Testing the Standard Model of Elementary Particle Physics II

6th lecture

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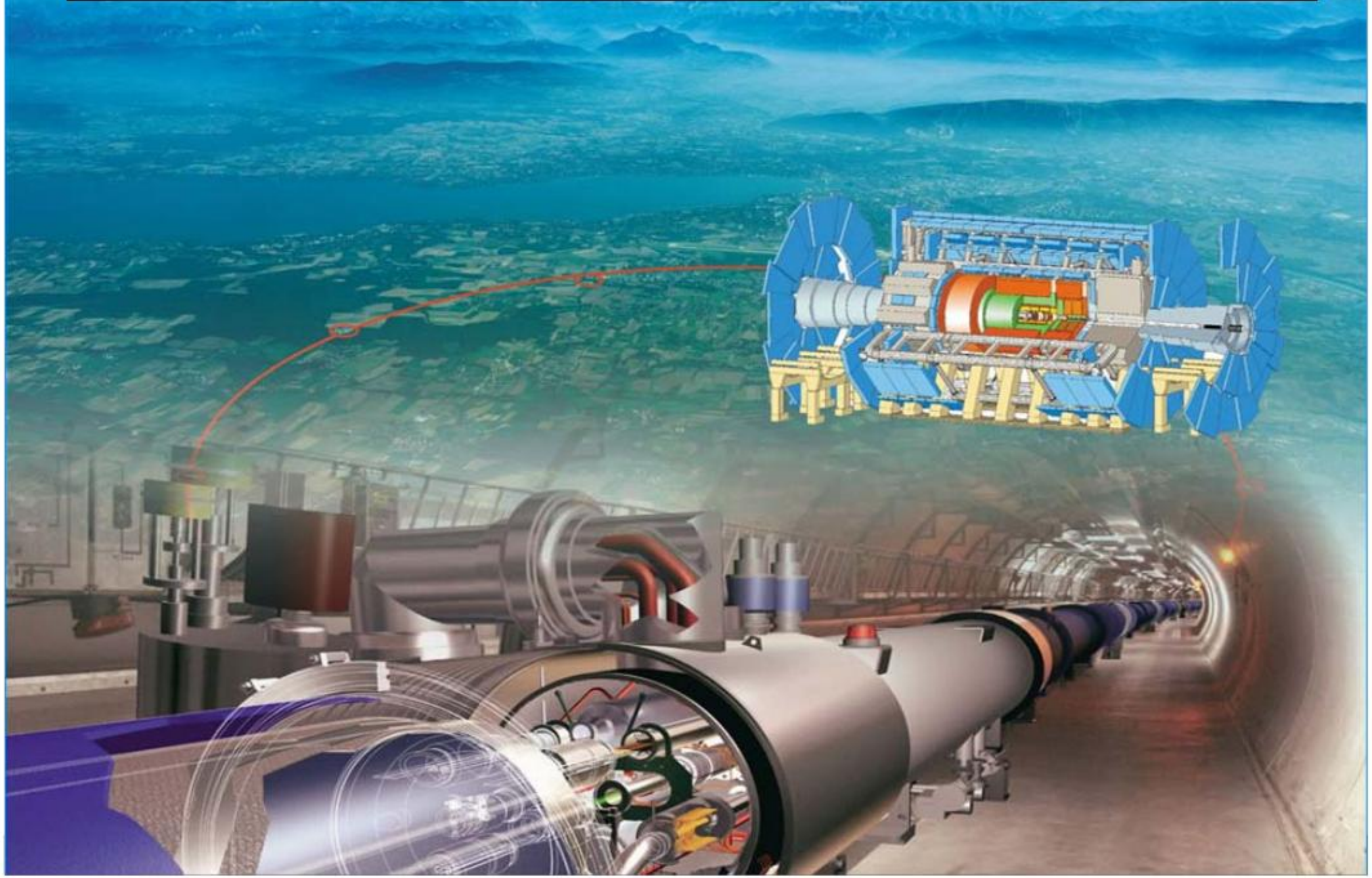
24th June 2021

4.5 Flavour Physics



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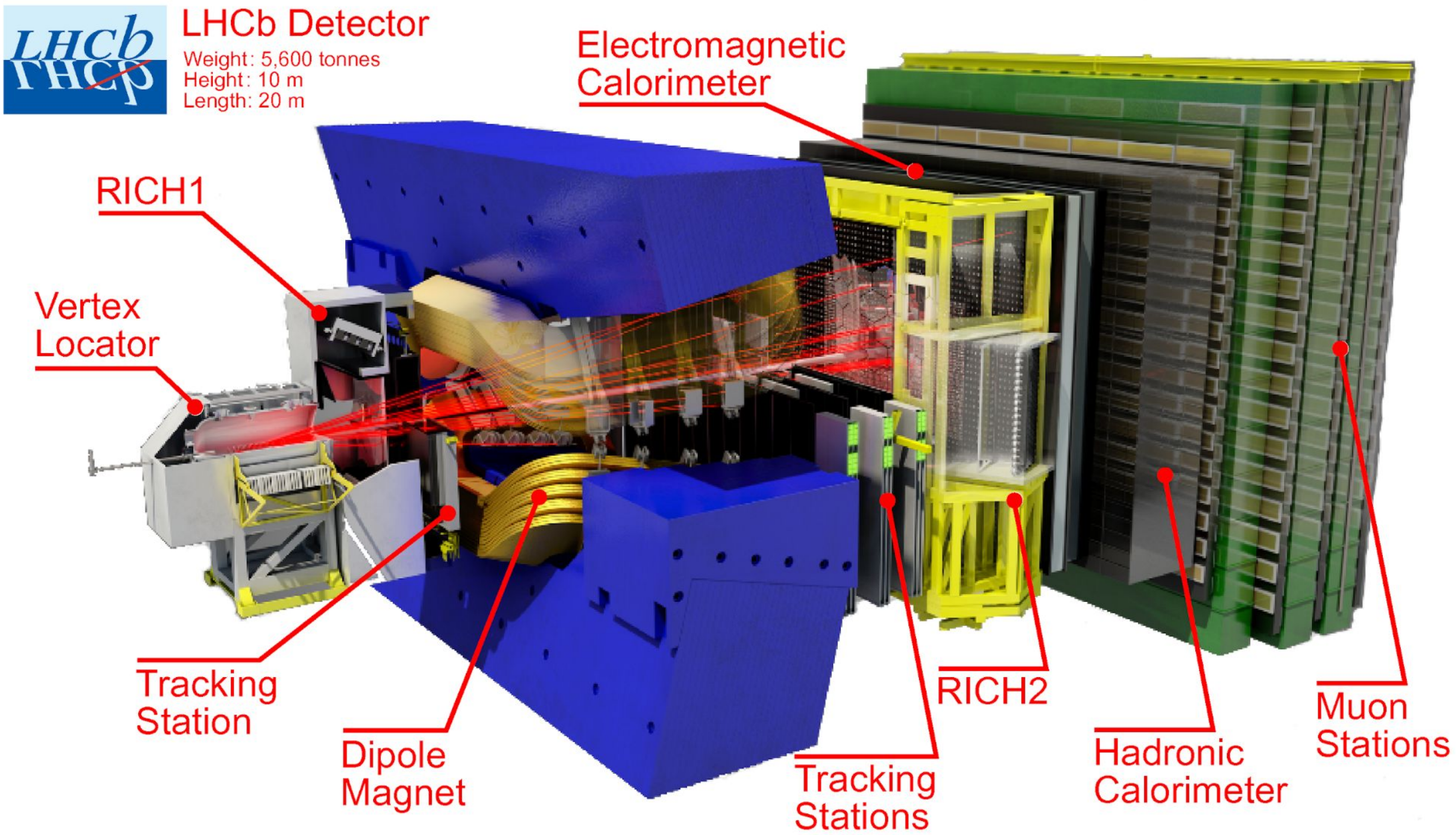
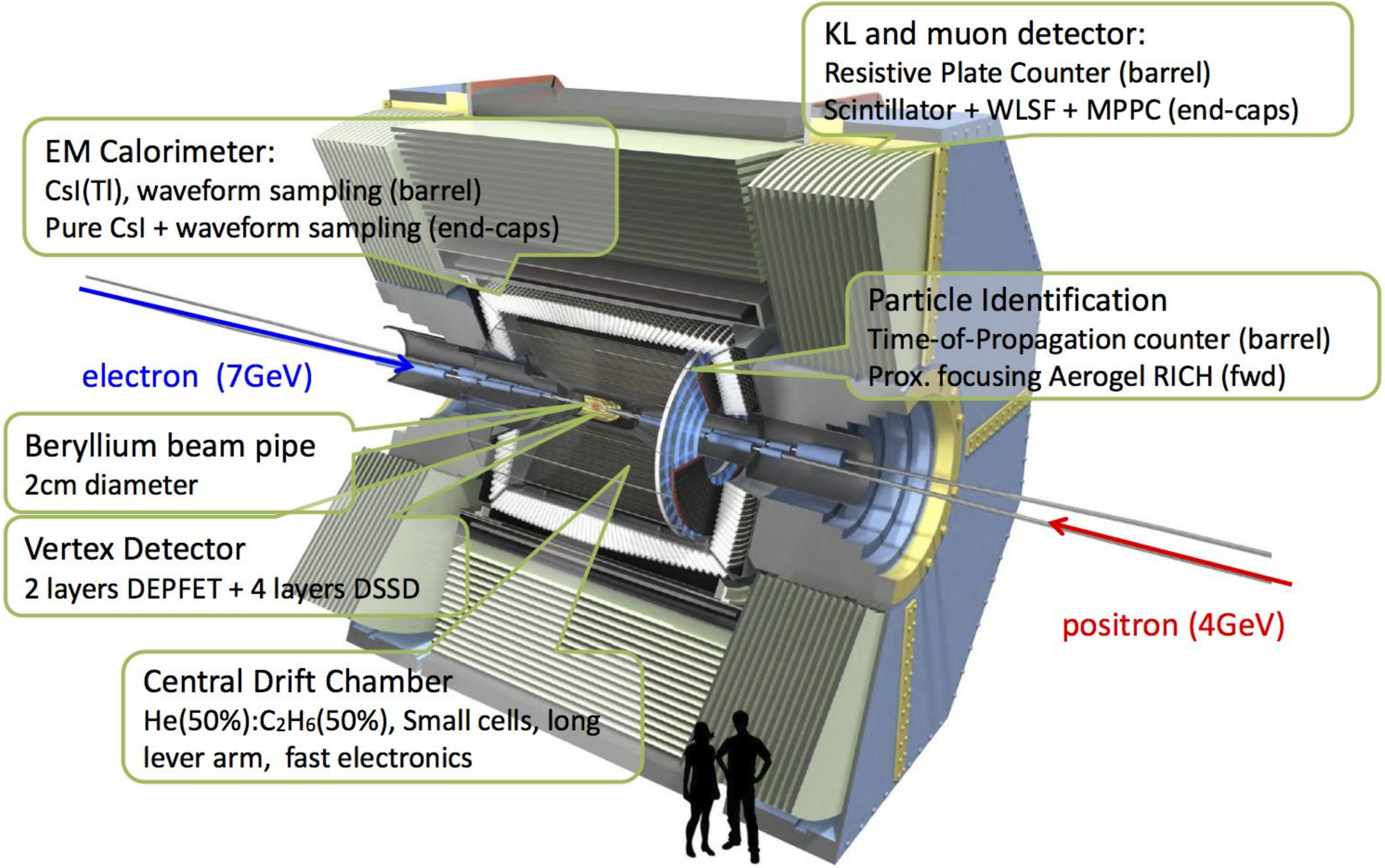
  - Lepton flavour universality tests
  - Pentaquarks

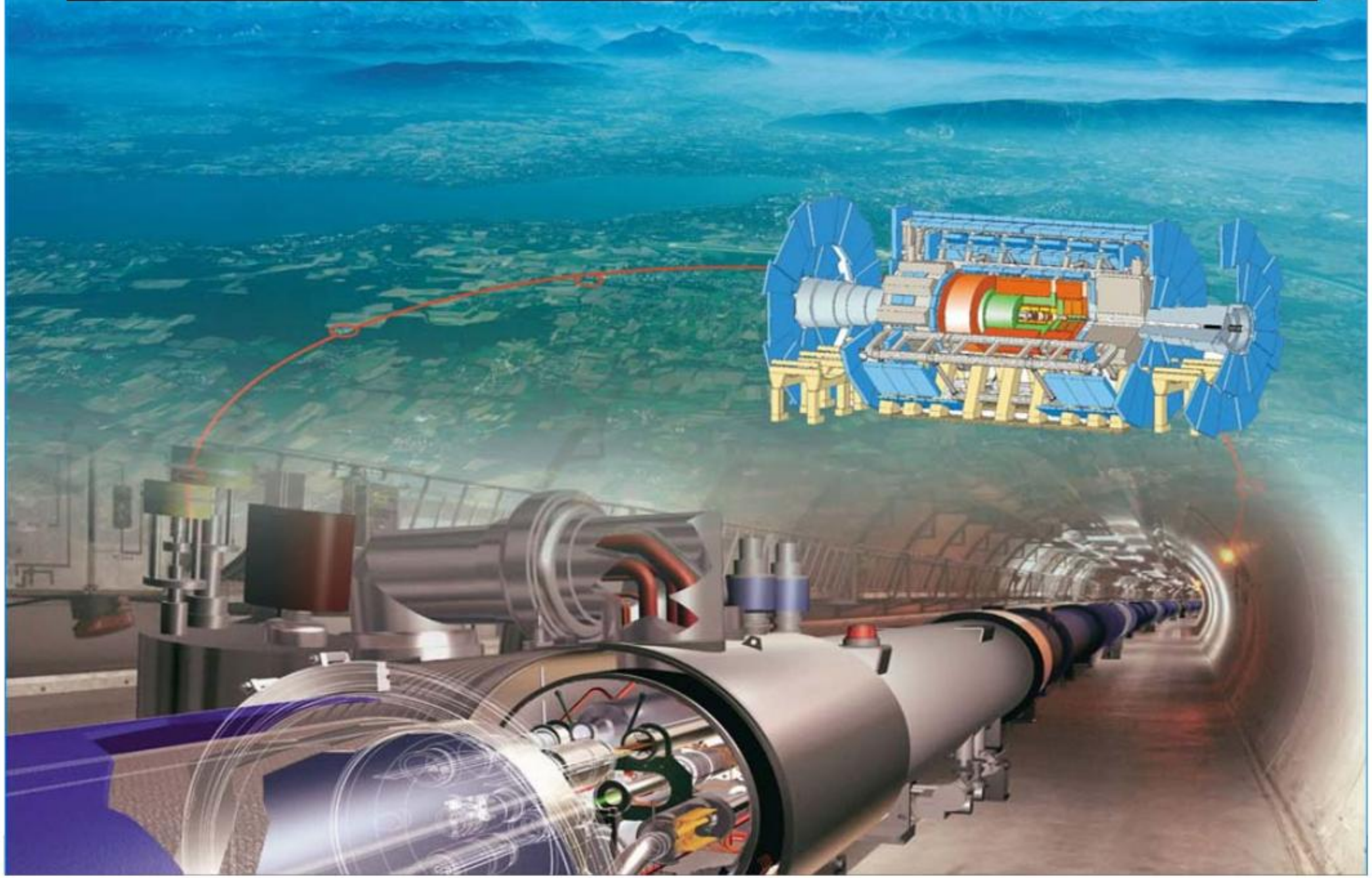

BABAR DETECTOR FOR THE PEP-II B FACTORY
Modeled by Daryl Oshatz • Berkeley Lab


LHCb
LHCb Detector
Weight: 5,600 tonnes
Height: 10 m
Length: 20 m
Electromagnetic
Calorimeter
RICH1
Vertex
Locator
Tracking
Station
Dipole
Magnet
Tracking
Stations
RICH2
Hadronic
Calorimeter
Muon
Stations

Belle II Detector



4.5.1 Quark-flavour mixing



Quark-flavour mixing

- Experimental evidence from weak decays of K, D and B mesons shows:
  - Mass eigenstates of quarks are different from weak eigenstates of quarks:
    - **Mass eigenstates:** mass operator is diagonal, fixed masses
    - **Weak eigenstates:** left-handed SU(2) doublet and right-handed SU(2) singlet
- The charged weak interaction mediates transitions between weak eigenstates within each generation
- The mass eigenstates

$$U'_L = (u', c', t')_L \quad \text{and} \quad D'_L = (d', s', b')_L$$

originate from the weak eigenstates U_L und D_L via the unitary transformations U_u (for up-type quarks) and U_d (for down-type quarks):

$$D'_L = U_d^\dagger D_L \quad \text{and} \quad U'_L = U_u^\dagger U_L$$

Quark-flavour mixing

- Charged weak currents are described via:

$$
\begin{aligned}
\mathcal{L}_{CC} &= -\frac{g}{\sqrt{2}}\left[j^{\mu+}_{CC} W^-_\mu + j^{\mu-}_{CC} W^+_\mu\right] \\
&= -\frac{g}{\sqrt{2}}\left[(\overline{U}_L \gamma^\mu \mathbb{1} D_L) W^-_\mu + (\overline{D}_L \gamma^\mu \mathbb{1} U_L) W^+_\mu\right] \\
&\equiv -\frac{g}{\sqrt{2}}\left[(\overline{U}'_L U^\dagger_u \gamma^\mu U_d D'_L) W^-_\mu + (\overline{D}'_L U^\dagger_d \gamma^\mu U_u U'_L) W^+_\mu\right] \\
&\equiv -\frac{g}{\sqrt{2}}\left[\underbrace{(\overline{U}'_L \gamma^\mu V_{CKM} D'_L)}_{=j^{\mu+}_{CC}} W^-_\mu + \underbrace{(\overline{D}'_L V^\dagger_{CKM} \gamma^\mu U'_L)}_{=j^{\mu-}_{CC}} W^+_\mu\right]
\end{aligned}
$$

where the unitary matrix $V_{CKM} = U^\dagger{}_u U_d$ (Cabibbo-Kobayashi-Maskawa matrix) describes charged weak transitions between the quark mass eigenstate

Quark-flavour mixing

- The CKM matrix describes the probability of a transition from one quark *i* to another quark *j*

$$\begin{pmatrix} u' \\ c' \\ t' \end{pmatrix} \longleftrightarrow \begin{pmatrix} d_C \\ s_C \\ b_C \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \cdot \begin{pmatrix} d' \\ s' \\ b' \end{pmatrix}$$

where the non-diagonal elements describe transitions between different generations (due to charged weak current)

Example:

$$u' \longleftrightarrow d_C = V_{ud} d' + V_{us} s' + V_{ub} b'$$

Quark-flavour mixing

For n = 2 generations:

- Until the discovery of the bottom quark:
  - Use a real 2 × 2 matrix with 1 real parameter (Cabibbo angle θ_c) and no complex phase:

Cabibbo − Matrix:

$$V = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix}$$

with:

$$\sin\theta_c \approx 0.23 \quad \text{and} \quad \cos\theta_c \approx 0.95$$

Quark-flavour mixing

For n = 3 generations:

- The CKM matrix can be parameterized by three mixing angles and the CP-violating complex phase. Of the many possible conventions, a standard choice has become:

$$V_{CKM} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \cdot \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \cdot \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -s_{23}c_{12} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

where $s_{ij} = \sin\theta_{ij}$, $c_{ij} = \cos\theta_{ij}$, and δ is the phase responsible for all CP-violating phenomena in flavor-changing processes in the SM. The angles θ_{ij} can be chosen to lie in the first quadrant, so $s_{ij}, c_{ij} \geq 0$.

