Testing the Standard Model of Elementary Particle Physics II

7th lecture

Dr. Dominik Duda

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4.6 Neutrinos (Masses and Oscillation)



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4.6.1 Neutrino Oscillation



Neutrino masses and oszillation

• In the Standard Model, neutrinos are massless

- Only left-handed neutrinos (and right handed anti-neutrinos)
- Right-handed neutrinos do not participate in the weak interaction

• Neutrino oscillation:

- First observed in 1998
- Implies that neutrino must have nonzero mass
- As well as violation of lepton flavour conservation, as for quarks

	Mass	Measurements	Discovery
v _e	< 1.1 eV	KATRIN	Cowan. Reines 1956 (inverse β decay)
ν _μ	< 190 KeV	PSI Zürich	Ledermann. Schwartz, Steinberger 1962
V _T	< 18.2 MeV	ALEPH (LEP)	DONUT Experiment (FNAL) 2001

Neutrino masses and oszillation

- Massive neutrinos couple in the same way to the Higgs-field as quarks do:
 - The mass eigenstates of neutrinos differ from their flavour eigenstates
 - Mass matrix is not diagonal for the flavour eigenstates
 - Flavour and mass eigenstates are connected via a unitary transformation
- The unitary transformation for left-handed neutrino states:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}_L = U_u \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_L = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_L$$

with the mass eigenstates v_i (i = 1, ..., 3) and the flavour eigenstates v_{α} (α = e, μ , τ):

$$\nu_{i} = \sum_{\alpha} U_{\alpha i}^{*} \nu_{\alpha} ; \quad \bar{\nu}_{i} = \sum_{\alpha} U_{\alpha i} \bar{\nu}_{\alpha}$$
$$\nu_{\alpha} = \sum_{i} U_{\alpha i} \nu_{i} ; \quad \bar{\nu}_{\alpha} = \sum_{i} U_{\alpha i}^{*} \bar{\nu}_{i}$$

Neutrino masses and oszillation

- As a consequence:
 - \circ Time dependent oscillation between mixed states \rightarrow Neutrino oscillation
- U_{ii} is the **Pontecorvo-Maki-Nakagawa-Sakata** (PMNS) mixing matrix
 - If there are only n = 3 Majorana neutrinos, U_u is a 3 × 3 matrix analogous to the CKM matrix for the quarks
 - It depends on six independent parameters: three mixing angles and three phases

$U_u = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$	0 ^c 23 —s ₂₃	0 ⁵ 23 ^C 23	$) \cdot \begin{pmatrix} c_{13} \\ 0 \\ -s_{13}e^{i\delta} \end{pmatrix}$	0 1 0	s ₁₃ e ^{−iδ} 0 ^c 13) .	(-	^c 12 - <i>s</i> 12 0	512 612 0	0 0 1) · (∕e ^{iφ} 1 0 √0	0 e ^{iφ} 2 0	0 0 1)
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where sij = sin θ ij > 0, cij = cos θ ij > 0, a CP violating phase δ as well as the two Majorana phases Φ_1 and $\Phi_2 \in [0, 2\pi]$

More information can be found in: https://pdg.lbl.gov/2020/reviews/rpp2020-rev-neutrino-mixing.pdf

• After travelling a distance L, neutrino states evolve as:

$$\ket{
u_lpha(t)} = \sum_{i=1}^n U^*_{lpha i} \ket{
u_i(t)}$$

• These neutrinos can then undergo a charged-current (CC) interaction producing a charge lepton ℓ_{β} , $v_{\alpha}(t) N' \rightarrow \ell_{\beta} N$, with a probability:

$$egin{split} P_{lphaeta} &= \left| \langle
u_eta |
u_lpha(t)
angle
ight|^2 = \left| \sum_{i=1}^n \sum_{j=1}^n U^*_{lpha i} U_{eta j} \left\langle
u_j |
u_i(t)
ight
angle
ight|^2 \end{split}$$

• Assume that $|v_i > is a plane wave:$

$$|\nu_i(t)\rangle = e^{-iE_it} |\nu_i(0)\rangle$$
 with: $E_i = \sqrt{p_i^2 + m_i^2} \approx p + \frac{m_i^2}{2p}$

More information can be found in: https://pdg.lbl.gov/2020/reviews/rpp2020-rev-neutrino-mixing.pdf



