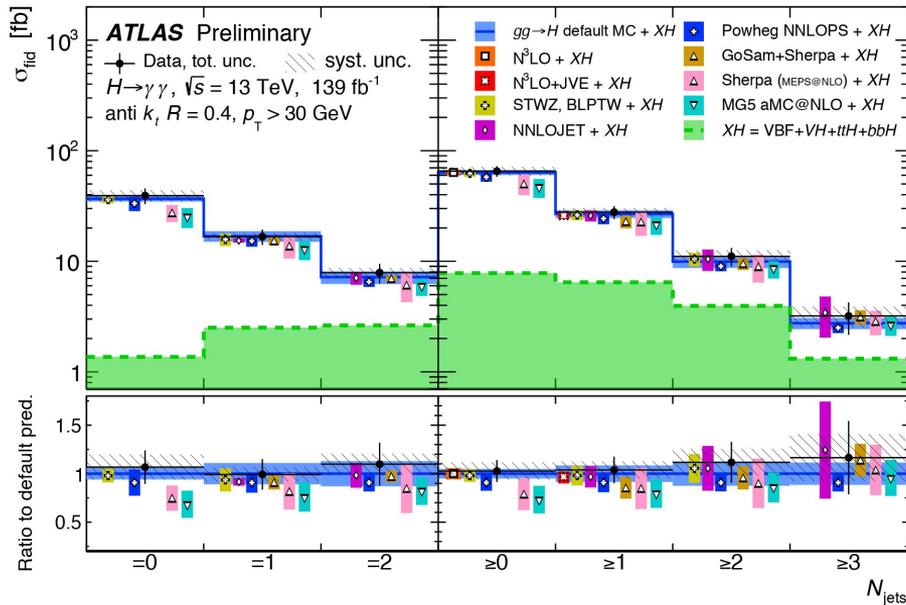
Estimation of contributions from signal and background processes



Monte Carlo simulations

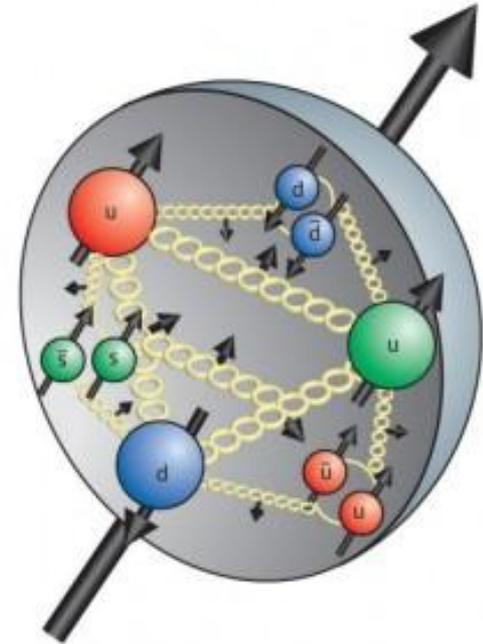
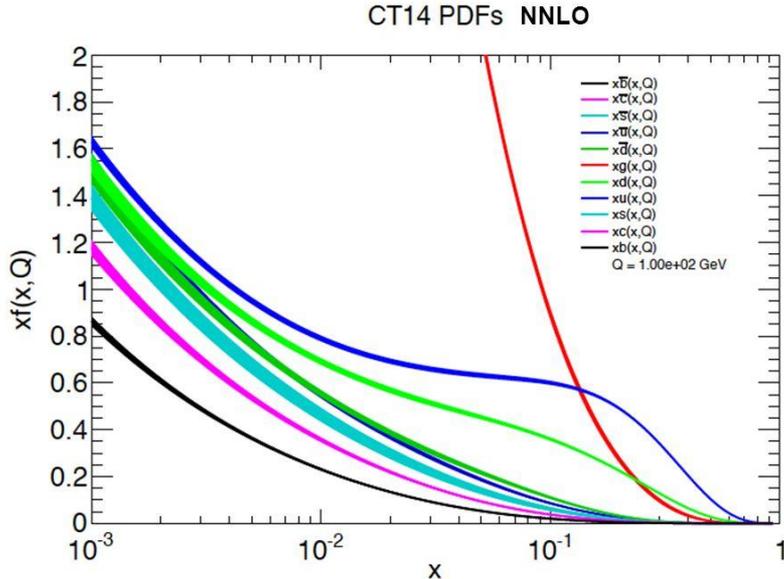
- Need good description of SM processes
 - To be sensitive to new physics

→ Tune Monte Carlo models



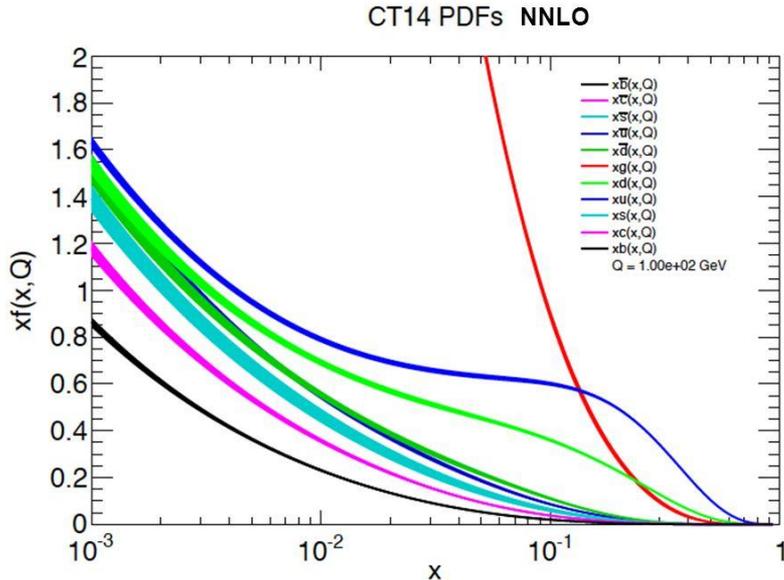
Monte Carlo simulations

- Observations in data are compared to SM predictions (Monte Carlo simulations)
- **Use factorisation approach:**
 - Parton distribution functions (PDF)
 - Hard process (matrix element/scattering amplitude)
 - Parton shower (fragmentation, hadronization, decay of unstable particles)
 - Detector simulation (including overlay with pile-up)