Quark-flavour mixing

- Elements of the CKM matrix are free parameters in the SM (they need to be determined via experiments)
  - Very active field, particularly in heavy quark physics (c, b, t)

- The complex phase of V_{CKM} allows CP-violation within the Standard Model
  - Occurs for weak interaction
  - Originates from Higgs--fermion coupling

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

Quark-flavour mixing

$\lvert V_{ud}\rvert = 0.97446 \pm 0.00010$	via nuclear β-decays
$\lvert V_{us}\rvert = 0.22452 \pm 0.00044$	via semileptonic kaon decays (e.g. $K \to \pi e \nu_e$)
$\lvert V_{ub}\rvert = 0.00365 \pm 0.00012$	via semileptonic B decays (e.g. $B \to X_u \ell \nu_\ell$)
$\lvert V_{cd}\rvert = 0.22438 \pm 0.00044$	extracted from semileptonic charm decays
$\lvert V_{cs}\rvert = 0.97359^{+0.00010}_{-0.00011}$	from semileptonic D or leptonic D_s decays
$\lvert V_{cb}\rvert = 0.04214 \pm 0.00076$	semileptonic decays of B mesons to charm
$\lvert V_{td}\rvert = 0.00896^{+0.00024}_{-0.00023}$	via B-$\bar{B}$ oscillations
$\lvert V_{ts}\rvert = 0.04133 \pm 0.00074$	via B-$\bar{B}$ oscillations
$\lvert V_{tb}\rvert = 0.999105 \pm 0.000032$	from top quark decays

Latest measurements of CKM matrix elements taken from: https://pdg.lbl.gov/2019/reviews/rpp2019-rev-ckm-matrix.pdf

Quark-flavour mixing

- An alternative representation of the CKM matrix is the Wolfenstein parameterisation:

$$V_{CKM} = \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

- The Wolfenstein parameters can be translated via:

$$s_{12} = \lambda = \frac{|V_{us}|}{\sqrt{|V_{ud}|^2 + |V_{us}|^2}} \qquad s_{23} = A\lambda^2 = \lambda \left|\frac{V_{cb}}{V_{us}}\right| \qquad s_{13}e^{i\delta} = V_{ub}^* = A\lambda^3(\rho + i\eta)$$

- Parameters have been measured to be:

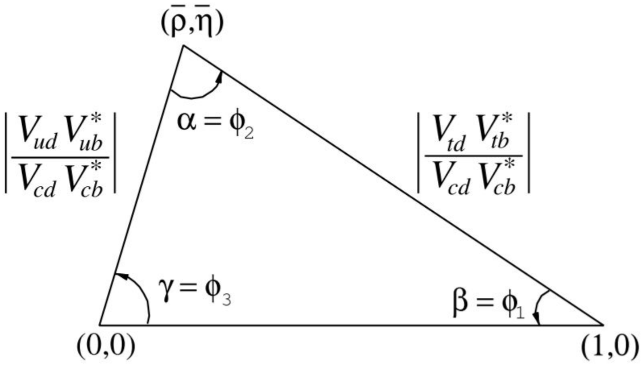
$$\lambda \equiv s_{12} = 0.2205 \pm 0.0018$$
$$A \equiv s_{23}/\lambda^2 = 0.82 \pm 0.06$$
$$\sqrt{\rho^2 + \eta^2} \equiv |V_{ub}|/A\lambda^3 = 0.36 \pm 0.09$$

Quark-flavour mixing

- The unitarity of the CKM matrix imposes:

$$\sum_i V_{ij} V_{ik}^* = \partial_{jk}$$

$$\sum_j V_{ij} V_{kj}^* = \partial_{ik}$$



- The six vanishing combinations can be represented as triangles (i.e. by the **unitarity triangles**) in a complex plane
- The most commonly used unitarity triangle arises from:

$$V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^* = 0 \qquad \textbf{(1)}$$

by dividing each side by the best-known one, $V_{cd} V_{cb}^*$

- **Phases of CKM elements:**

$$\beta = \Phi_1 = \arg\left(-\frac{V_{cd} V_{cb}^*}{V_{td} V_{tb}^*}\right)$$

$$\alpha = \Phi_2 = \arg\left(-\frac{V_{td} V_{tb}^*}{V_{ud} V_{ub}^*}\right)$$

$$\gamma = \Phi_3 = \arg\left(-\frac{V_{ud} V_{ub}^*}{V_{cd} V_{cb}^*}\right)$$

Quark-flavour mixing

- Other unitarity triangles are described via:

$$V_{ud}^* V_{us} + V_{cd}^* V_{cs} + V_{td}^* V_{ts} = 0$$
$$V_{us}^* V_{ub} + V_{cs}^* V_{cb} + V_{ts}^* V_{tb} = 0$$

- The unitarity triangles vanish if the phase δ and/or the element V_{ub} are equal to zero i.e. in the absence of CP violation.
- The unitarity relation (1) can be rewritten as:

$$\underbrace{\frac{|V_{ud} V_{ub}^*|}{|V_{cd} V_{cb}^*|}}_{R_b} e^{-i\beta} + \underbrace{\frac{|V_{td} V_{tb}^*|}{|V_{cd} V_{cb}^*|}}_{R_t} e^{-i\gamma} = 1$$

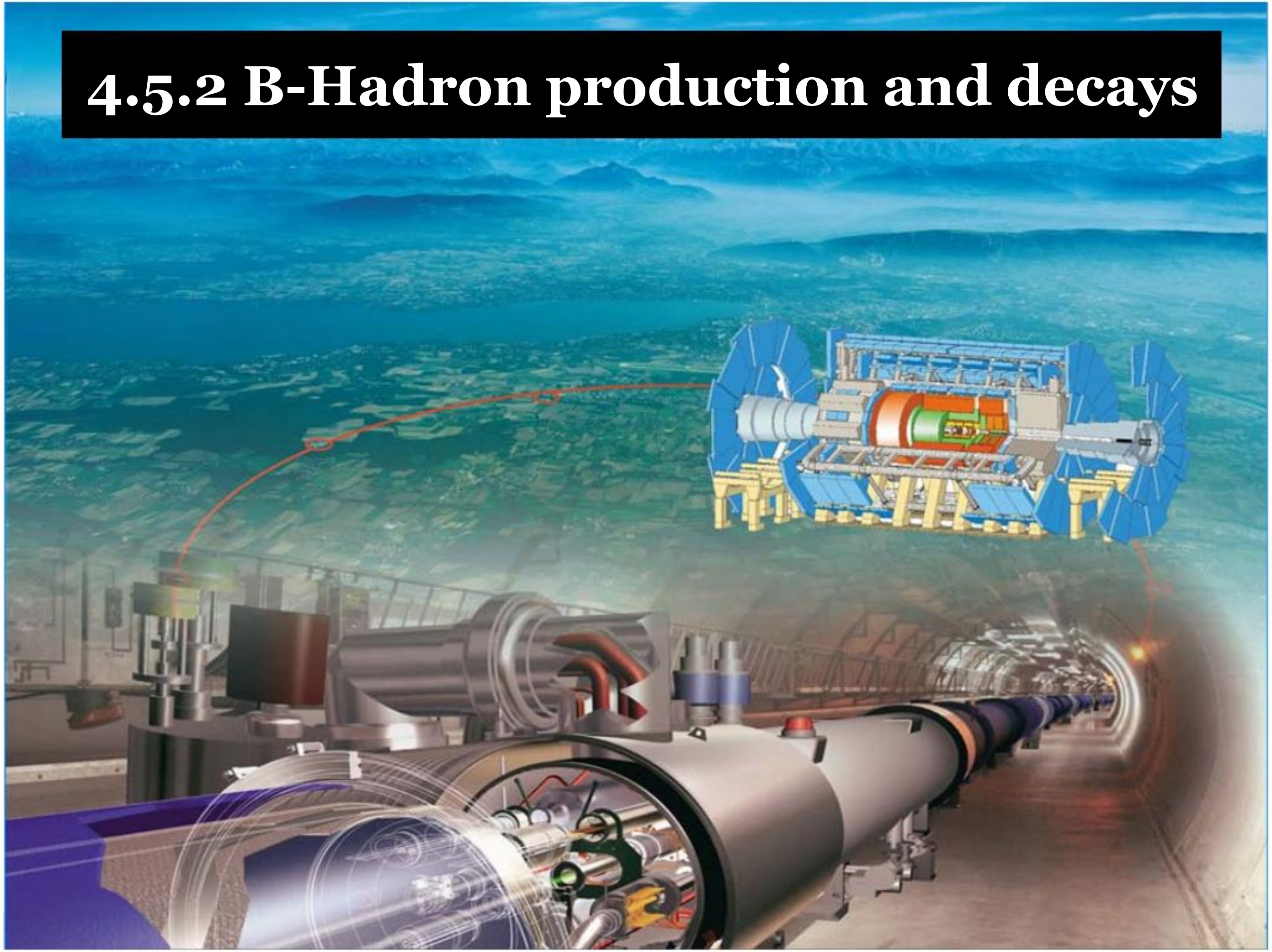
with:

$$V_{td} = |V_{td}| e^{-i\beta}$$

$$V_{ub} = |V_{ub}| e^{-i\gamma}$$

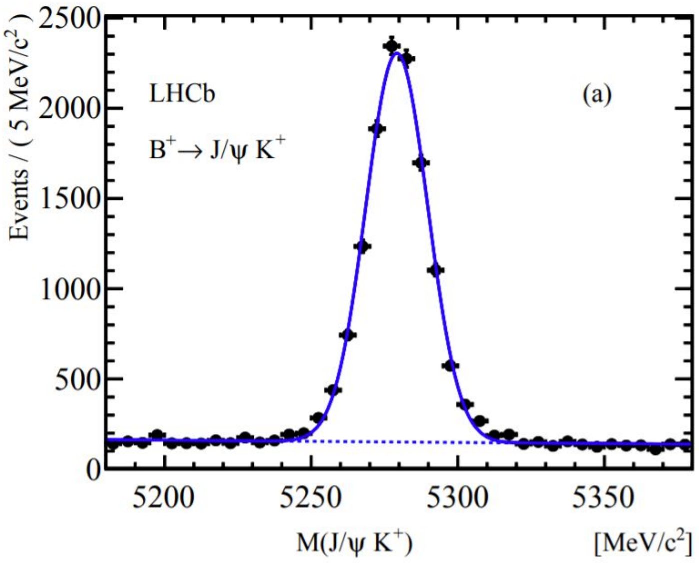
$$\alpha + \beta + \gamma = 180^\circ$$

4.5.2 B-Hadron production and decays



B-Hadron production

- b-hadrons are mainly produced in pairs at colliders (since ee → bb or pp → bb)
- Masses of b-hadrons are measured with high precision



Hadron	Quark content	Mass [MeV]
B^+	$u\bar{b}$	5279
B^0	$d\bar{b}$	5279
B_s^0	$s\bar{b}$	5366
B_c^+	$c\bar{b}$	6274
Λ_b^0	udb	5619
Ω_b^-	ssb	6046

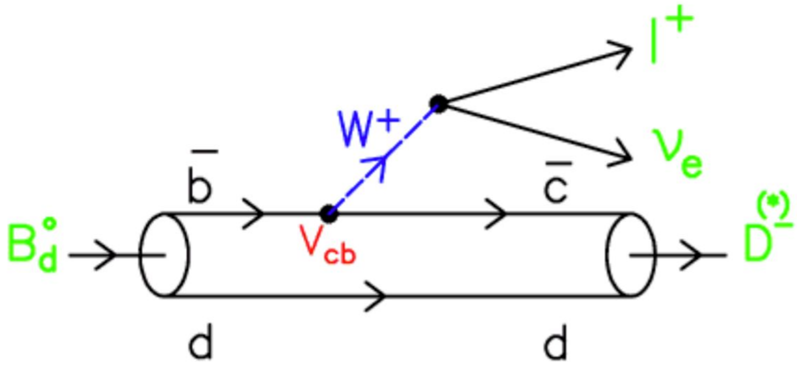
Taken from: https://arxiv.org/pdf/1112.4896.pdf

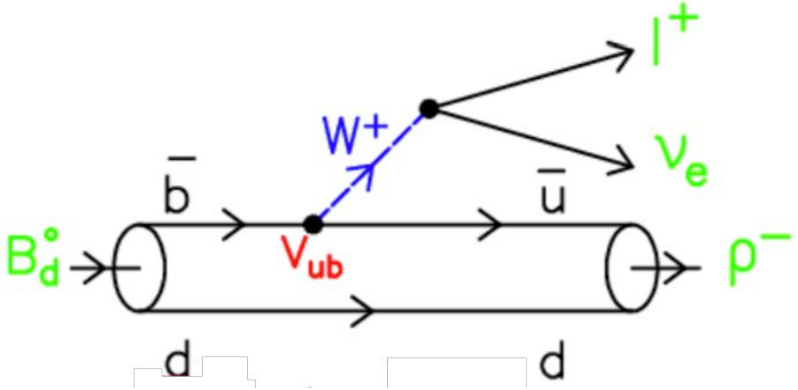
Weak B-hadron decays

- 20% of all B-hadrons decay semi-leptonically (with ℓ = e, μ)
  - E.g.:
    - **b → cℓv**
    - **b → uℓv**
  - Leptons are usually relative soft (i.e. they tend to have relative low transverse momenta)
- **Sensitive to CKM matrix elements**
  - **E.g.:**

$B^0 \rightarrow D^{(*)-} \ell^+ v$ → $|V_{cb}| = (42.1 \pm 0.7) \cdot 10^{-3}$

$B^0 \rightarrow \rho^- \ell^+ v$ → $|V_{ub}| = (3.65 \pm 0.12) \cdot 10^{-3}$





B-Hadron decays

- Precise knowledge of the lifetime is crucial for determining the CKM matrix elements and the quark-flavour mixing parameters
- Lifetime differences are dominated by:
  - Mass differences
  - whether or not the other quark(s) decay weekly

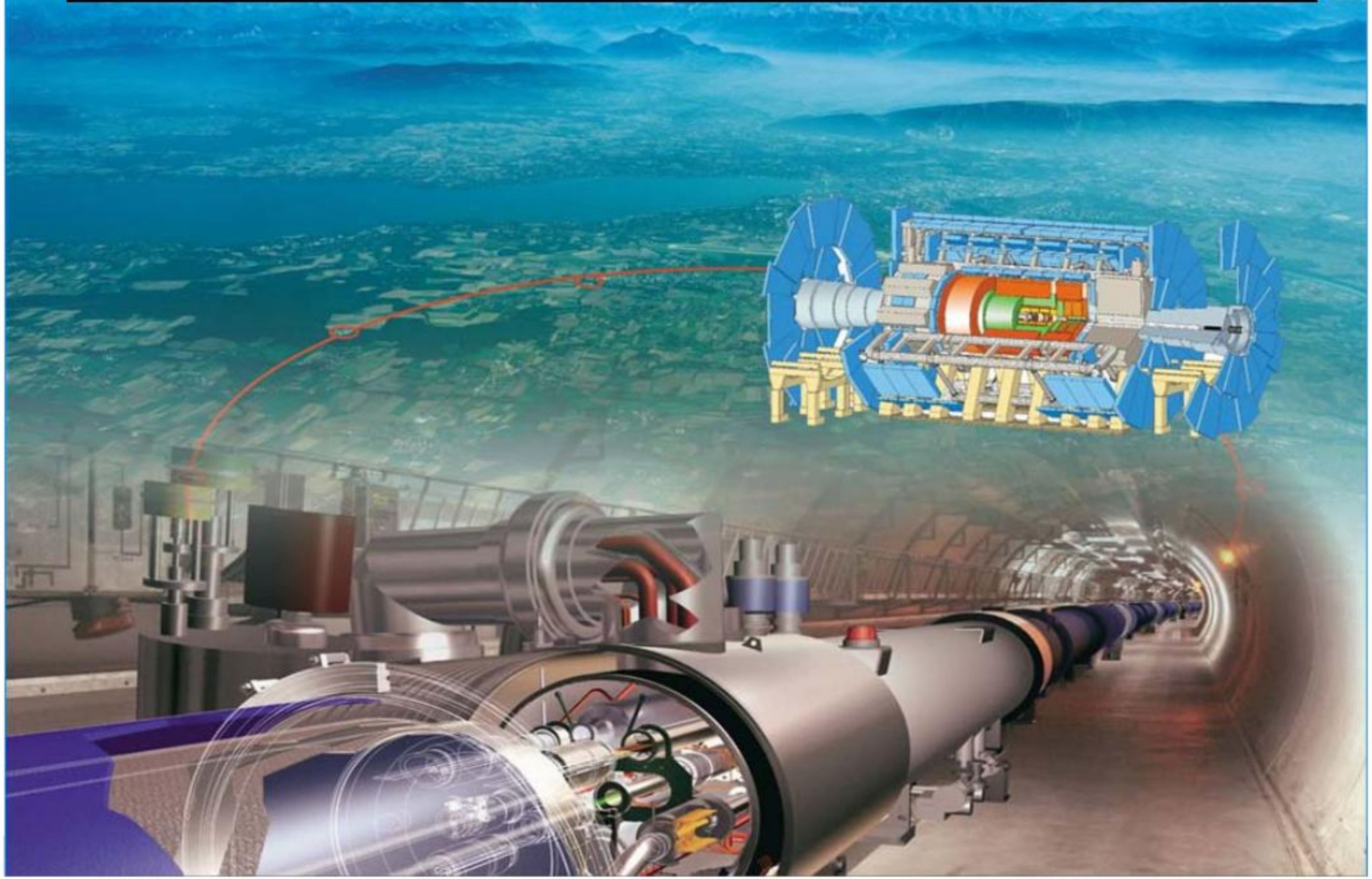
→ B-hadrons are long-lived particles that travel macroscopic distances in the detector