- Same derivation for antineutrino states would give a similar expression but with the exchange U → U^{*}
- With the oscillation lengths:

$$L_{ij}^{osc.} = \frac{4\pi E}{|\Delta m_{ij}^2|}$$

For n = 2 generations

• Mixing between two neutrino generations (with mixing angle θ and oscillation frequency $\Delta m^2 = m_2^2 - m_1^2$):

$$\left(\begin{array}{c}\nu_{e}\\\nu_{\mu}\end{array}\right) = \left(\begin{array}{cc}\cos\theta & \sin\theta\\-\sin\theta & \cos\theta\end{array}\right) \cdot \left(\begin{array}{c}\nu_{1}\\\nu_{2}\end{array}\right) = U \cdot \left(\begin{array}{c}\nu_{1}\\\nu_{2}\end{array}\right)$$

• Transition probability:

$$P_{lphaeta} = \delta_{lphaeta} - (2\delta_{lphaeta} - 1)\sin^2 2 heta\sin^2 X$$

Characteristic values of L and E for experiments performed using various neutrino sources and the corresponding ranges of $|\Delta m^2|$ to which they can be most sensitive to flavour oscillations in vacuum. SBL stands for Short Baseline and LBL for Long Baseline.

Experiment		L (m)	E (MeV)	$ \Delta m^2 \text{ (eV}^2)$
Solar		10^{10}	1	10^{-10}
Atmospheric		$10^4 - 10^7$	$10^2 - 10^5$	$10^{-1} - 10^{-4}$
Reactor	SBL	$10^2 - 10^3$	1	$10^{-2} - 10^{-3}$
	LBL	$10^4 - 10^5$		$10^{-4} - 10^{-5}$
Accelerator	SBL	10^{2}	$10^{3} - 10^{4}$	> 0.1
	LBL	$10^5 - 10^6$	$10^3 - 10^4$	$10^{-2} - 10^{-3}$

- Neutrinos propagating in a dense medium can interact with the particles in the medium
- The probability of an incoherent inelastic scattering is very small
 - The characteristic cross section for v-proton scattering is of the order:



- When neutrinos propagate in dense matter, they can also interact coherently with the particles in the medium
 - In coherent interactions, the medium remains unchanged so it is possible to have interference of the forward scattered and the unscattered neutrino waves which enhances the oscillation effects

More information can be found in: https://pdg.lbl.gov/2020/reviews/rpp2020-rev-neutrino-mixing.pdf

- For neutrinos with relatively low energy (i.e. neutrinos from nuclear fusion in the sun), matter is transparent such that only elastic scattering with electrons give a significant cross section:

 - $E_v \le 0.42$ MeV (pp-fusion) $E_v \le 14.6$ MeV (⁸B-fusion)
- **Representative Feynman-diagrams:**



For n = 2 generations

• Oscillation probability, taking varying potential (which depends on the density and composition of the matter) into account:

$$P_{\alpha\beta} = \left| \sum_{i} \tilde{U}_{\alpha i}(0) \tilde{U}_{\beta i}(L) \exp\left(-\frac{i}{2E} \int_{0}^{L} \mu_{i}^{2}(x') dx'\right) \right|^{2}$$

with the effective masses:

$$\mu_{1,2}^2(x) = \frac{m_1^2 + m_2^2}{2} + E[V_\alpha + V_\beta] \mp \frac{1}{2} \sqrt{[\Delta m^2 \cos 2\theta - A]^2 + [\Delta m^2 \sin 2\theta]^2}$$

Potential
$$A = 2E(V_\alpha - V_\beta)$$

Resonance condition:

$$A_R = \Delta m^2 \cos 2\theta$$

• The Mihheev-Smirnov-Wolfenstein Effect for Solar Neutrinos:

- Matter effects are relevant for solar neutrinos
 - In particular if the sun produces v_e in its core (have to consider the propagation of a $v_e v_x$ neutrino system, with X indicating superposition of µ and τ)
- The density of solar matter decreases monotonically as a function of the distance R from the center of the sun (for R < 0.9 R_{\odot}):

$$n_e(R) = n_e(0)e^{-10.5R/R_{\odot}}$$

- Distinguish between three relevant cases
 - After crossing the sun, the composition of the neutrino state exiting the sun will depend on the relative size of $\Delta m^2 \cos 2\theta$ versus $A_0 = 2 E G_F n_{e,0}$ (with $n_{e,0}$ being the density at the point of neutrino production)

1) If the relevant matter potential at the production point is well below the resonant value, $A_R = \Delta m^2 \cos 2\theta >> A_0$, matter effects are negligible (propagation occurs as in vacuum):

$$P_{ee}\left(\Delta m^2\cos 2 heta >> A_0
ight) = 1 - rac{1}{2}\sin^2 2 heta > rac{1}{2}$$