Monte Carlo simulations

- Observations in data are compared to SM predictions (Monte Carlo simulations)
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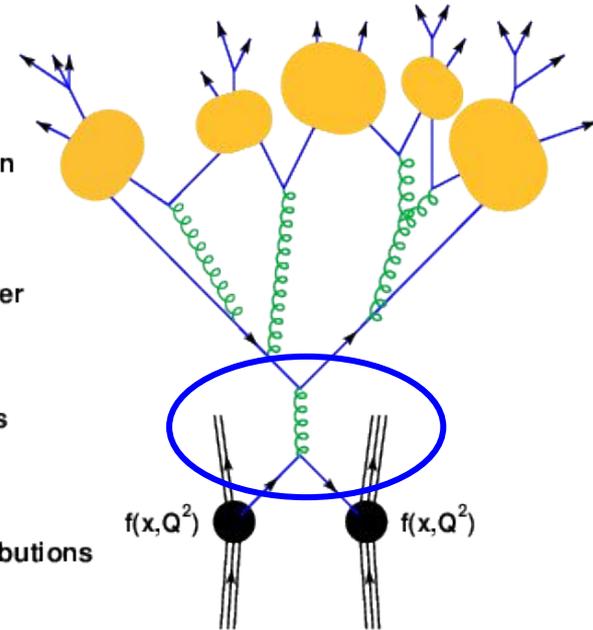
Decay of
unstable
particles

Hadronization

Parton Shower

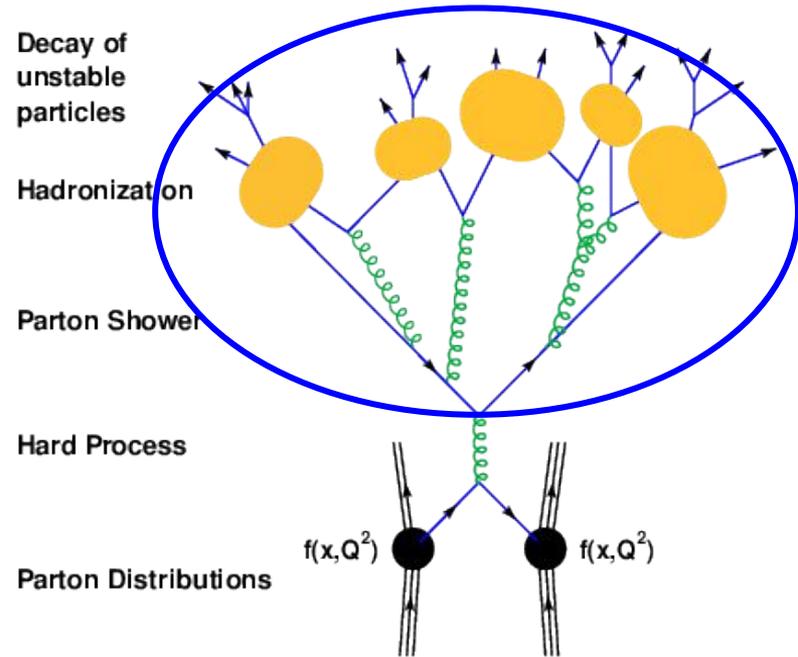
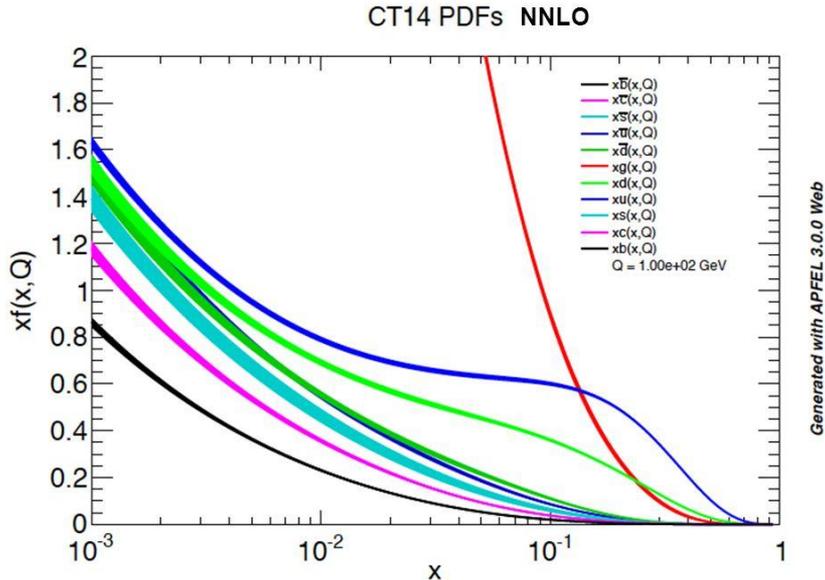
Hard Process

Parton Distributions



Monte Carlo simulations

- Observations in data are compared to SM predictions (Monte Carlo simulations)
- **Use factorisation approach:**
 - Parton distribution functions (PDF)
 - Hard process (matrix element/scattering amplitude)
 - **Parton shower (fragmentation, hadronization, decay of unstable particles)**
 - Detector simulation (including overlay with pile-up)



Monte Carlo simulations

- **Initial state radiation:**

- Radiative emissions from the initial state partons

- **Final state radiation:**

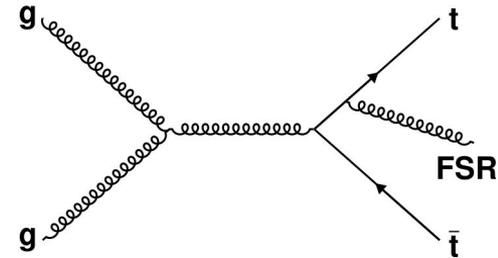
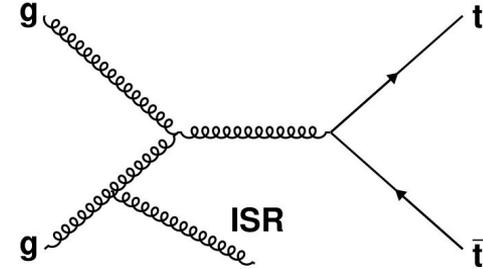
- Radiative emissions from the final state partons