→ Tracking is crucial for B-hadron identification & measurements

Particle	Lifetime [ps]
B^+	1.638 ± 0.004
B^0	1.519 ± 0.004
B^0_s	1.515 ± 0.004
B^0_{sL}	1.423 ± 0.005
B^0_{sH}	1.620 ± 0.007
B^+_c	0.510 ± 0.009
Λ^0_b	1.471 ± 0.009
Ξ^-_b	1.572 ± 0.040
Ξ^0_b	1.480 ± 0.030
Ω^-_b	$1.64^{+0.18}_{-0.17}$

Taken from: https://pdg.lbl.gov/2020/mobile/reviews/pdf/rpp2020-rev-b-meson-prod-decay-m.pdf:

4.5.3 Quark-flavour oscillation



Quark-flavour oscillation

- Particle-antiparticle oscillations of neutral Mesons:

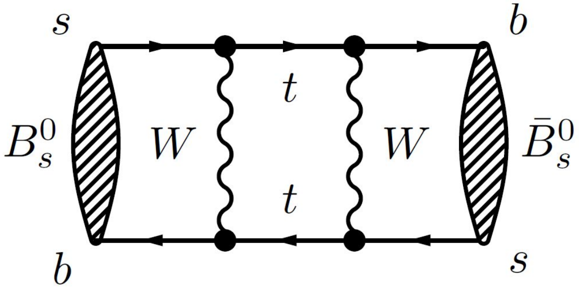
$$K^0 = (d\bar{s}) \longleftrightarrow \overline{K}^0 = (\bar{d}s) \quad (|\Delta S| = 2)$$

$$D^0 = (c\bar{u}) \longleftrightarrow \overline{D}^0 = (\bar{c}u) \quad (|\Delta C| = 2)$$

$$B_d^0 = (d\bar{b}) \longleftrightarrow \overline{B}_d^0 = (\bar{d}b) \quad (|\Delta B| = 2)$$

$$B_s^0 = (s\bar{b}) \longleftrightarrow \overline{B}_s^0 = (\bar{s}b) \quad (|\Delta B| = 2)$$

- Weak interaction violates the conservation of flavour-quantum numbers



Quark-flavour oscillation

- Prediction of flavour oscillation by Gell-Mann and Pais (1955) for K^0 mesons:
  - **Flavour-eigenstates of neutral mesons are not equal to CP - eigenstates:**

$$K^0 \ (S=-1),\ \overline{K}^0 \ (S=+1)$$ (with distinct masses $m_{S,L}$ and lifetimes)

    - **Lifetimes:**

$$K_S^0 \ \text{(short-lived)}: \ \tau_S \approx 10^{-10} \ \text{s}; \ K_S^0 \to \pi^+\pi^-, \ \pi^0\pi^0 \ (CP=+1)$$

$$K_L^0 \ \text{(long-lived)}: \ \tau_L \approx 10^{-7} \ \text{s}; \ K_L^0 \to \pi^+\pi^-\pi^0, \ \pi^0\pi^0\pi^0 \ (CP=-1)$$

Quark-flavour oscillation

- Time development of the states:

$$|\phi(t)\rangle = a(t)\left|K^0\right\rangle + b(t)\left|\overline{K}^0\right\rangle = \begin{pmatrix} a(t) \\ b(t) \end{pmatrix} \quad \text{with} \left|K^0\right\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \text{ and } \left|\overline{K}^0\right\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

are described via the Schrödinger equation:

$$i\frac{\partial}{\partial t}|\phi\rangle = H|\phi\rangle$$

with the Hamiltonian:

with:

$$m_{11} = m_{22}$$

$$m_{12} = m_{21} = m_{12}^*$$

Mass matrix

Decay width matrix

$$\begin{aligned} H &= \widehat{M} - \frac{i\widehat{\Gamma}}{2} \\ &= \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} - \frac{i}{2}\begin{pmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{21} & \Gamma_{22} \end{pmatrix} \\ &= \begin{pmatrix} m_K & m_{12} \\ m_{12} & m_K \end{pmatrix} - \frac{i}{2}\begin{pmatrix} \Gamma_K & \Gamma_{12} \\ \Gamma_{12} & \Gamma_K \end{pmatrix} \end{aligned}$$

with:

$$m_K = (m_S + m_L)/2$$

$$\Gamma_K = (\Gamma_S + \Gamma_L)/2$$

Quark-flavour oscillation

- Via diagonalization of the mass - decay width matrix one obtains the mass eigenstates:

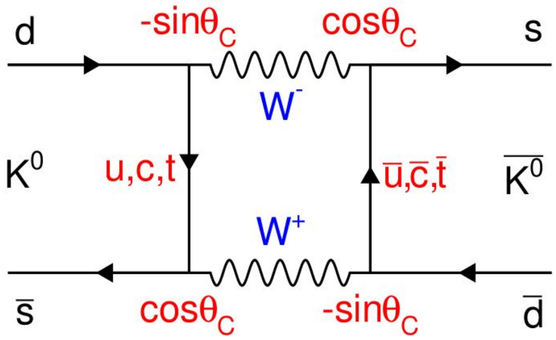
$$
\begin{aligned}
K_S^0 &= \frac{1}{\sqrt{2}}\left(K^0 + \overline{K}^0\right) \\
K_L^0 &= \frac{1}{\sqrt{2}}\left(K^0 - \overline{K}^0\right)
\end{aligned}
$$

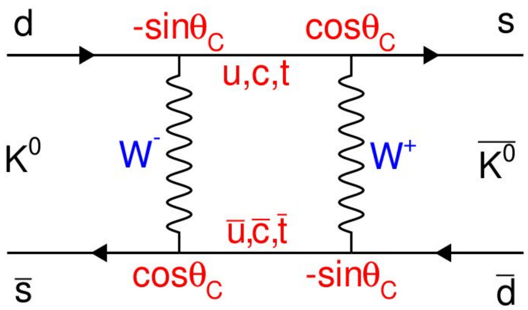
and the eigenvalues:

$$
\begin{aligned}
m_{S,L} &= m_K \pm \mathcal{R}e\sqrt{\left(m_{12} - \frac{i}{2}\Gamma_{12}\right)\left(m_{12}^* - \frac{i}{2}\Gamma_{12}^*\right)} \\
\Gamma_{S,L} &= \Gamma_K \mp \mathcal{I}m\sqrt{\left(m_{12} - \frac{i}{2}\Gamma_{12}\right)\left(m_{12}^* - \frac{i}{2}\Gamma_{12}^*\right)} = \tau_{S,L}^{-1}
\end{aligned}
$$

Quark-flavour oscillation

- Kaon transition is described at 2. order of the weak interaction (Δ**S = 2**)





- Contribution from charm-quark exchange is dominiert, since $m_c \gg m_u$
- Contribution from top-quark exchange is suppressed due to V_{CKM} factor, but dominant for B^0 mixing

$$\Longrightarrow \Delta m \approx \frac{G_F^2}{4\pi^2} f_K^2 m_K m_c^2 \cos^2\theta_C \sin^2\theta_C$$

Kaon decay constant f_K **Cabibbo angle θ_C**

Quark-flavour oscillation

- The time development of the mass-eigenstates is described via:

$$K_S^0(t) = \mathcal{N} e^{-(im_S + \frac{\Gamma_S}{2})t} K_S(0)$$

$$K_L^0(t) = \mathcal{N} e^{-(im_L + \frac{\Gamma_L}{2})t} K_L(0)$$

- The time development of the flavour-eigenstates is described via:

$$K^0(t) = \mathcal{N} e^{-(im_K + \frac{\Gamma_K}{2})t} \left[\cos(\Delta m t/2) K^0 + \sin(\Delta m t/2) \overline{K}^0\right]$$

$$\overline{K}^0(t) = \mathcal{N} e^{-(im_K + \frac{\Gamma_K}{2})t} \left[\sin(\Delta m t/2) K^0 + \cos(\Delta m t/2) \overline{K}^0\right]$$

$K^0_L - K^0_S$ mass difference Δm = oscillation frequency ($h\omega = \Delta mc^2$)

$\Delta m = (3.489 \pm 0.008)\cdot 10^{-6}$ eV = $(0.530 \pm 0.001)\cdot 10^{10}$ Hz (measured first in 1964 at BNL)

Quark-flavour oscillation

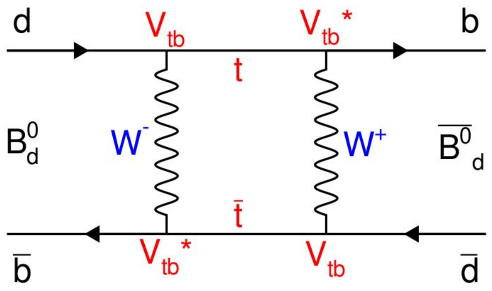
- Mixing of neutral B-mesons due to second order weak interaction (as for neutral K-mesons)

$$\Delta m_d = \frac{G_F^2}{6\pi^2} M_{B_d} m_{\text{top}}^2 F\left(\frac{m_{\text{top}}^2}{M_W^2}\right) \eta_{QCD} (f_{B_d}^2 B_{B_d}) |V_{td} V_{tb}^*|^2$$

$$\Longrightarrow \frac{\Delta m_d}{\Delta m_s} = \frac{M_{B_d}}{M_{B_s}} \cdot \frac{f_{B_d}^2 B_{B_d}}{f_{B_s}^2 B_{B_s}} \cdot \frac{|V_{td}|^2}{|V_{ts}|^2}$$

$$\approx (0.88 \pm 0.04)^2 \cdot \frac{|V_{td}|^2}{|V_{cb}|^2}$$

with $|V_{ts}| \approx |V_{cb}|$



$$\boldsymbol{\Delta m_d = 0.507 \pm 0.005 \ \mathrm{ps}^{-1}}$$

$$\boldsymbol{\Delta m_s = 17.77 \pm 0.12 \ \mathrm{ps}^{-1}}$$

$$|\boldsymbol{V_{td}}| = 0.009 \pm 0.002$$

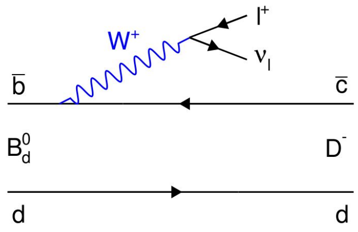
$$|\boldsymbol{V_{td}}| = 0.008 \pm 0.0003$$

Quark-flavour oscillation

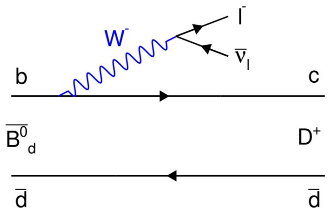
- Distinguish B^0 and $\bar{B}^0$ by measuring the lepton charge in semileptonic B decays
- Characteristics of oscillations can be determined by studying the fraction of same-sign lepton pairs as a function of the B-meson lifetime t = d/βγc (with d being the decay length)

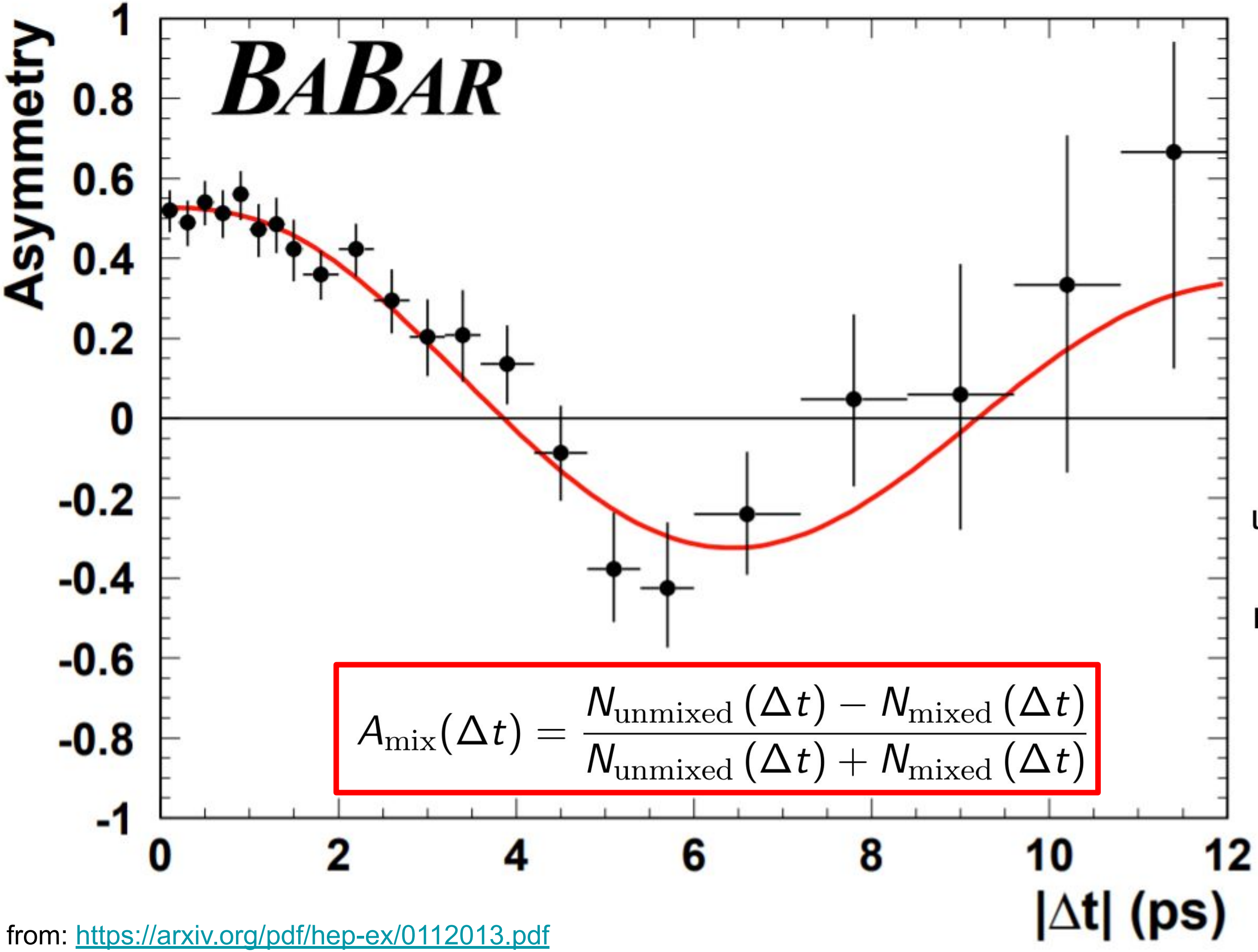
Probability of a B^0 oscillating into a $\bar{B}^0$

$$\frac{N(\ell^\pm\ell^\pm)[t]}{N_{\mathrm{tot}}(\ell\ell)[t]} = \frac{\mathcal{P}(B^0 \to \overline{B^0})[t]}{\mathcal{P}(B^0 \to B^0)[t] + \mathcal{P}(B^0 \to \overline{B^0})[t]} = \sin^2(\Delta m \cdot t/2)$$



$$A_{\mathrm{mix}}(\ell^\pm\ell^\mp - \ell^\pm\ell^\pm) = \frac{\mathcal{P}(B^0 \to B^0)[t] - \mathcal{P}(B^0 \to \overline{B^0})[t]}{\mathcal{P}(B^0 \to B^0)[t] + \mathcal{P}(B^0 \to \overline{B^0})[t]} = \cos(\Delta m \cdot t)$$





$$A_{\mathrm{mix}}(\Delta t) = \frac{N_{\mathrm{unmixed}}(\Delta t) - N_{\mathrm{mixed}}(\Delta t)}{N_{\mathrm{unmixed}}(\Delta t) + N_{\mathrm{mixed}}(\Delta t)}$$