- **Multiparton interactions (MPI) / underlying event (UE):**

- Particle production not associated with the leading hardest parton-parton interaction (same protons)

- **Pile-up:**

- Particle production not associated with the leading hardest parton-parton interaction (different protons)



Monte Carlo simulations

$\mu \approx \lambda_{QCD}$

Decay of unstable particles

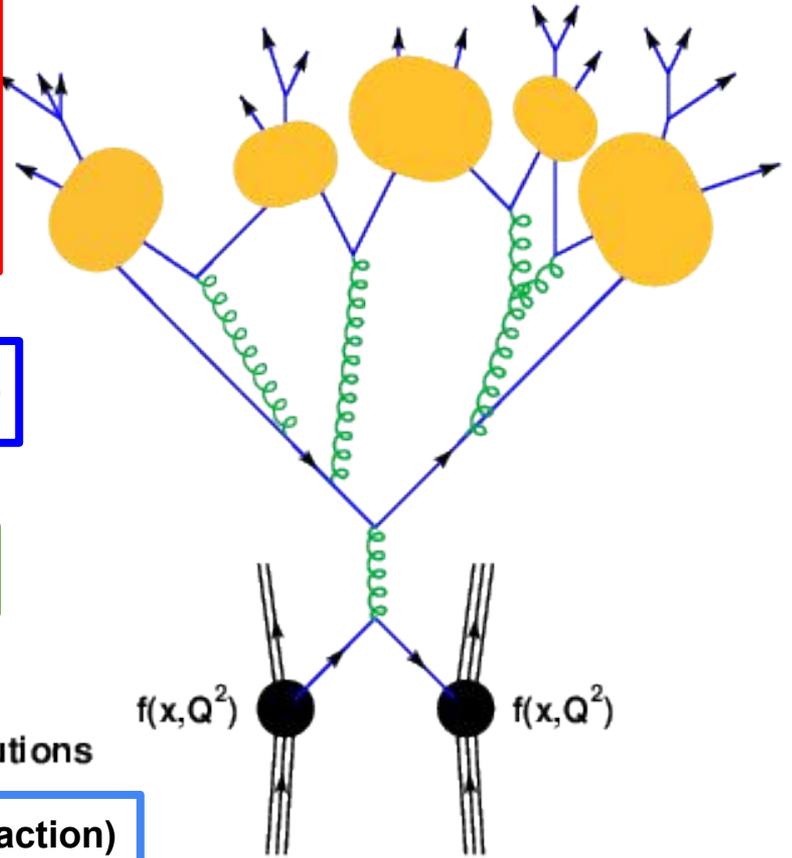
Hadronization

$\lambda_{QCD} < \mu < Q$ Parton Shower

$\lambda_{QCD} \ll \mu \approx Q$ Hard Process

Parton Distributions

(Multiparton Interaction)



Monte Carlo simulations

Parton distribution functions (PDFs)



Determined from data (as PDF sets)

Hard scattering

$$\lambda_{QCD} \ll \mu \approx Q$$



Hard process computed using fixed order (FO) or mixed order perturbation theory at LO, NLO, NNLO, N3LO,...

Parton shower

$$\lambda_{QCD} < \mu < Q$$



Hierarchy of scales appearing as large logarithms in the calculation. Computed using resummation or MC parton showers (PS)

Hadronization and hadron decay

$$\mu \approx \lambda_{QCD}$$



Cannot be computed directly: non-perturbative models tuned to e^+e^- data

Multiparton interaction



Monte Carlo simulations

Parton distribution functions (PDFs)

Determined from data (as PDF sets)

Hard scattering

$$\lambda_{QCD} \ll \mu \approx Q$$

Hard process computed using fixed order (FO) or mixed order perturbation theory at LO, NLO, NNLO, N3LO,...

+ ME and PS matching (different order \rightarrow different schemes)

Parton shower

$$\lambda_{QCD} < \mu < Q$$

Hierarchy of scales appearing as large logarithms in the calculation. Computed using resummation or MC parton showers (PS)

Hadronization and hadron decay

$$\mu \approx \lambda_{QCD}$$

Cannot be computed directly: non-perturbative models tuned to e^+e^- data

Multiparton interaction

Master formula for hadron collisions

$$\sigma_{h_1+h_2 \rightarrow X} = \sum_{a,b} \int \underbrace{dx_1 dx_2 d\Phi}_{\text{phase - space integral}} \underbrace{f_a(x_1, \mu_F) f_b(x_2, \mu_F)}_{\text{Parton distribution function}} \underbrace{\hat{\sigma}_{a+b \rightarrow X}(\hat{S}, \mu_F, \mu_R)}_{\text{Parton-level cross section}}$$

- The parton-level fixed order cross section can be computed as a series in perturbation theory, using the coupling constant as an expansion parameter

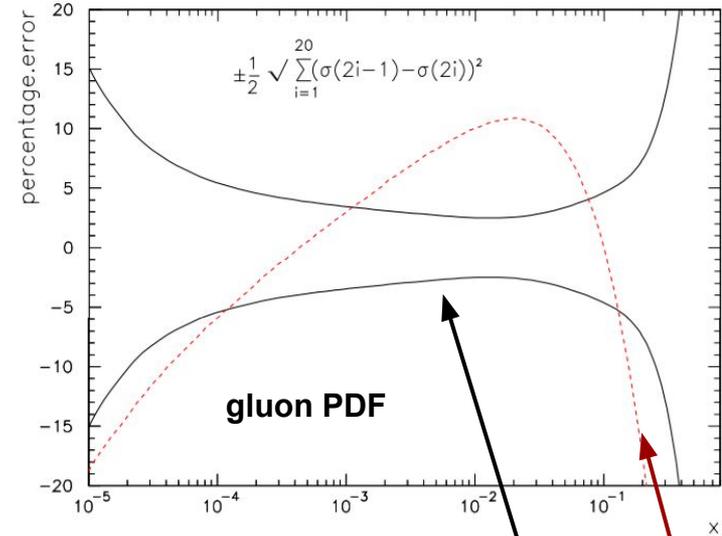
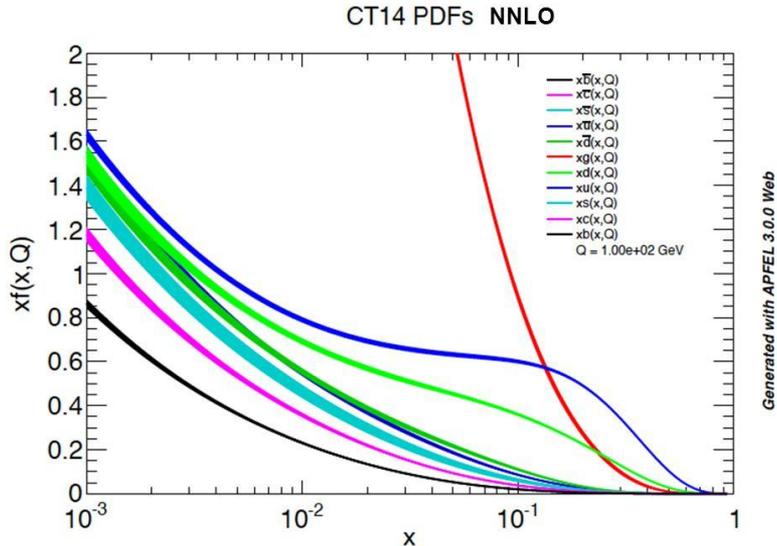
$$\hat{\sigma} = \sigma^{\text{Born}} \left(1 + \frac{\alpha_S}{2\pi} \sigma^{(1)} + \left(\frac{\alpha_S}{2\pi} \right)^2 \sigma^{(2)} + \left(\frac{\alpha_S}{2\pi} \right)^3 \sigma^{(3)} + \dots \right)$$

Coefficients of the perturbative series

- Including higher corrections improves the predictions and reduces theoretical uncertainties

Particle distribution functions (PDFs)

- Quantify the probability density for finding a parton with a certain flavour and momentum fraction
- Obtained from fits to data
- Crucial source of uncertainties for both searches and measurements



Relative variations of the MSTW set

Variation of CTEQ6 set wrt central values of the MSTW set

Taken from <https://arxiv.org/pdf/1207.2389v4.pdf>

Particle distribution functions (PDFs)

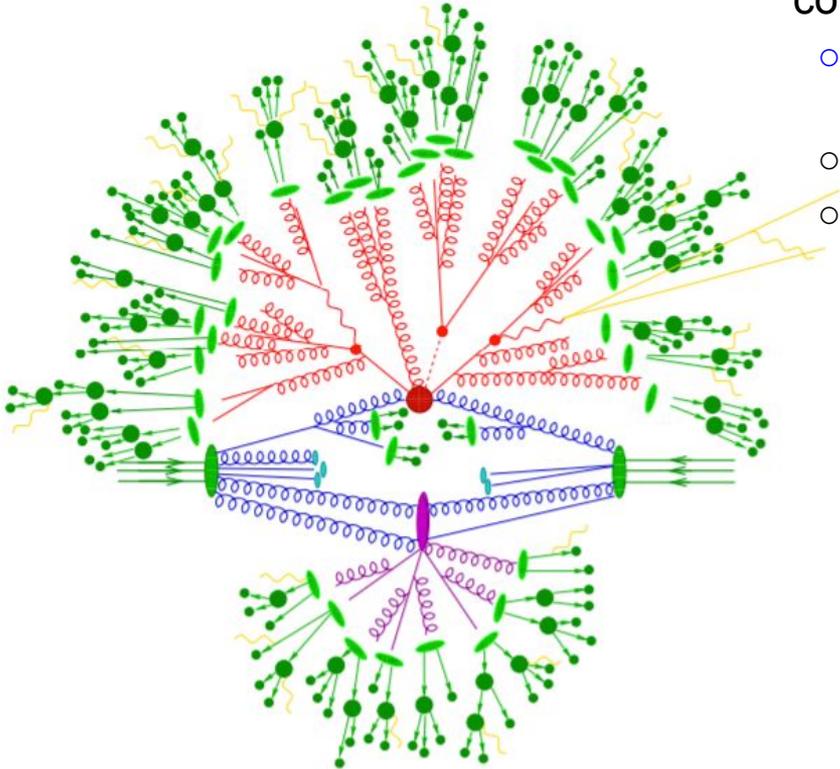
- PDF sets can be downloaded from **LHAPDF** page (<https://lhapdf.hepforge.org/pdfsets>):

LHAPDF ID	Set name	Number of set members
251	GRVPI0	1
252	GRVPI1	1
270	xFitterPI_NLO_EIG	8
280	xFitterPI_NLO_VAR	6
10000	cteq6	41
10042	cteq6l1	1
10150	cteq61	41
10550	cteq66	45
10770	CT09MCS	1
10771	CT09MC1	1
10772	CT09MC2	1
10800	CT10	53

260000	NNPDF30_nlo_as_0118	101
260200	NNPDF30_nlo_as_0118_nf_3	101
260400	NNPDF30_nlo_as_0118_nf_4	101
260600	NNPDF30_nlo_as_0118_nf_6	101
260800	NNPDF30_nlo_as_0118_mc	101
261000	NNPDF30_nnlo_as_0118	101
261200	NNPDF30_nnlo_as_0118_nf_3	101
261400	NNPDF30_nnlo_as_0118_nf_4	101
261600	NNPDF30_nnlo_as_0118_nf_6	101
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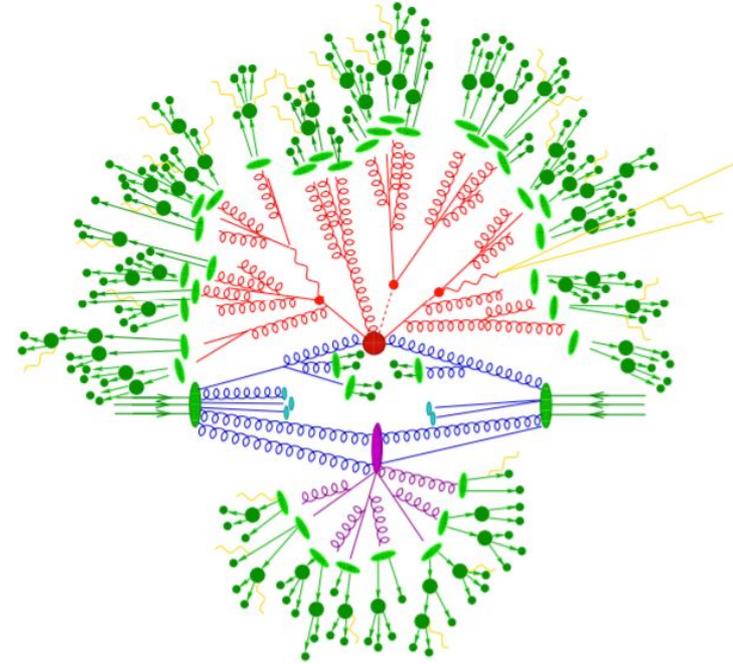
Parton shower