unmixed: $B^0 \bar{B}^0$

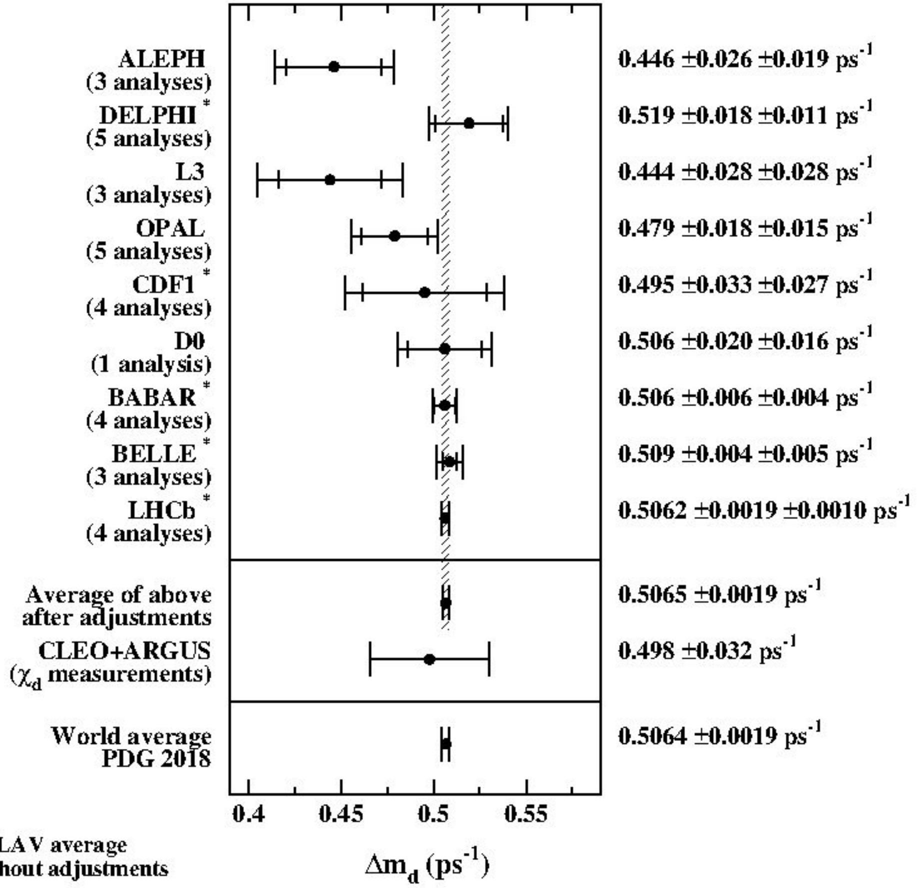
mixed: $B^0 B^0$, $\bar{B}^0 \bar{B}^0$

Taken from: https://arxiv.org/pdf/hep-ex/0112013.pdf

Quark-flavour oscillation

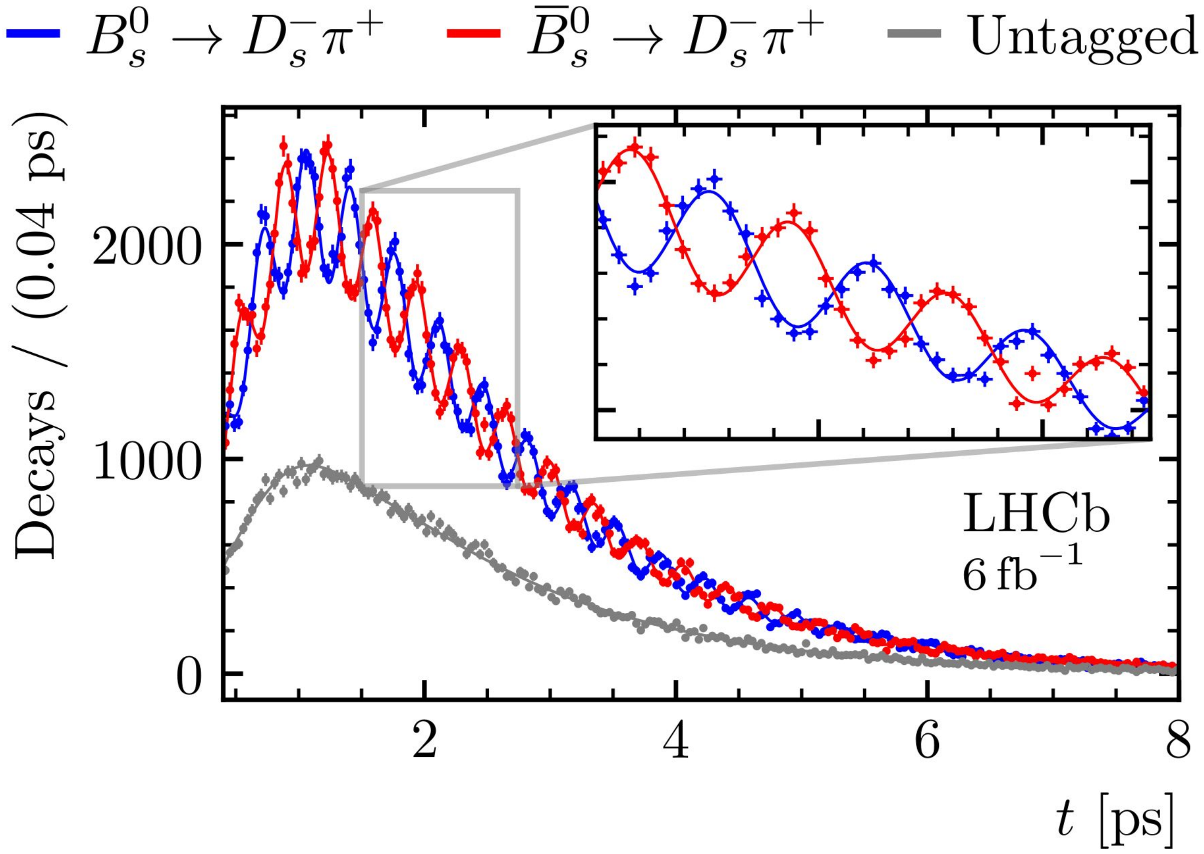
Combinations:

$$\Delta m_d = 0.5065 \pm 0.0019\ \text{ps}^{-}$$

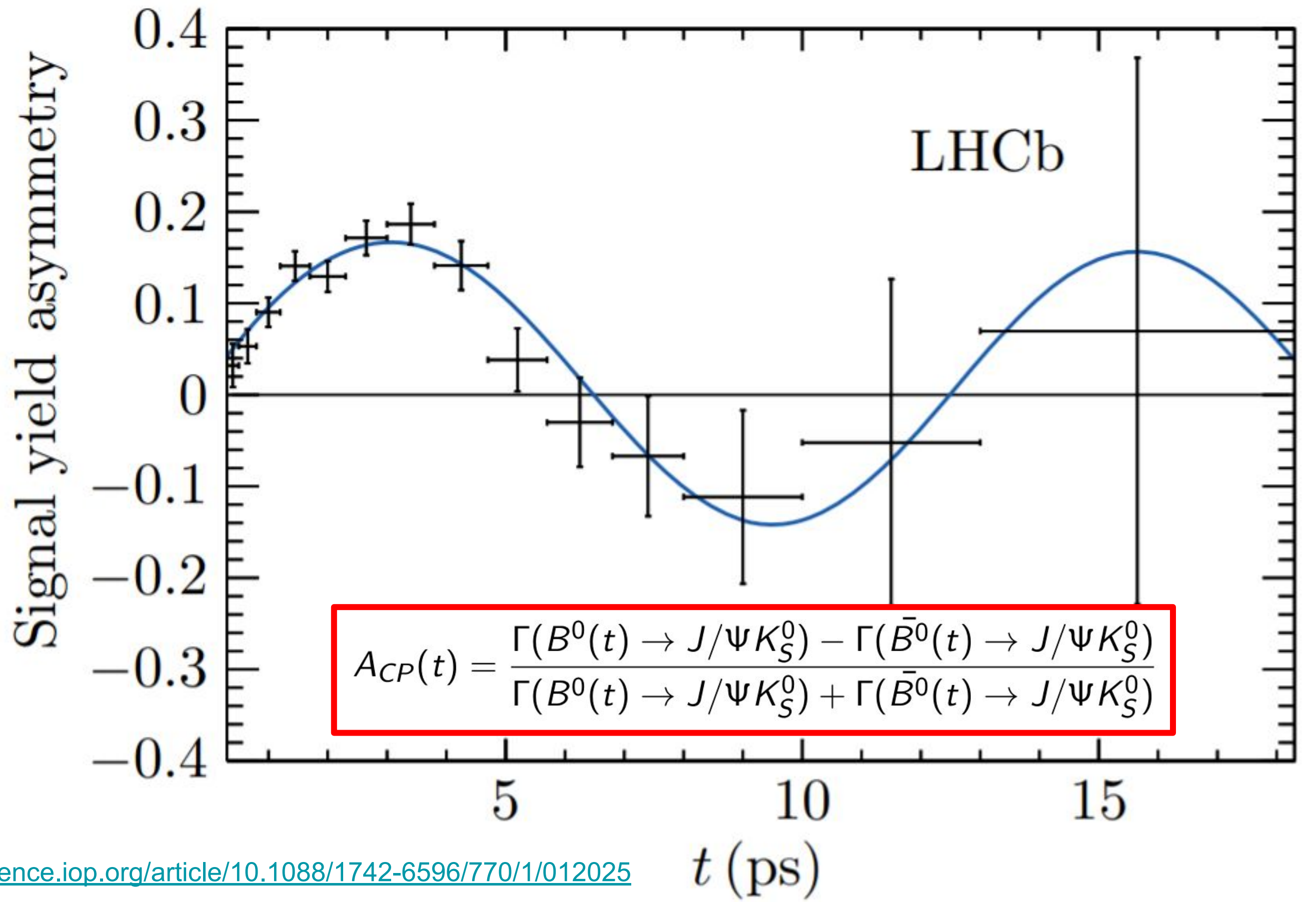


Taken from: https://hflav-eos.web.cern.ch/hflav-eos/osc/PDG_2018/

Quark-flavour oscillation



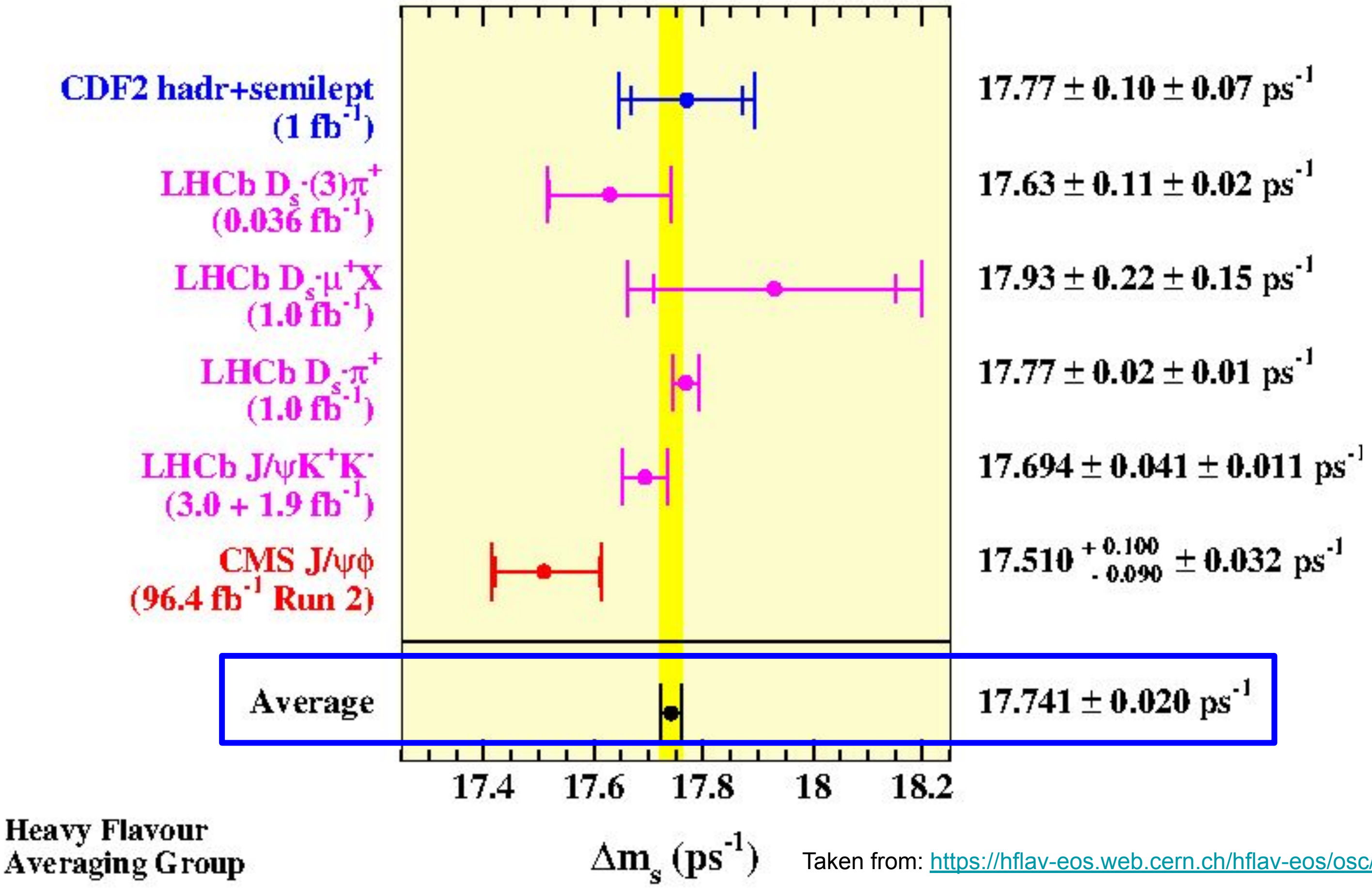
Quark-flavour oscillation



$$A_{CP}(t) = \frac{\Gamma(B^0(t) \to J/\Psi K_S^0) - \Gamma(\bar{B}^0(t) \to J/\Psi K_S^0)}{\Gamma(B^0(t) \to J/\Psi K_S^0) + \Gamma(\bar{B}^0(t) \to J/\Psi K_S^0)}$$

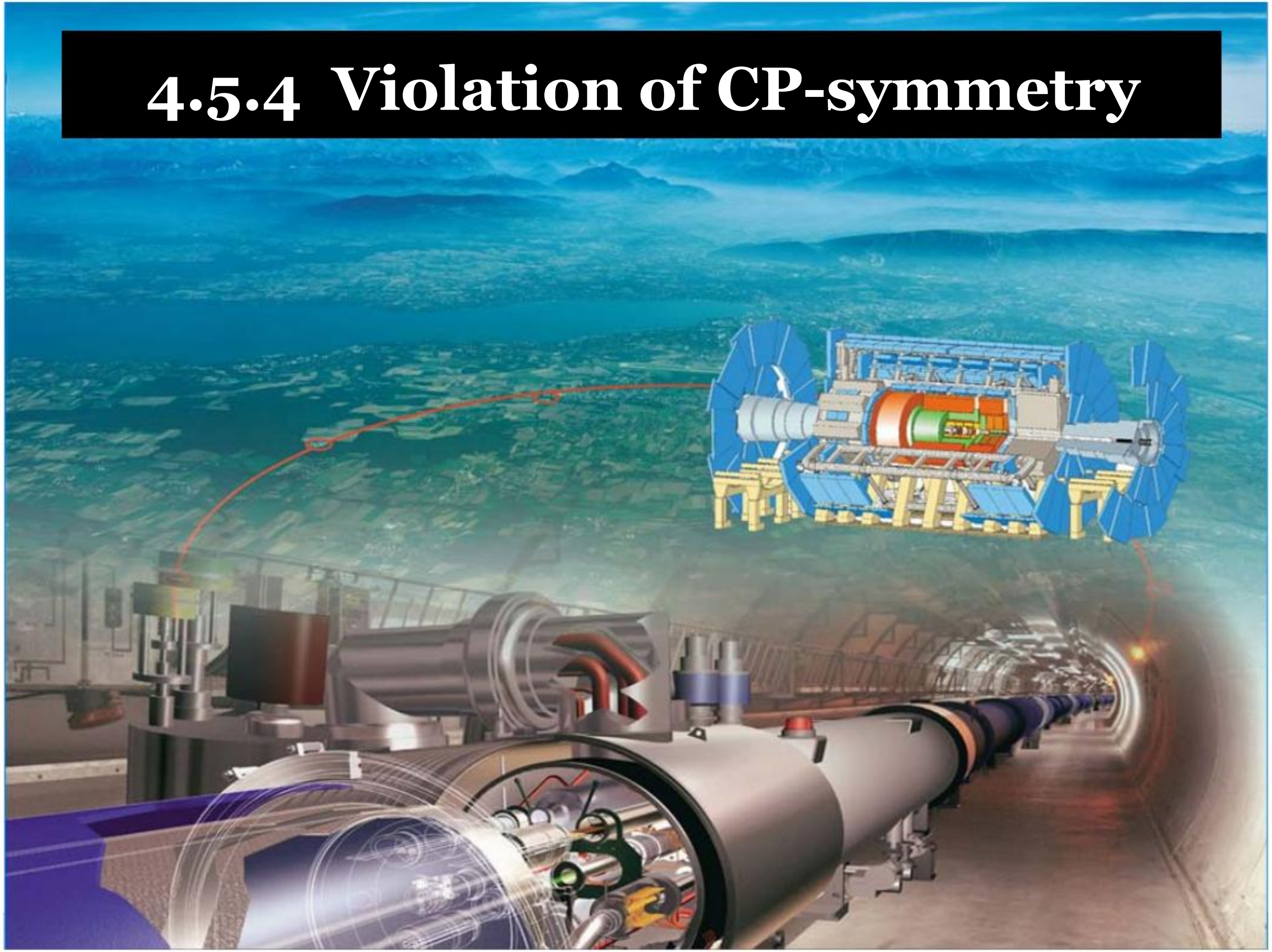
Taken from: https://iopscience.iop.org/article/10.1088/1742-6596/770/1/012025

Quark-flavour oscillation



Taken from: https://hflav-eos.web.cern.ch/hflav-eos/osc/PDG_2018/

4.5.4 Violation of CP-symmetry



Violation of CP-symmetry

- **Direct vs. indirect CP violation:**
  - Indirect CP violation:
    - CP violation in mixing:

      Probability of B^0 oscillating to $\bar{B}^0$ is different from probability of $\bar{B}^0$ oscillating to B^0

$$P\left(B \to \bar{B}\right) \neq P\left(\bar{B} \to B\right)$$

  - Direct CP violation:
    - CP violation in decay (direct)

$$BR\left(B \to f + X\right) \neq BR\left(\bar{B} \to f + X\right)$$

      Different decay rates between CP-conjugate states

Violation of CP-symmetry

- Neutral kaons provide a perfect experimental system for testing CP invariance.
  - By using a long enough beam, one can produce an arbitrarily pure sample of the long-lived neutral kaon species.
    - If at this point, 2π decays are observed, we can conclude that CP has been violated.
  - As it was experimentally confirmed, the long-lived (and also short-lived) neutral kaon is not a perfect CP eigenstate:

$$K_S^0 = pK^0 - q\overline{K}^0 = \frac{K_+^0 - \varepsilon K_-^0}{\sqrt{1+|\varepsilon|^2}} \approx K_+^0$$

$$K_L^0 = pK^0 + q\overline{K}^0 = \frac{K_-^0 + \varepsilon K_+^0}{\sqrt{1+|\varepsilon|^2}} \approx K_-^0$$

$$|p|^2 + |q|^2 = 1$$

$$\frac{q}{p} = \sqrt{\frac{M_{12}^* - i\Gamma_{12}^*/2}{M_{12} - i\Gamma_{12}^*/2}} = \frac{1-\varepsilon}{1+\varepsilon}$$

→ **mass eigenstates $K^0_{S,L}$ are linear combinations of the CP-eigenstates $K^0_{\pm}$ (CP = ±1)**