- Parton showers approximate higher-order real-emission corrections to the hard scattering
 - Simulation of the branching of a single external parton into two partons.
 - Local conservation of flavor and four momenta
 - Respect unitarity (i.e. a parton may either split into two partons, or not).



Hadronisation/Fragmentation

- To complete the simulation of realistic event topologies:
 - Quarks and gluons from the hard scattering simulations, parton showers and multiple scattering simulations must be transformed into **color-neutral final states**.
- Two different models are used nowadays:
 - **String model** (Lund model)
 - **Cluster model**



Monte Carlo generators

- **The Monte Carlo Method:**
 - Monte Carlo (MC) techniques are based on a repeated random sampling of numerical estimations of variables following complicated probability density functions
 - Based on the implementation of (B)SM predictions
- **Monte Carlo Event Generators** try to give the **best full description** (according our current knowledge) of a collision combining theoretical predictions for the **different stages** of an event and providing a **fully exclusive final state in terms of hadrons and leptons** which is as close as possible to what is measured in a real experiment
- Predictions are usually fed into a detector-simulation software to emulate the reconstruction effects of our real world detectors

Some Monte Carlo generators

- MadGraph_aMC@NLO (<https://launchpad.net/mg5amcnlo>):
 - Tool for calculation of cross sections for SM and BSM processes and event generation (LO or NLO)
- POWHEG (<http://powhegbox.mib.infn.it/>):
 - MC generator for hard processes at NLO
- Sherpa (<https://sherpa-team.gitlab.io/>):
 - MC event generator for the simulation of $\ell\ell$, $\ell\gamma$, $\gamma\gamma$, ℓh and hh collisions
- Pythia (<http://home.thep.lu.se/Pythia/>):
 - *Multi-purpose MC generator (for event generation and/or parton shower)*
 - Supports Lund string fragmentation model
- ALPGEN (<https://arxiv.org/pdf/hep-ph/0206293.pdf>):
 - MC generator for hard multiparton processes in hadronic collisions
- HERWIG (<https://herwig.hepforge.org/>):
 - *Multi-purpose MC generator (for event generation and/or parton shower)*
 - Supports angular-ordered and dipole showers as well as MPI
- MCFM (<https://mcfm.fnal.gov/>):
 - Tool dedicated to calculate cross sections of various processes at NLO (and NNLO) in QCD
 - Can also be used as event generator for some of these processes
- JHU (<https://spin.pha.jhu.edu/>):
 - Event generator for $pp \rightarrow X \rightarrow VV$, VBF , $X+JJ$, $pp \rightarrow VX$, $ee \rightarrow VX$

Detector simulations

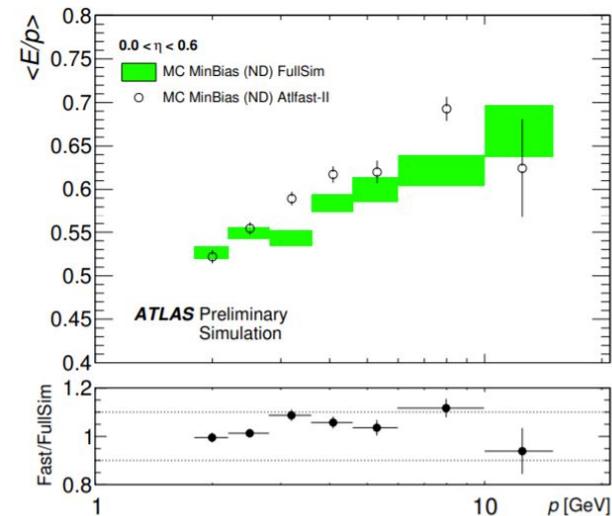
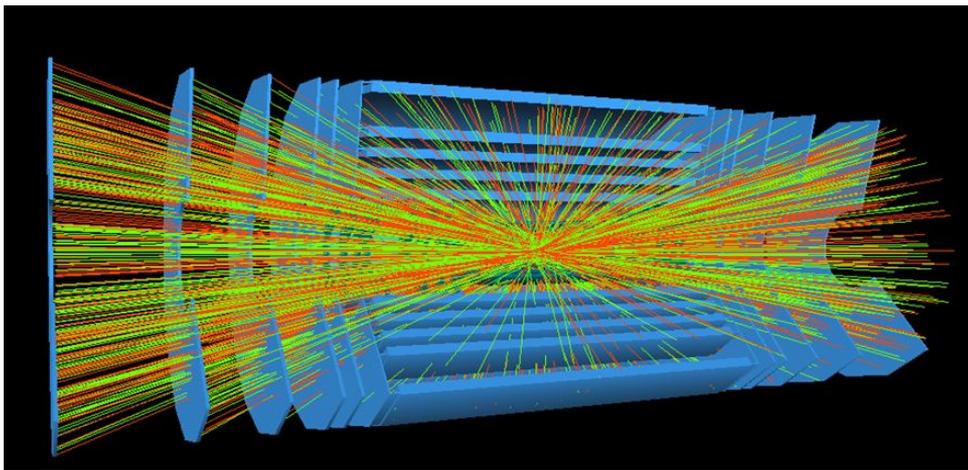
- Two different approaches:

Taken from: <https://cds.cern.ch/record/1300517/files/ATL-PHYS-PUB-2010-013.pdf>

1) Full simulation: Includes a detailed description of the detector geometry and of the simulation of particle interactions in the detector material with **GEANT4**