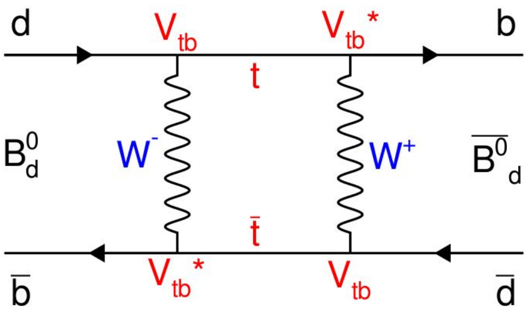
From Measurements: $|\varepsilon| = (2.271 \pm 0.017) \cdot 10^{-3}$ (small effect)

Violation of CP-symmetry

- **Indirekte CP-violation** in mixing of neutral B-mesons
  - Mass eigenstates:

$$B_H^0 = pB^0 - q\overline{B}^0 = \frac{B_+^0 - \varepsilon_B B_-^0}{\sqrt{1+|\varepsilon_B|^2}} \approx B_+^0$$

$$B_L^0 = pB^0 + q\overline{B}^0 = \frac{B_-^0 + \varepsilon_B B_+^0}{\sqrt{1+|\varepsilon_B|^2}} \approx B_-^0$$



width: $\frac{q}{p} = \left|\frac{q}{p}\right| e^{-i\phi_{\mathrm{mix}}}$ and $\phi_{\mathrm{mix}} \equiv arg\left(M_{12}/\Gamma_{12}\right)$

$$V_{td} = |V_{td}|e^{-i\beta}$$

In the standard model:

$$\left|\frac{q}{p}\right| = \left|\frac{1-\varepsilon_B}{1+\varepsilon_B}\right| = 1 + \frac{1}{2}\left|\frac{\Gamma_{12}}{M_{12}}\right| \sin\phi_{\mathrm{mix}} \neq 1 \longrightarrow \varepsilon_B \neq 0$$

Violation of CP-symmetry

- CP-violating asymmetry, arising via the complex phase of flavour-mixing, can be studied in semileptonic B^0 decays:

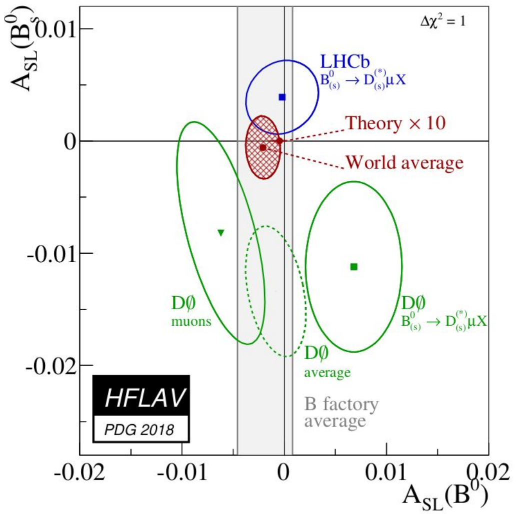
$$A_{sl} = \frac{\Gamma\left(\overline{B}^0 \to B^0(t) \to \ell^+ \nu X\right) - \Gamma\left(B^0 \to \overline{B}^0(t) \to \ell^- \nu X\right)}{\Gamma\left(\overline{B}^0 \to B^0(t) \to \ell^+ \nu X\right) + \Gamma\left(B^0 \to \overline{B}^0(t) \to \ell^- \nu X\right)}$$

$$= \frac{|p/q|^2 - |q/p|^2}{|p/q|^2 + |q/p|^2} \approx 4\mathcal{R}e\varepsilon_B$$

- Studied by multiple experiments
  - Definitive evidence still to be reached
  - Some tension between different measurements (D0 / LHCb)

$$A_{sl}(B^0_d) = (-2.1 \pm 1.7) \times 10^{-3}$$

$$A_{sl}(B^0_s) = (-0.6 \pm 2.0) \times 10^{-3}$$



Violation of CP-symmetry

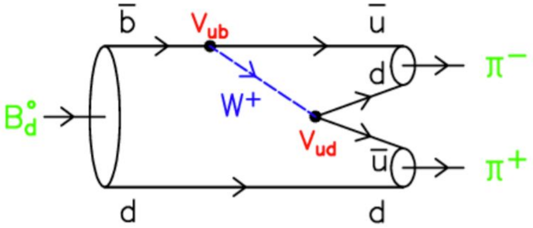
- **Direkte CP-violation** in B-meson decays ($B^0_d \to \pi^+\pi^-$) originates from interference between tree-level and penguin diagrams

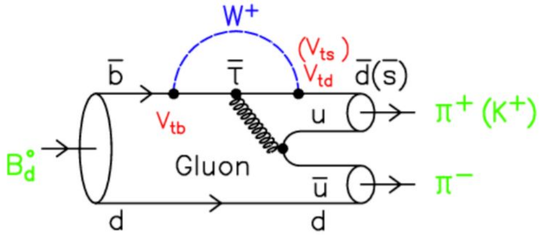
- First observed by **BaBar** and **Belle** via asymmetry:

$$A_{K\pi} = \frac{\Gamma(\overline{B}^0 \to K^-\pi^+) - \Gamma(B^0 \to K^+\pi^-)}{\Gamma(\overline{B}^0 \to K^-\pi^+) + \Gamma(B^0 \to K^+\pi^-)}$$

  - Later measured by CDF and LHCb

$$\mathcal{A}_{\overline{B}^0 \to K^-\pi^+} = -0.084 \pm 0.004$$
$$\mathcal{A}_{\overline{B}^0_s \to K^+\pi^-} = +0.213 \pm 0.017$$





Taken from: https://pdg.lbl.gov/2019/reviews/rpp2019-rev-cp-violation.pdf

Violation of CP-symmetry

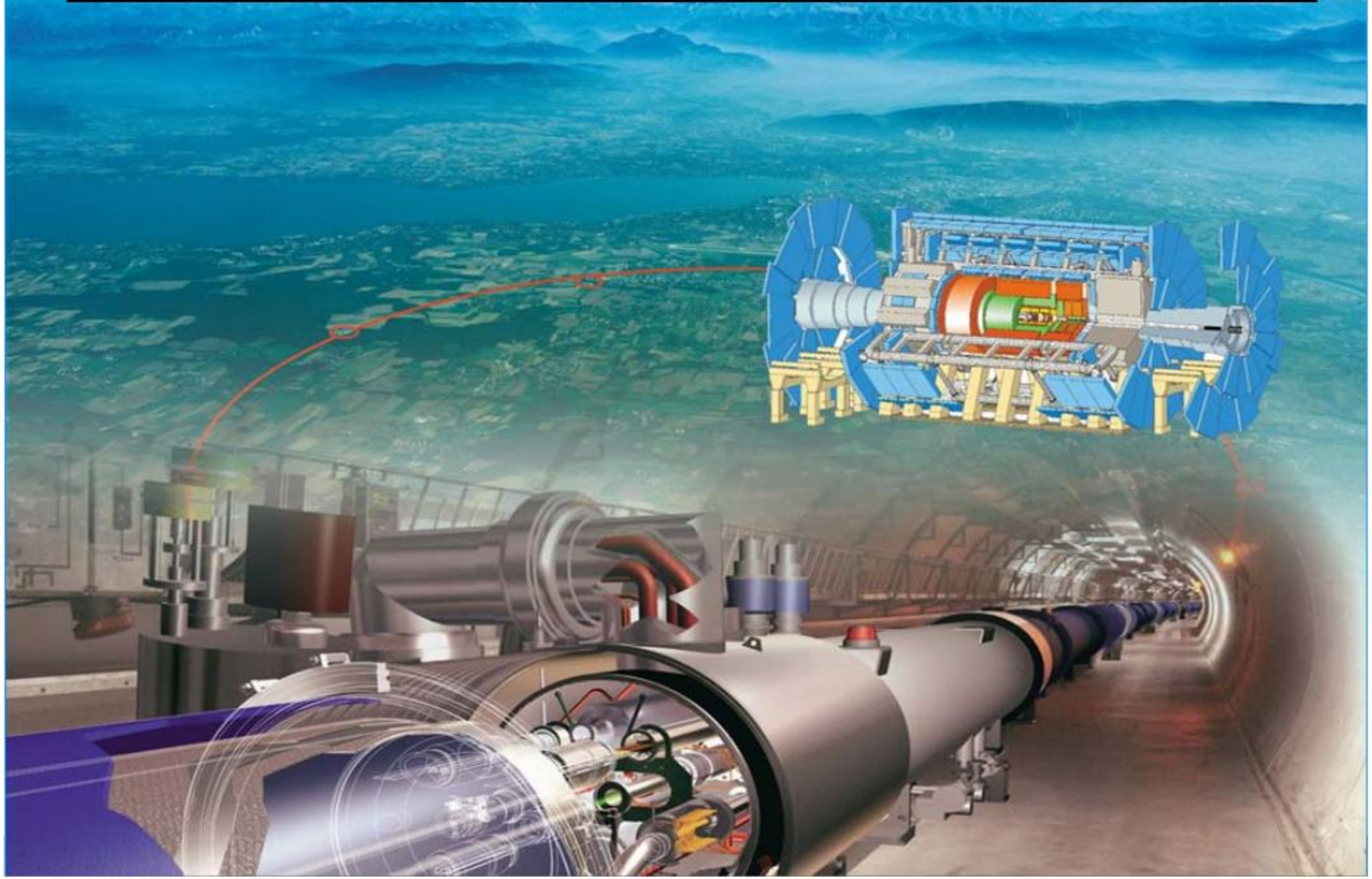
- Combination of direct and indirect CP violation:
  - Interference between CP-violating effects in decay and mixing
  - Resulting asymmetry has periodic time dependence
  - CP-violating asymmetries in both decay and mixing (time dependent)

$$A_f(t) = \frac{\Gamma\left(B^0 \to \overline{B}^0(t) \to f\right) - \Gamma\left(\overline{B}^0 \to B^0(t) \to f\right)}{\Gamma\left(B^0 \to \overline{B}^0(t) \to f\right) + \Gamma\left(\overline{B}^0 \to B^0(t) \to f\right)} = \boxed{S_f \sin \Delta m_d t - C_f \cos \Delta m_d t} \quad \textbf{(2)}$$

$S_f > 0$ or $S_f < 0$ gives **indirect** CP-violation

$C_f > 0$ or $C_f < 0$ gives **direct** CP-violation

4.5.5 Measurement of CKM matrix elements



Measurement of $|V_{cb}|$ and $|V_{ub}|$

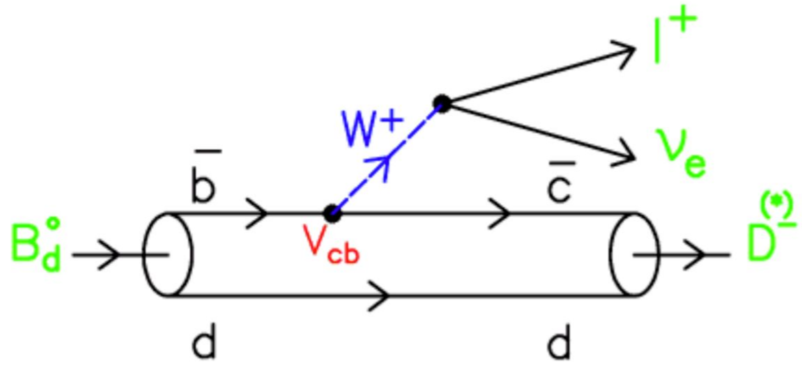
- Via branching ratios of semi-leptonic B-meson decays:

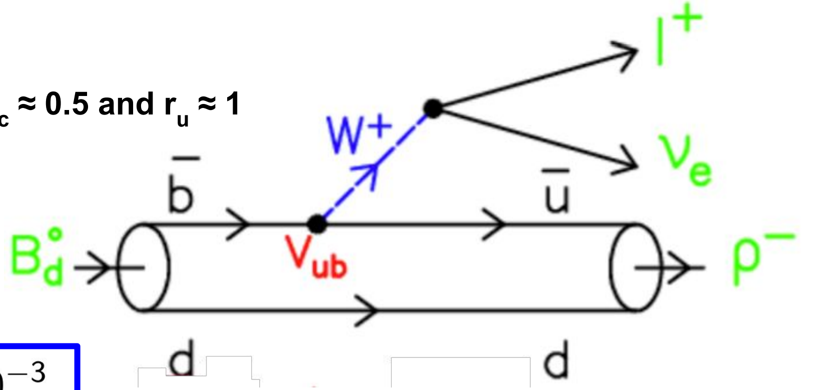
$$
\begin{aligned}
BR(B \to X\ell\nu_\ell) &= \frac{\Gamma(B \to X\ell\nu_\ell)}{\Gamma^B_{tot}} \\
&= \frac{G_F^2 m_b^5}{192\pi^3}\left(r_c(x)|V_{cb}|^2 + r_u(x)|V_{ub}|^2\right) \\
&\cdot \quad (1+\delta_{QCD}) \cdot \tau_B
\end{aligned}
$$

width: $\Gamma^B_{tot} = \tau_B^{-1}$

Phase space factors $r_c \approx 0.5$ and $r_u \approx 1$

QCD correction factors





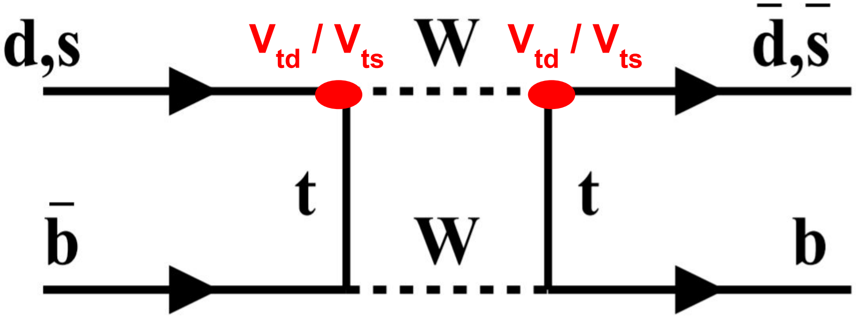
$|V_{cb}| = (42.1 \pm 0.7) \cdot 10^{-3}$

$|V_{ub}| = (3.65 \pm 0.12) \cdot 10^{-3}$

Measurement of $|V_{td}|$ and $|V_{ts}|$

- The $|V_{td}|$ and $|V_{ts}|$ are not likely to be precisely measurable in tree-level processes involving top quarks
  - Instead exploit neutral B-meson oscillations (i.e. box diagrams)
  - Uncertainties can be reduced by simultaneously measuring the ratio $|V_{td}/V_{ts}|$

$$\frac{\Delta m_d}{\Delta m_s} \approx (0.88 \pm 0.04)^2 \cdot \frac{|V_{td}|^2}{|V_{ts}|^2} \quad \Rightarrow \quad |V_{td}/V_{ts}| = 0.205 \pm 0.001 \pm 0.006$$



Measurement of $|V_{tb}|$

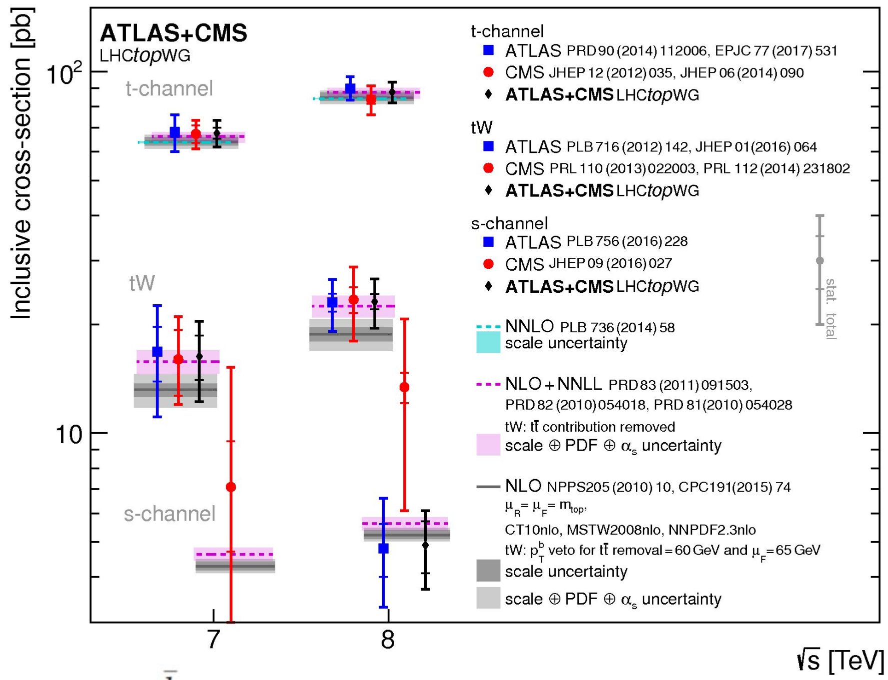
- From top-quark decays:

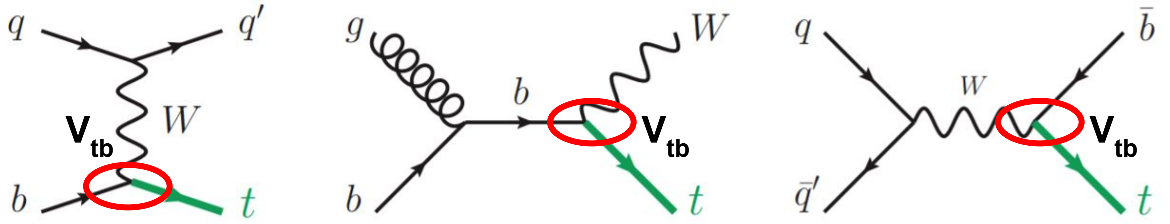
$$R = \frac{BR(t \to Wb)}{BR(t \to Wq)} = \frac{|V_{tb}|^2}{\sum_q |V_{tq}|^2} = |V_{tb}|^2$$