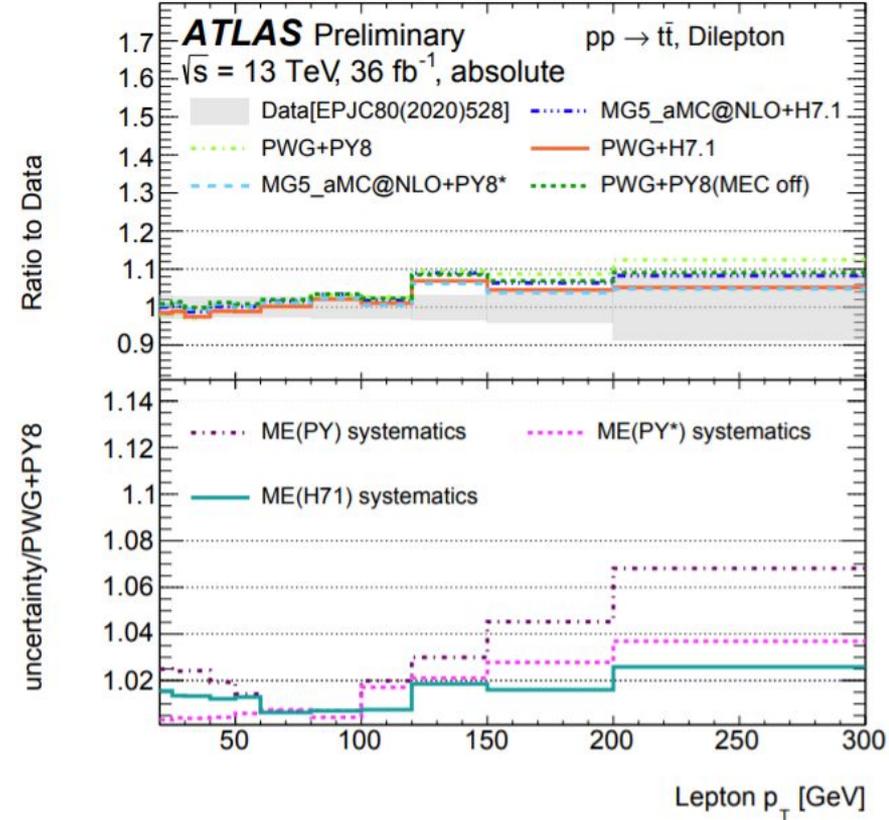
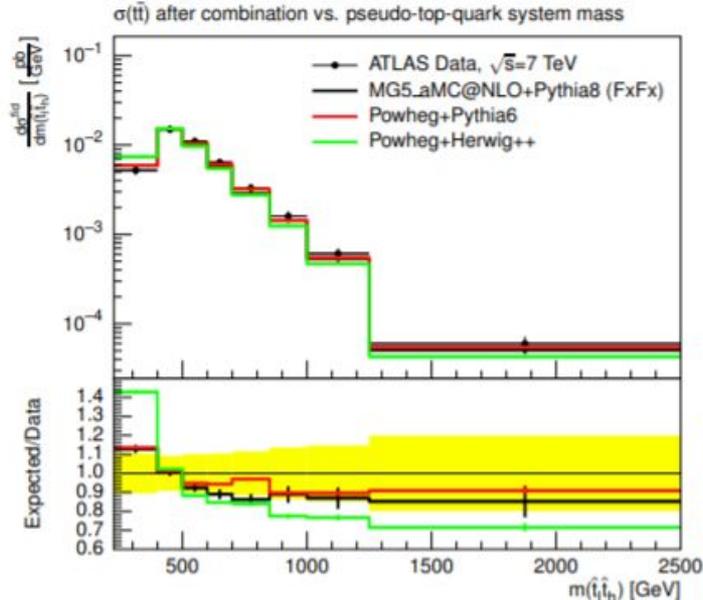
- The drawback of such a detailed simulation is a CPU time requirement of several minutes per event, of which more than 90% is spent inside the calorimeter systems.

2) Approximation (or better parameterized simulation) of the particle energy response and of the energy distribution in within the calorimeter system of (ATLAS, CMS,...)



Tuning of Monte Carlo simulations

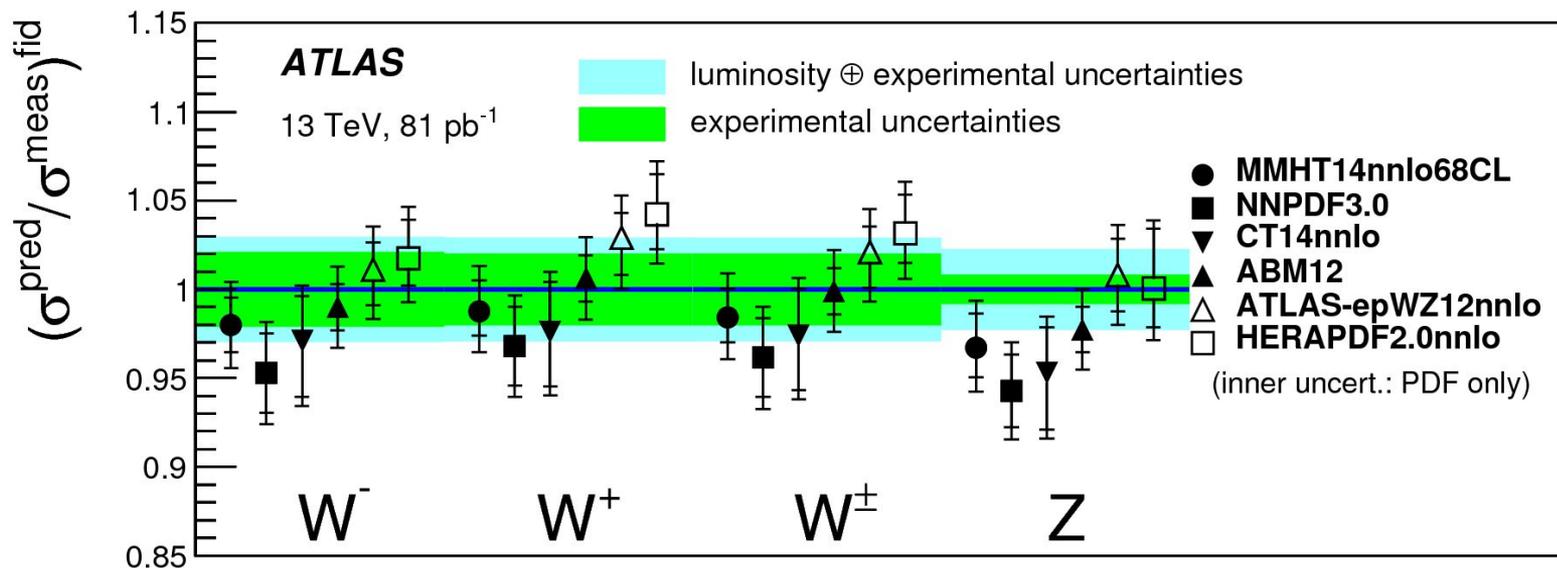
- Differential measurements are used:
 - To Compare different MC generator setups
 - To tune Monte Carlo models
 - In particular parton shower models



Tuning of Monte Carlo simulations

- Predictions from PDF sets are compared to data
 - To motivate choice of one particular set over the other

From: <https://arxiv.org/abs/1603.09222>



W & Z boson production cross section measurements are sensitive to the PDF sets
→ Can constrain them

4.2.2 Statistics for Pedestrians



Probability

- **What is probability?**

- **Frequentist approach:**

- **Definition:** If an experiment is performed N times with a specific outcome A occurring M times, then:

$$P(A) = \lim_{N \rightarrow \infty} \frac{M}{N}$$

- **Interpretation:** If the experiment is performed once more, the outcome A will occur with the probability $P(A)$.

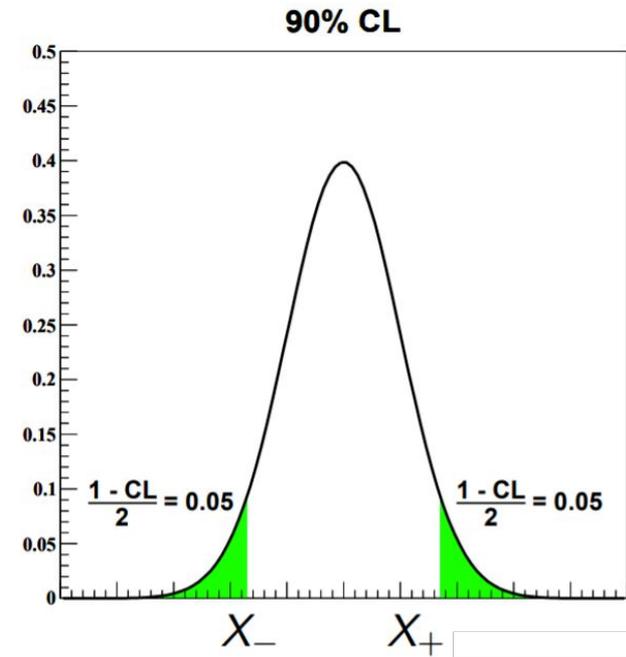
- **Bayesian approach (subjective probability):**

- **Definition of Conditional probability $P(A|B)$:** The probability of A to occur under the condition that B has occurred.

$$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}$$

Confidence Levels

- **Confidence level intervals:**
 - Given a precisely known true value μ of a certain property we can ask:
 - What is the range into which a certain amount (e.g. 90%) of measurements x_i will fall?



Example:

- Suppose cereal packets are produced according to a Gaussian distribution of mean 520g and standard deviation 10g.
 - 68% of all packets will weigh more than 510g and less than 530g
 - If we say that the weight of a packet lies in the interval 510g to 530g, we will be correct 68% of the time.
 - We can make that statement with 68% confidence

Confidence Levels

- **Central confidence intervals for a Gaussian distributions:**

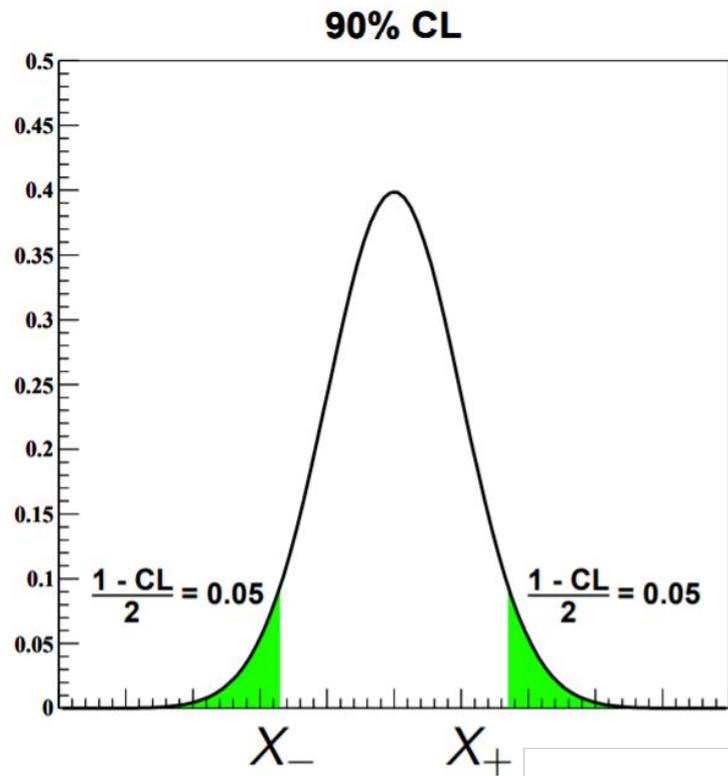
$$P(X_- \leq x \leq X_+) = \int_{X_-}^{X_+} P(x) dx = CL$$

x: measurement

X_{\pm} : limits of the confidence interval.