- From single top-quark cross section measurements:

$$|f_{LV} V_{tb}| = \sqrt{\frac{\sigma_{meas.}}{\sigma_{theo.}(V_{tb}=1)}}$$

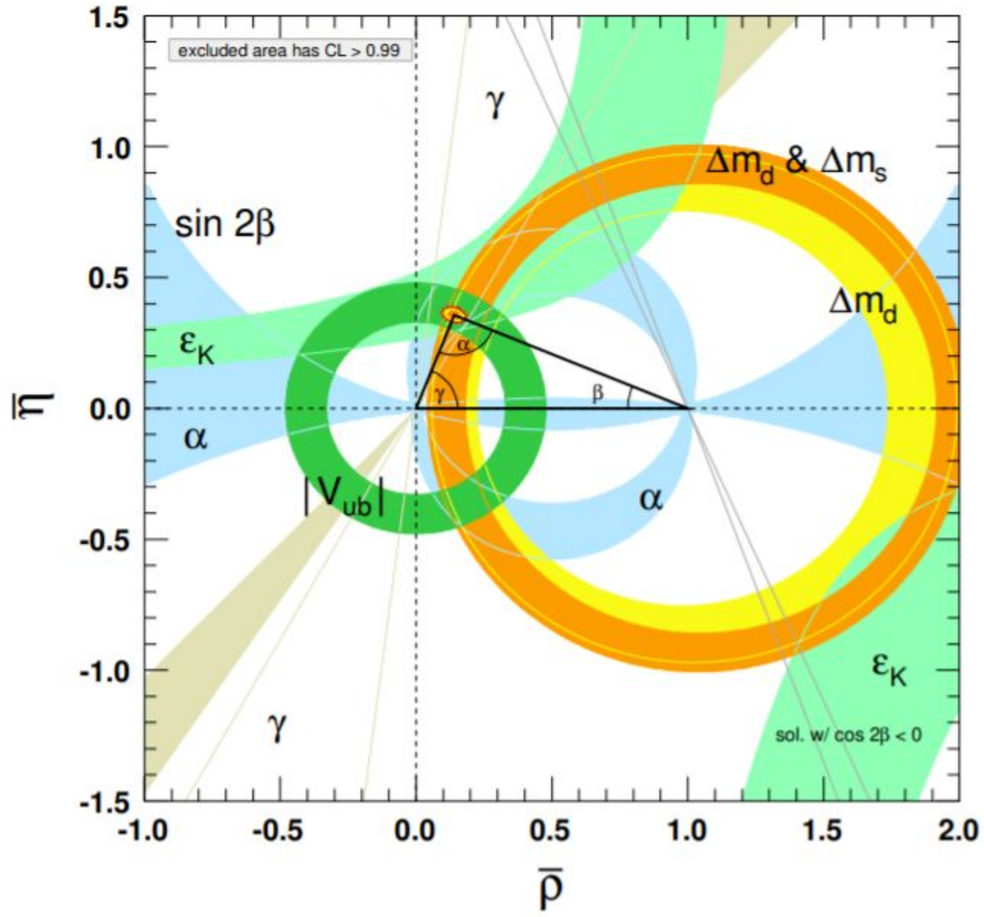
Equal to one in SM





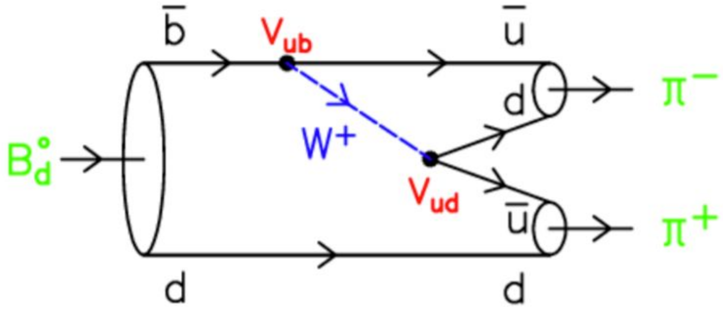
- World average: $|V_{tb}| = 1.013 \pm 0.030$

Determination of the unitarity triangle



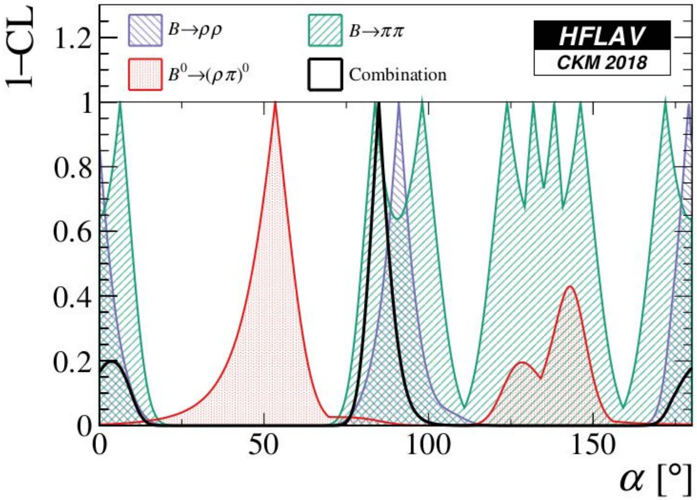
Determination of the unitarity triangle

$$a_{CP}(t) = \sin 2(\beta + \gamma) \sin \Delta m\, t$$
$$= \sin 2\alpha \sin \Delta m\, t$$



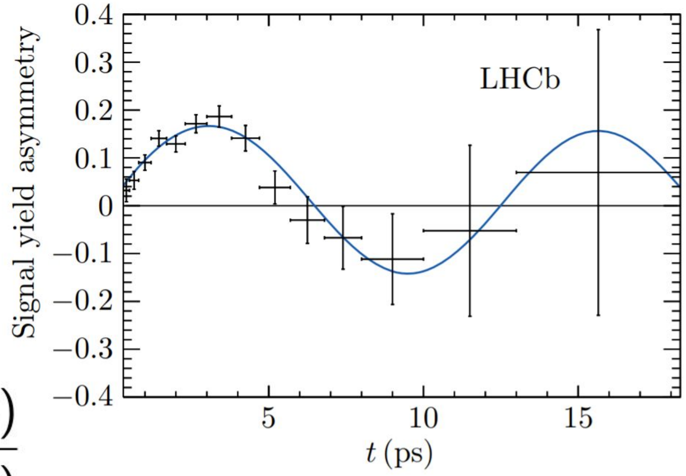
World average value:

$$\alpha = (84.9^{+5.1}_{-4.5})^\circ$$



Determination of the unitarity triangle

From B meson mixing



$$A_{CP}(t) = \frac{\Gamma(B^0(t) \to J/\Psi K_S^0) - \Gamma(\bar{B}^0(t) \to J/\Psi K_S^0)}{\Gamma(B^0(t) \to J/\Psi K_S^0) + \Gamma(\bar{B}^0(t) \to J/\Psi K_S^0)}$$
$$\approx -\sin 2\beta \sin \Delta m \cdot t$$

$$\sin 2\beta = 0.731 \pm 0.035\,(\text{stat.}) \pm 0.020\,(\text{syst.})$$

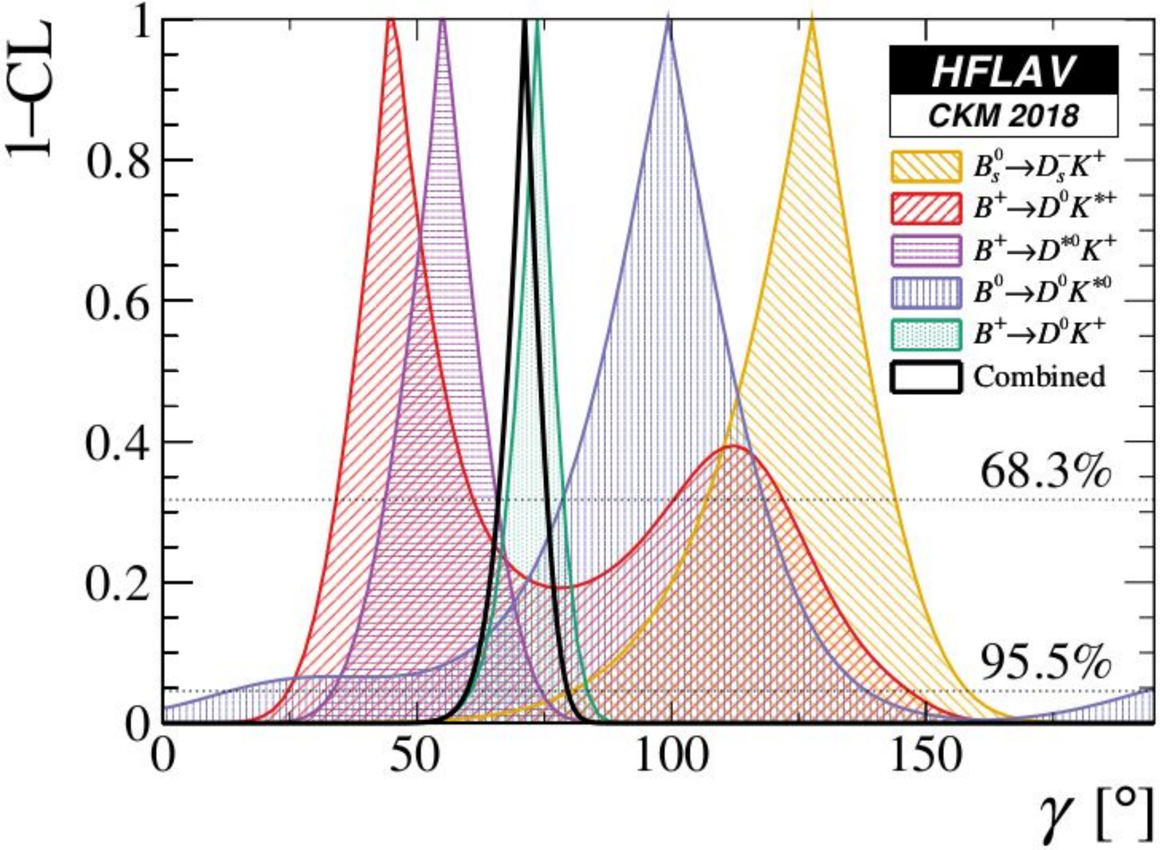
Taken from: https://journals.aps.org/prl/pdf/10.1103/PhysRevLett.115.031601

Determination of the unitarity triangle

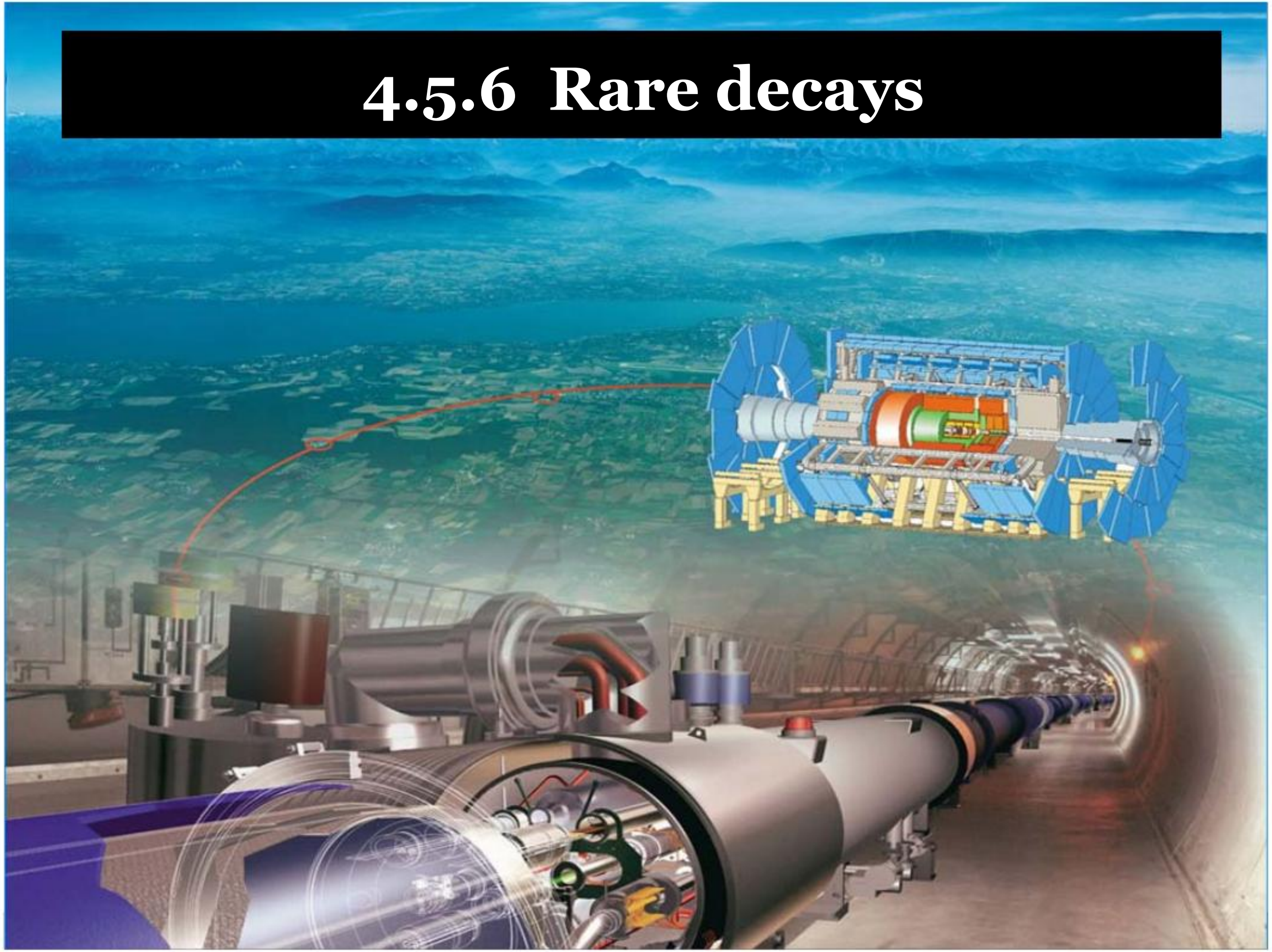
From CP asymmetries and decay rates in $B^{\pm} \to D^0 K^{\pm}$

World average value:

$$\gamma = (71.1^{+4.6}_{-5.3})^{\circ}$$



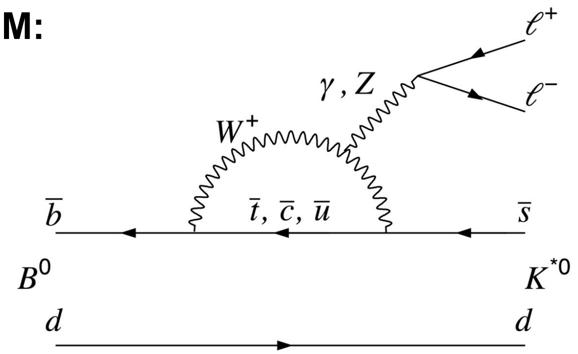
4.5.6 Rare decays



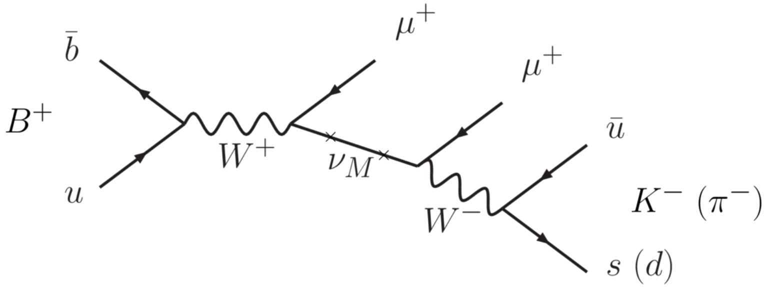
Rare decays

- Examples are:
  - **Fully leptonic decays:** $B \to \mu^+\mu^-$, $B_s \to \mu^+\mu^-$, $B_s \to \tau^+\tau$, $B \to \mu^+\mu^-\mu^+\mu^-$
  - **Electroweak penguin decays:** $B \to K^{*0}\mu^+\mu^-$, $B \to K^{*0}e^+e^-$, $\Lambda_b \to \Lambda\mu^+\mu^-$, …
  - **Radiative decays:** $B \to K^*\gamma$, $B_s \to \phi\gamma$, $\Lambda_b \to \Lambda^{(*)}\gamma$, $B \to \rho\gamma$
  - **Lepton flavour violation:** $\tau \to \mu\mu\mu$, $B \to \tau\mu$, $\Upsilon \to e\mu$
    - See: Dedicated section on Lepton Flavour Universality tests
  - **Lepton number violation:** $\tau \to p\mu\mu$, $B \to K\mu\mu$, $B \to D^{(*)}\mu\mu$ (same sign muons)

SM:



BSM:



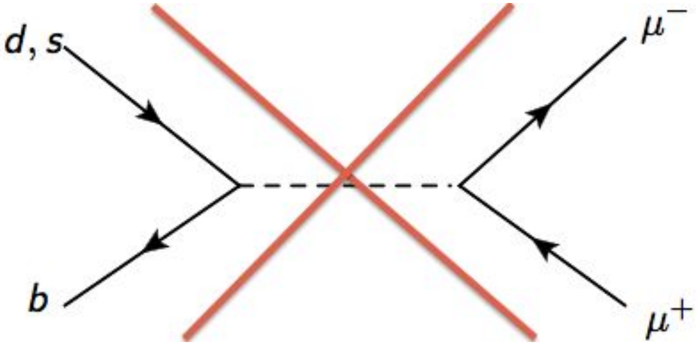
Newest LHCb results: https://lhcbproject.web.cern.ch/Publications/LHCbProjectPublic/Summary_RD.html

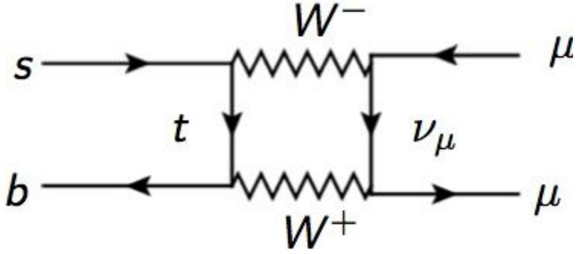
Rare decays

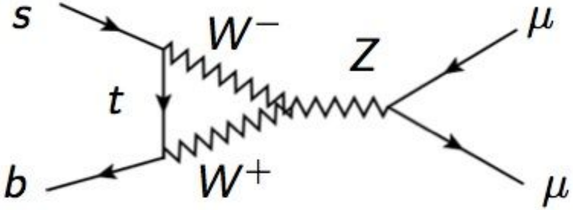
- Flavour-changing neutral-current processes are highly suppressed in the Standard Model.
- The branching fractions of the decays $B^0_s \rightarrow \mu^+\mu^-$ and $B^0 \rightarrow \mu^+\mu^-$ are also helicity suppressed:
    - They are predicted to be:
        - $BR(B^0_s \rightarrow \mu^+\mu^-) = (3.65 \pm 0.23) \times 10^{-9}$
        - $BR(B^0 \rightarrow \mu^+\mu^-) = (1.06 \pm 0.09) \times 10^{-10}$
    - The smallness and precision of these predicted branching fractions provide an ideal environment for observing contributions from new physics
    - Significant deviations from SM predictions could arise in models involving non-SM heavy particles:
        - Minimal Supersymmetric Standard Model
        - Minimal Flavour Violation
        - Two-Higgs-Doublet Models