- **Common values:**

1σ	$\hat{=}$	68.27%	1.6449σ	$\hat{=}$	90%
2σ	$\hat{=}$	95.45%	1.9600σ	$\hat{=}$	95%
3σ	$\hat{=}$	99.73%	2.5758σ	$\hat{=}$	99%
5σ	$\hat{=}$	99.99994%			



Confidence Levels

- **Central confidence intervals for a Gaussian distributions:**

$$P(x \geq X_+) = \int_{-\infty}^{X_+} P(x)dx = CL_{\text{upper}}$$

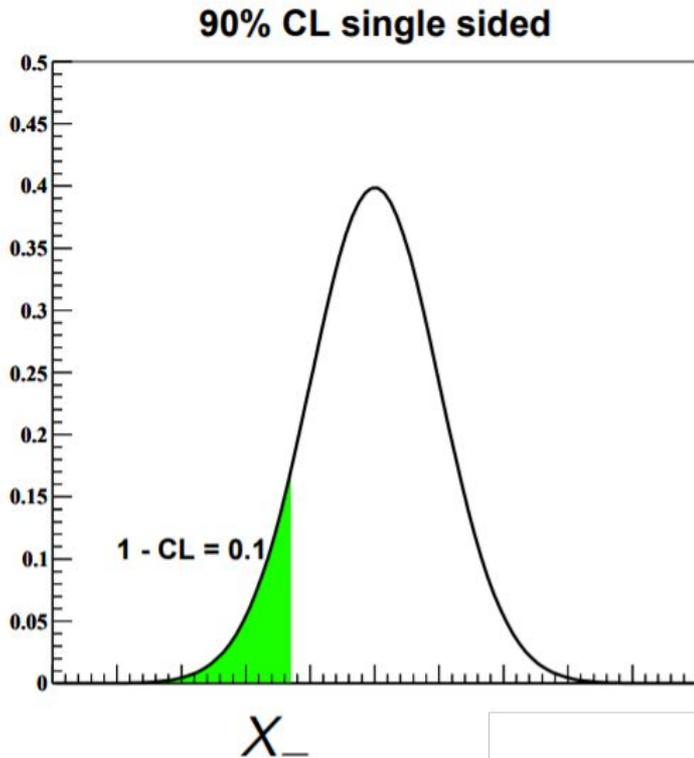
$$P(X_- \geq x) = \int_{X_-}^{\infty} P(x)dx = CL_{\text{lower}}$$

x: measurement

X_{\pm} : limits of the confidence interval.

- **Common values:**

1σ	$\hat{=}$	84.13%	1.2816σ	$\hat{=}$	90%
2σ	$\hat{=}$	97.72%	1.6449σ	$\hat{=}$	95%
3σ	$\hat{=}$	99.87%	2.3263σ	$\hat{=}$	99%
5σ	$\hat{=}$	99.99997%			



Use a **one-sided confidence interval** to obtain the tightest upper (lower) bound on a sample mean

Test statistics & p-values

- A test statistic is a quantity calculated from our sample of data. Its value can be used to estimate how probable is the result that we observe with respect to some **null hypothesis**.
 - Usually: Null hypothesis = 'background only' hypothesis
- Definition of tests statistics (simplified version in absence of uncertainties):

$$q = \frac{L(H_1)}{L(H_0)} = \frac{L(s + b)}{L(b)}$$

alternative hypothesis



via Likelihood ratio

- In a counting experiment μ_s and b would be the average number of the expected signal and background events and the Likelihoods would be derived from the data using Poisson statistics.

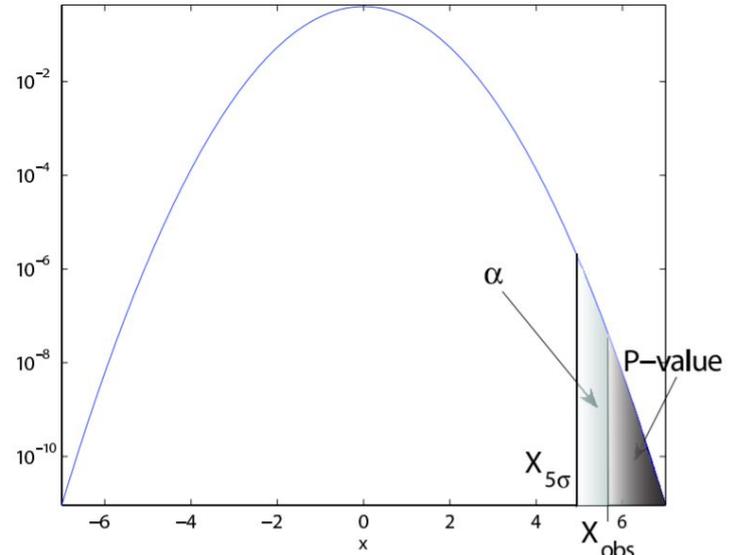
Test statistics & p-values

- Given the probability density function of the test statistic x for the background only hypothesis: $g(x | H_0)$
- If the observed result $x_{\text{obs}} > x_{5\sigma}$ then the probability to get a result which is as or less compatible with the background hypothesis is given by:

$$p = \int_{x_{\text{obs}}}^{\infty} g(x | H_0) dx$$

This probability is called the p - value

In the presence of a signal, the background-only hypothesis is rejected with a probability of $1 - p$.



Search procedure

- **The standard (frequentist) procedure to search for new physics processes follows:**
 1. Calculate a p value to test the null hypothesis that the data were generated by standard model processes alone.
 2. If $p \leq \alpha_1$ claim discovery and calculate a two-sided, 68% confidence level interval on the production cross section of the new process.
 3. If $p > \alpha_1$ calculate a 95% confidence level upper limit on the production cross section of the new process.
 - The purpose of reporting an upper limit when failing to claim a discovery is to exclude cross sections that the experiment is sensitive to and did not detect.
- Typical confidence levels are $\alpha_1 = 2.9 \times 10^{-7}$ (corresponds to 5σ)

Upper limit

- With the upper limit on a model **parameter of interest (POI)** μ , for a given observation N_{obs} we search for the largest value of μ for which the probability to make an observation of $N \leq N_{\text{obs}}$ is less than the value α .
 - In particle physics, we usually use $\alpha = 0.05$ (\rightarrow 95% CL limit)
- For a given observation find the largest value for μ where N_{obs} is still contained in the interval
- μ is usually the signal strength parameter:

$$\mu = \frac{\sigma_{\text{observed}}}{\sigma_{\text{theory}}}$$

but could also be any other model parameter

Limits on the upper production cross section at the 95% C.L.