Rare decays

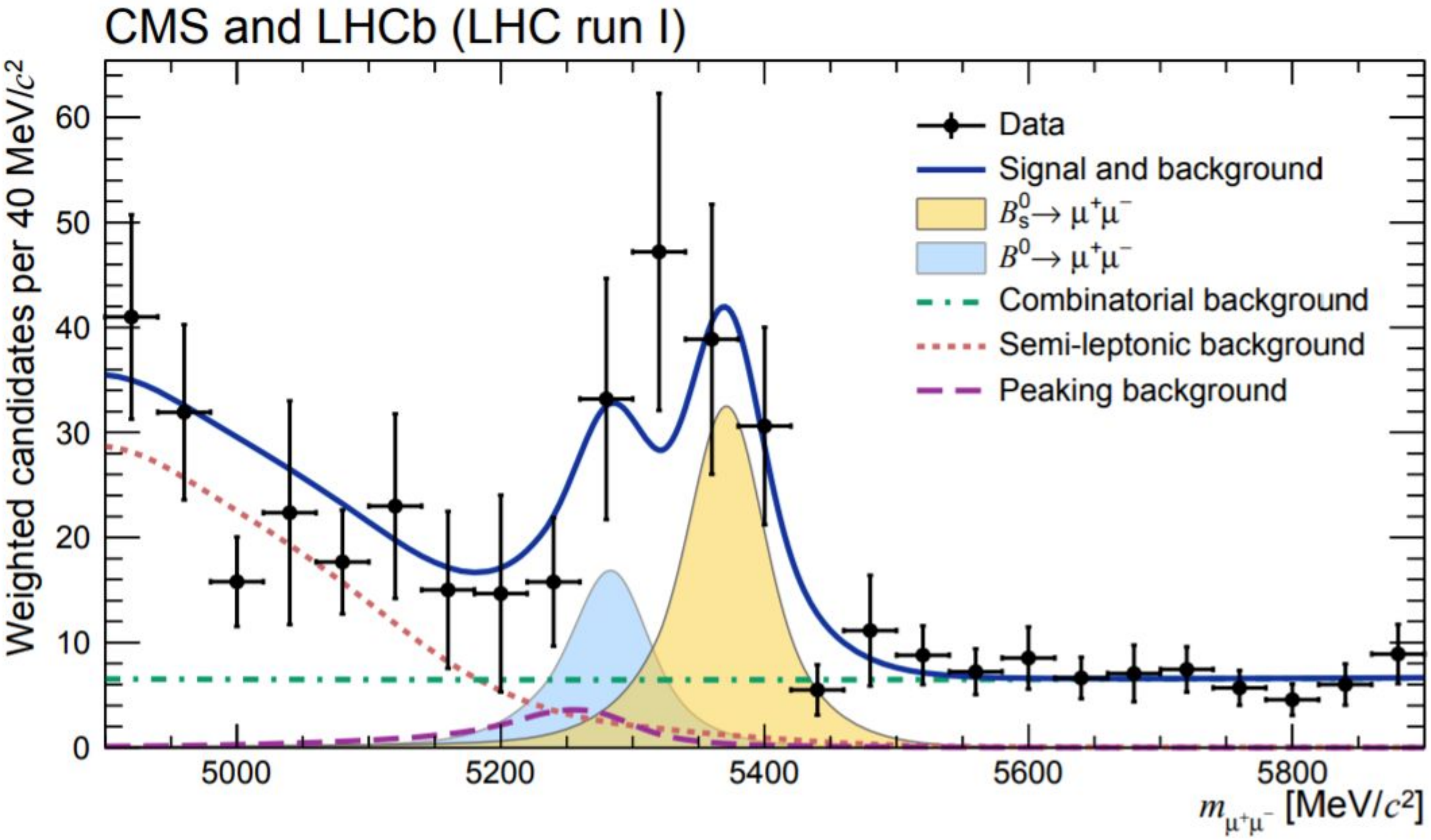
- The decays $B^0_s \rightarrow \mu^+\mu^-$ and $B^0 \rightarrow \mu^+\mu^-$ cannot proceed via tree-level processes, as they would involve flavor changing neutral currents.
  - Therefore, the process must proceed at a higher order than tree level





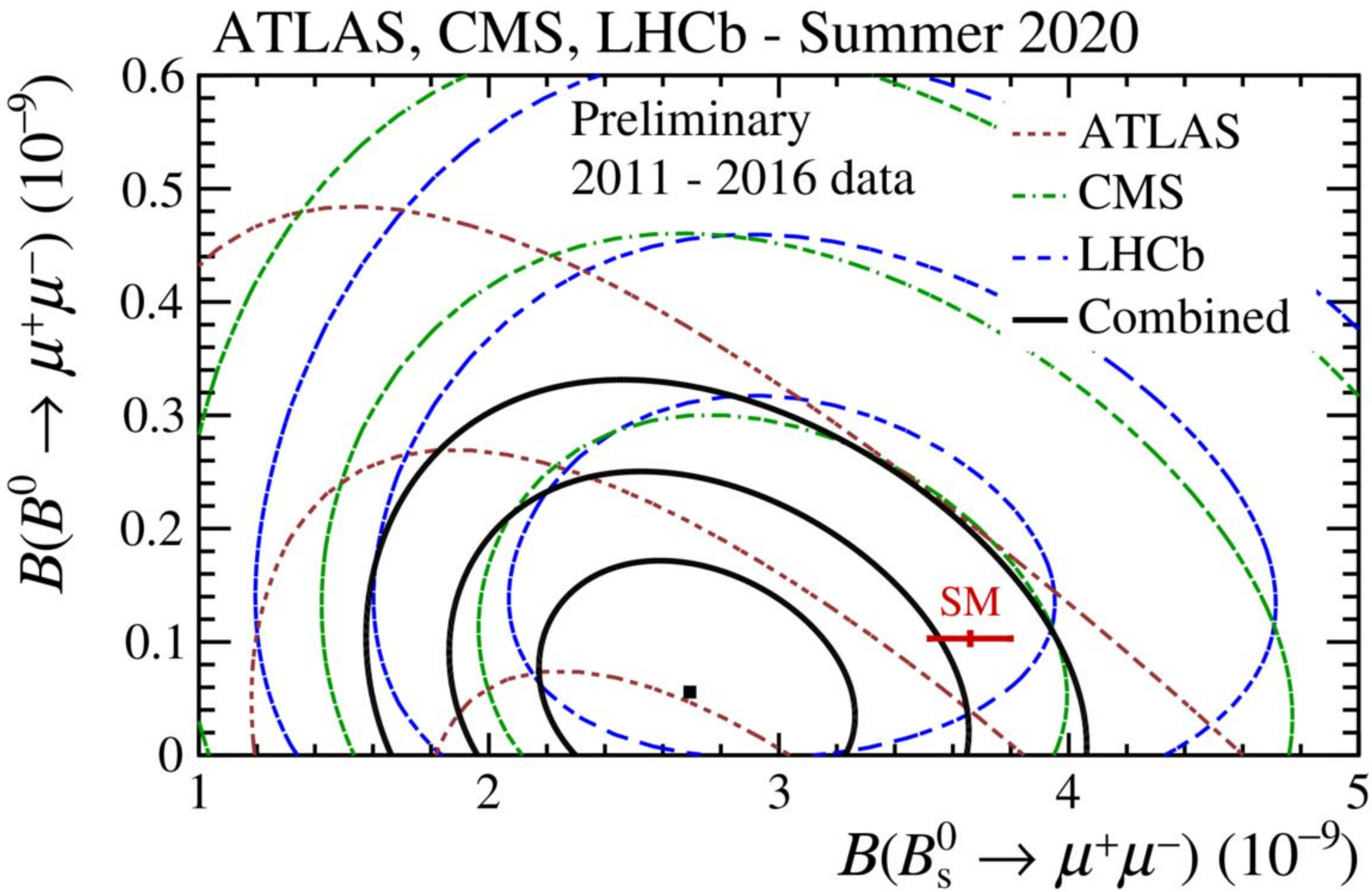


Rare decays

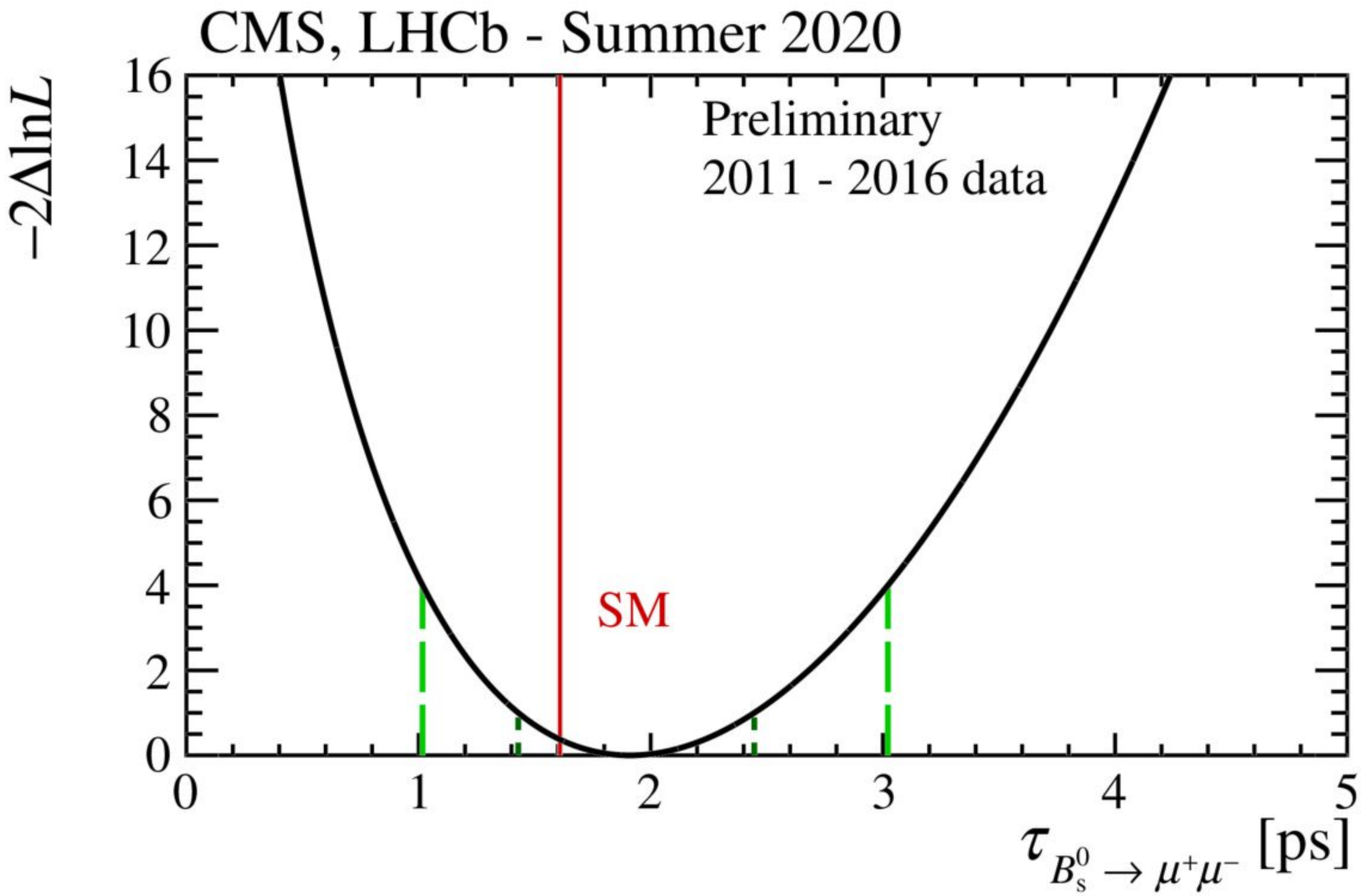


CMS and LHCb (LHC run I)
Weighted candidates per 40 MeV/c^2
$m_{\mu^+\mu^-}$ [MeV/c^2]
Data
Signal and background
$B_s^0 \to \mu^+\mu^-$
$B^0 \to \mu^+\mu^-$
Combinatorial background
Semi-leptonic background
Peaking background

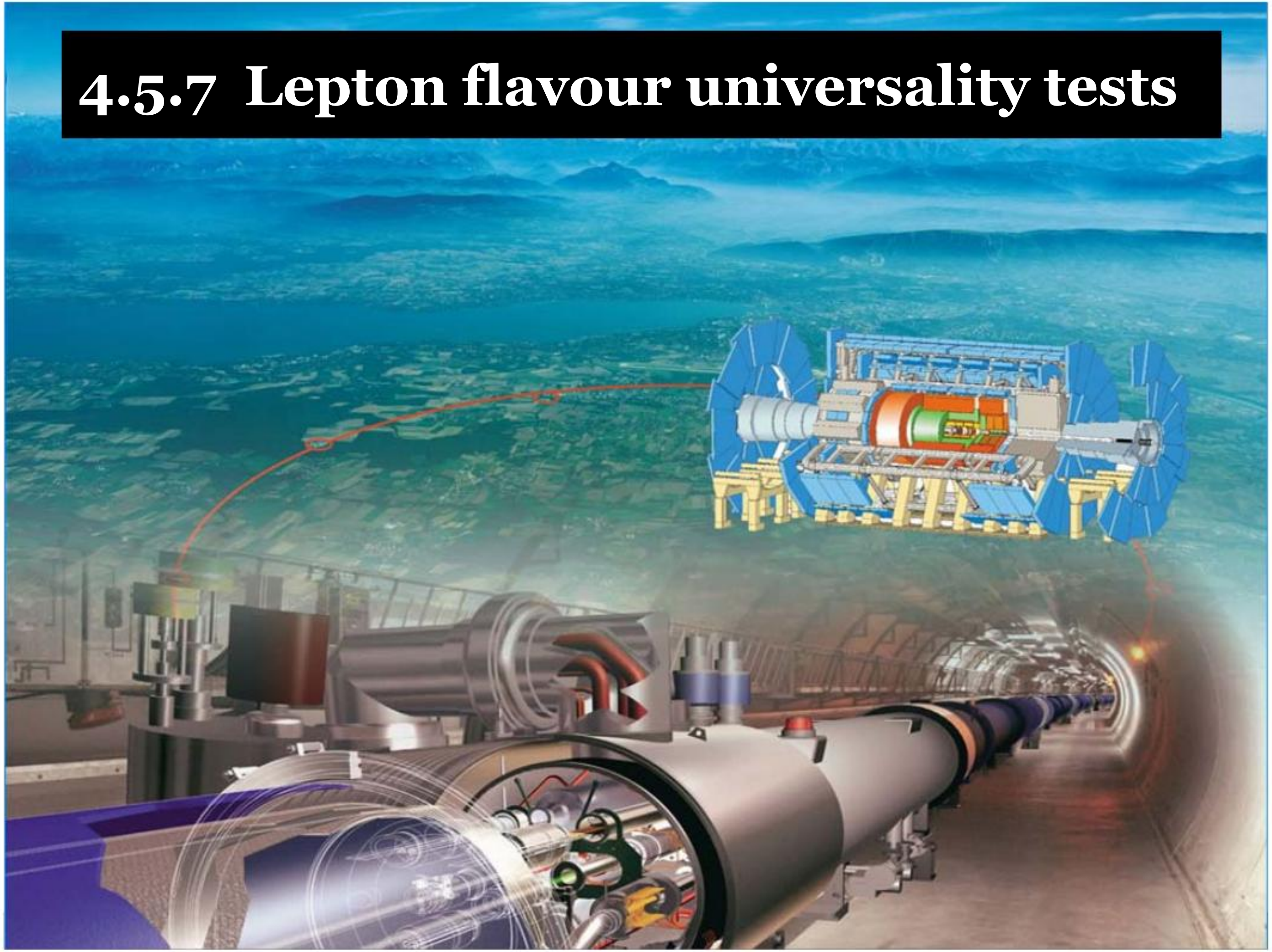
Rare decays: ATLAS, CMS & LHCb combinations



Rare decays: ATLAS, CMS & LHCb combinations



4.5.7 Lepton flavour universality tests

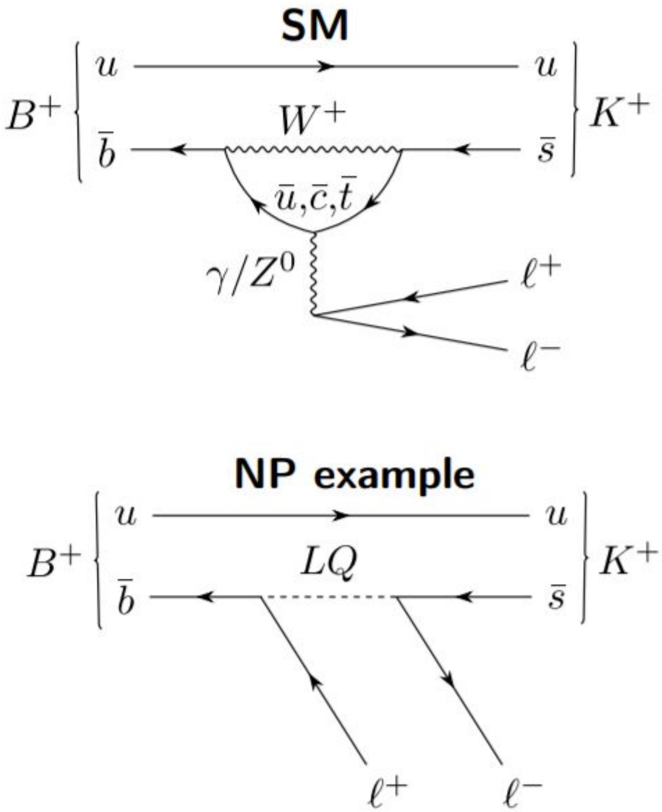


Lepton Flavour Universality tests

- In the SM couplings of gauge bosons to leptons are independent of lepton flavour
  - Branching fractions differ only by phase space and helicity-suppressed contributions
- LHCb is performing LFU tests in B hadron decays:

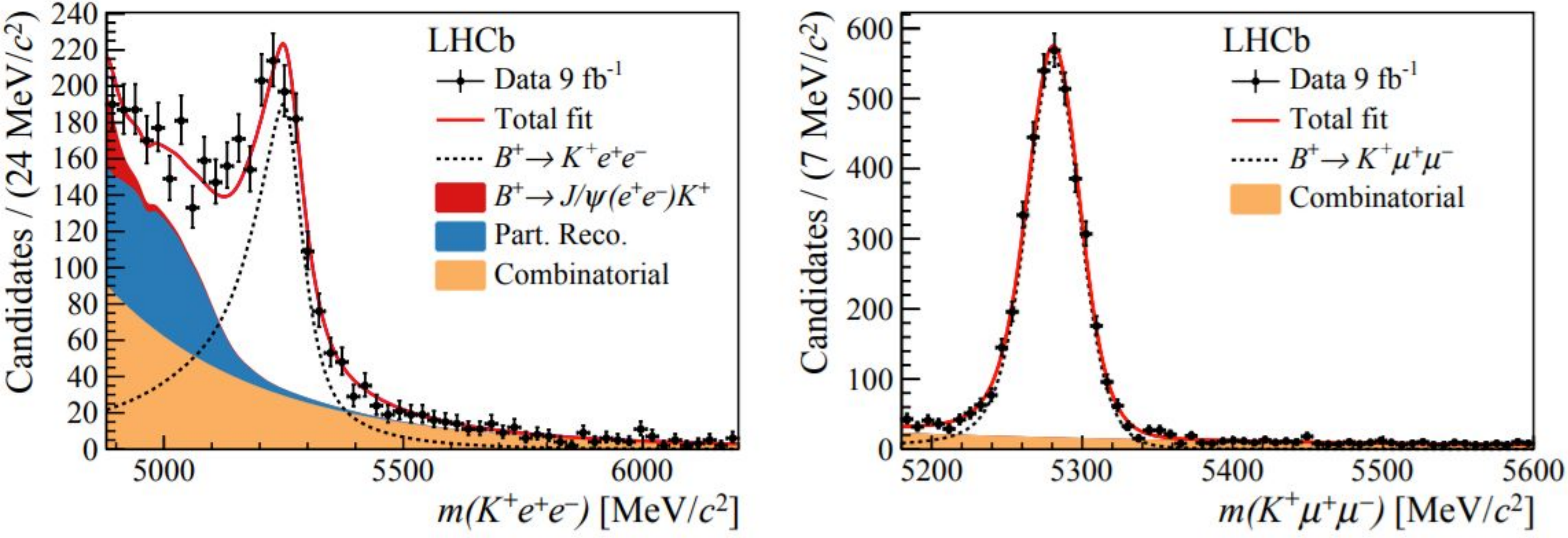
$$R_{K^{(*)}} = \frac{\mathcal{B}\left(B \to K^{(*)}\mu^+\mu^-\right)}{\mathcal{B}\left(B \to K^{(*)}e^+e^-\right)} \overset{\text{SM}}{\cong} 1$$

→ **Any significant deviation would be a smoking gun for New Physics.**



Taken from: https://indico.cern.ch/event/976688/attachments/2213706/3748404/RK_CernSeminar_Tue23rdMar2021.pdf

Lepton Flavour Universality tests

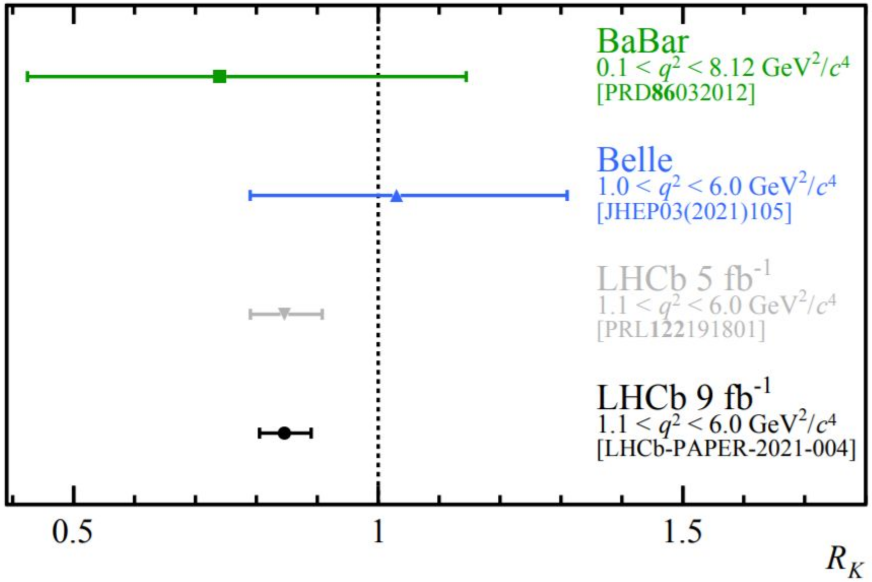


Measurement is based on study of invariant $K^+\ell\ell$ distribution and relies on excellent knowledge of electron/muon reconstruction efficiencies

Taken from: https://indico.cern.ch/event/976688/attachments/2213706/3748404/RK_CernSeminar_Tue23rdMar2021.pdf

Lepton Flavour Universality tests

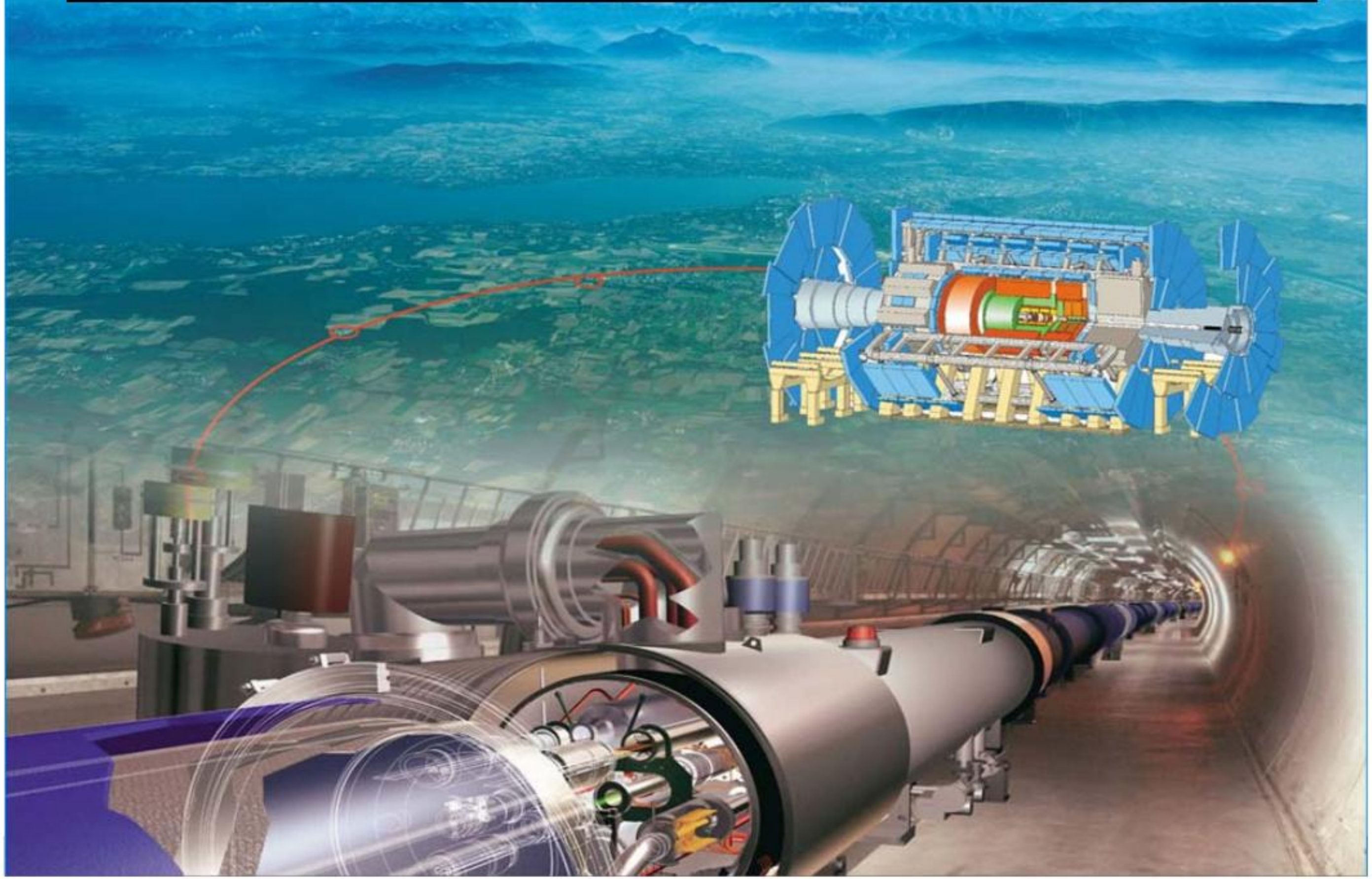
Newest results from LHCb



$$R_K = 0.846\ ^{+0.042}_{-0.039}\ (\mathrm{stat})\ ^{+0.013}_{-0.012}\ (\mathrm{syst})$$

→ Evidence of LFU violation at 3.1σ

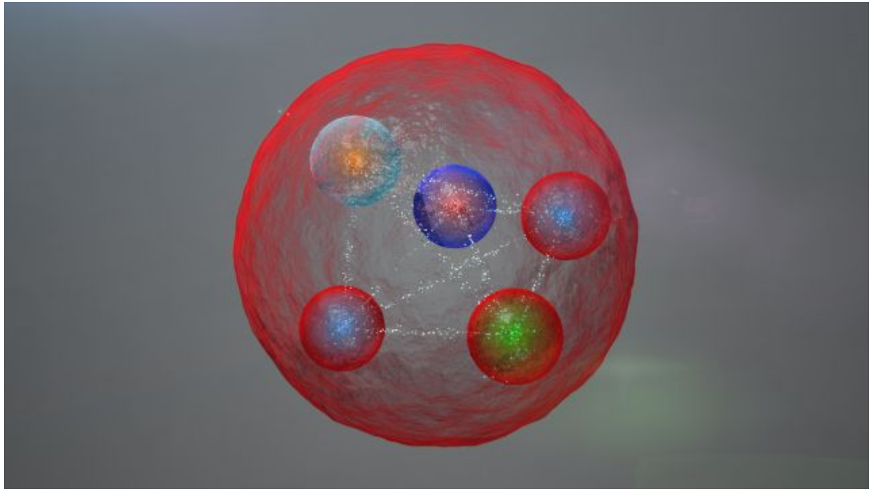
4.5.8 Pentaquarks



Pentaquarks

Pentaquarks = hadrons composed of **four quarks and one antiquark**

Observation of pentaquark states by LHCb in 2015 and 2019



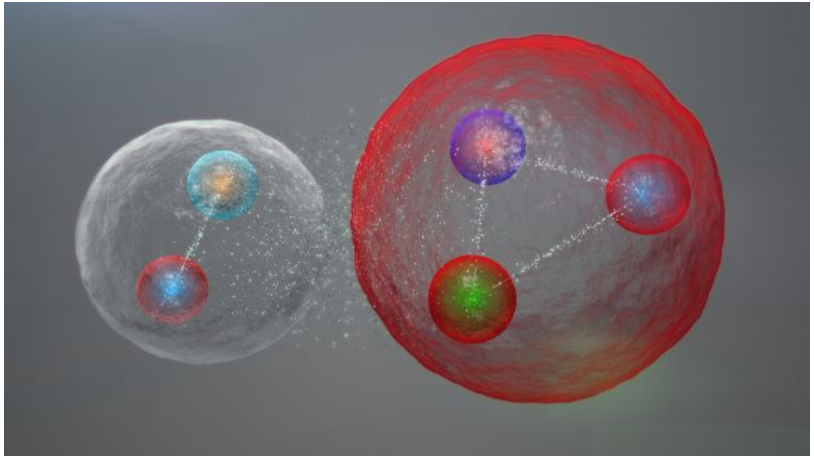


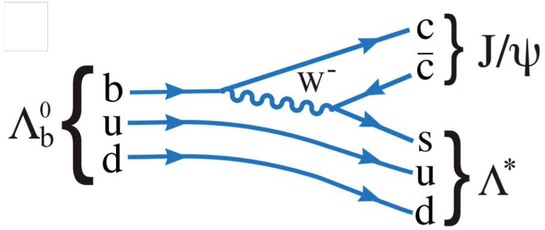
Illustration of the possible layout of the quarks in a pentaquark particle such as those discovered at LHCb. The five quarks might be assembled into a meson (one quark and one antiquark) and a baryon (three quarks), weakly bound together

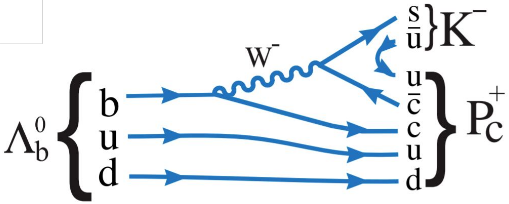
The five quarks might be tightly bonded

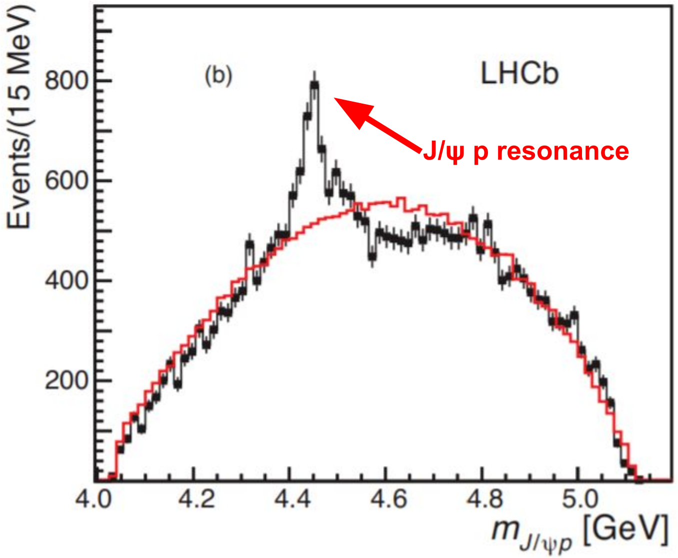
Taken from: https://home.cern/news/news/physics/lhcb-experiment-discovers-new-pentaquark

Pentaquarks

- The prospect of hadrons with more than the minimal quark content (qq or qqq) was proposed by Gell-Mann in 1964 [1] and Zweig [2]
  - Followed by a quantitative model for two quarks plus two antiquarks (Tetraquarks) [3].
  - The idea was expanded [4] to include hadrons composed of **four quarks plus one antiquark**
- Large yields of $\boldsymbol{\Lambda^0_b \rightarrow J/\psi\, K^- p}$ decays are available at LHCb
  - Expected to be dominated by $\boldsymbol{\Lambda^* \rightarrow K^- p}$
  - It could also have exotic decays via: $\boldsymbol{\Lambda_b^0 \rightarrow K^-\ P_C^+}$
- $\boldsymbol{P_C^+}$ mainly decays via: $\boldsymbol{P_C^+ \rightarrow J/\psi\, p}$

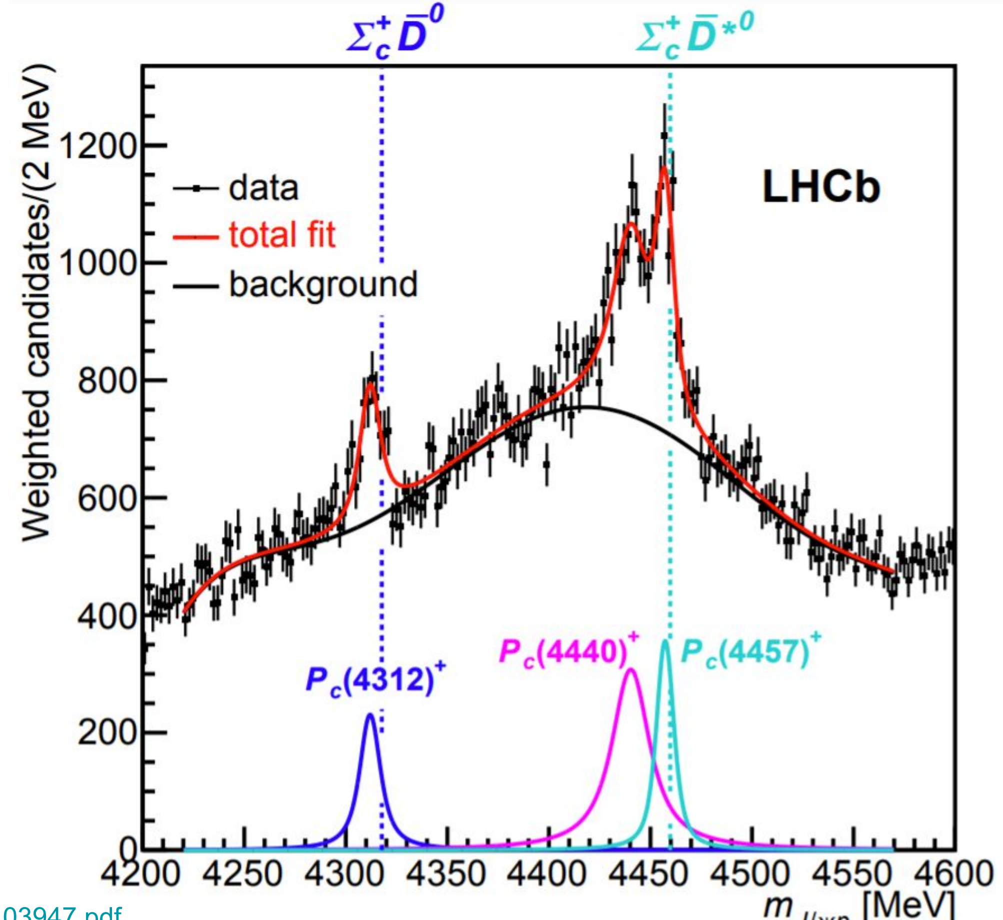






Taken from: https://arxiv.org/pdf/1507.03414.pdf

Pentaquark (spectroscopy)



Taken from: https://arxiv.org/pdf/1904.03947.pdf