2) If the relevant matter potential at production is only slightly below the resonant value, $A_R = \Delta m^2 \cos 2\theta \sim A_0$, the neutrino does not cross a region with resonant density, but matter effects are sizable enough to modify the mixing:

$$P_{ee}\left(\Delta m^2\cos 2\theta \geq A_0\right) = \cos^2\theta_{m,0}\cos^2\theta + \sin^2\theta_{m,0}\sin^2\theta = \frac{1}{2}\left[1 + \cos 2\theta_{m,0}\cos 2\theta\right]$$

3) In the case that $A_R = \Delta m^2 \cos 2\theta < A_0$, the neutrino can cross the resonance on its way out:

$$P_{ee}\left(\Delta m^2\cos 2\theta < A_0\right) = \frac{1}{2}\left[1 + \cos 2\theta_{m,0}\cos 2\theta\right] = \sin^2\theta$$





Taken from: https://arxiv.org/abs/1111.0507

Atmospheric neutrinos

• Atmospheric neutrino flux

- The primary cosmic rays, mostly protons, interact with molecules of the atmosphere and produce pions and kaons.
 - Neutrinos are created by the decay of $\pi/K \to \mu + v_{\mu}$ and also by the subsequent decay of $\mu \to e + v_{\mu} + v_{e}$
 - They have a broad range of energy (~0.1 GeV to >TeV) and long travel distances before detection (~10 to 1.3 × 10⁴ km)
- Experiments:
 - Early experiments:
 - Kamiokande or IMB (based on water Cherenkov detectors)
 - Frejus or NUSEX (based on iron tracking calorimeters)
 - Recent experiments:
 - Super-Kamiokande
 - OPERA
 - ANTARES and IceCube (although they mainly target high energy neutrinos)

Detection of Atmospheric Neutrinos

- How to detect atmospheric neutrinos (with Super-Kamiokande):
 - $\circ \quad v + N \rightarrow I + N' (+ \pi/K)$

• Super-Kamiokande Detector:

- \circ The 50 kton water
- 11000 photomultiplier tubes
- Placed 1000m underground
- Events in Super-K are classified into different event types:
 - \circ Upward going muons (Up-µ):
 - \circ Upward stopping muons (Stop- μ)
 - Others ...

Taken from: https://link.springer.com/content/pdf/10.1140/epjc/s10052-019-6796-2.pdf





Atmospheric neutrinos



- In the Sun, electron neutrinos are produced in the thermonuclear reactions which generate the solar energy
 - These reactions occur via two main processes, the **pp chain** and the **CNO cycle**
 - Result in the overall fusion of protons into:

$$4p \rightarrow He^4 + 2e^+ + 2v_e + 26.73 \text{ MeV}$$

where most of the energy is radiated through photons and only a small fraction is carried by neutrinos $\langle E_{2v} \rangle = 0.59 \text{ MeV}$

- In addition, electron capture on ¹³N, ¹⁵O, and ¹⁷F produces line spectra of neutrinos called ecCNO neutrinos
- At the Earth, the pp solar neutrino flux is determined to be $6 \times 10^{10} \text{ cm}^{-2} \text{s}^{-1}$
 - Based on the Standard Solar Model (SSM)



Taken from: <u>https://iopscience.iop.org/article/10.1088/1742-6596/1056/1/012050f</u>



Solar neutrino experiments

List of former and ongoing solar neutrino experiments

Name	Target material	Energy threshold (MeV)	Mass (ton)	Years
Homestake	C_2Cl_4	0.814	615	1970 - 1994
SAGE	Ga	0.233	50	1989 -
GALLEX	$GaCl_3$	0.233	100 [30.3 for Ga]	1991 - 1997
GNO	$GaCl_3$	0.233	100 [30.3 for Ga]	1998 - 2003
Kamiokande	H_2O	6.5	3,000	1987 - 1995
Super-Kamiokande	H_2O	3.5	50,000	1996 -
SNO	D_2O	3.5	1,000	1999 - 2006
KamLAND	Liquid scintillator	0.5/5.5	1,000	2001 - 2007
Borexino	Liquid scintillator	0.19	300	2007 -

Detection of solar neutrinos and the solar neutrino problem

• Early solar neutrino experiments (e.g. Homestake):

Based on:
$$v_e^{+37}CI \rightarrow e^{-} + {}^{37}Ar$$

- Most relevant fluxes are from the ⁷Be and ⁸B neutrinos (because the process energy threshold of 814 keV)
- The detector contained ~ 615 t of C_2Cl_4
- The produced ³⁷Ar, which has a half life of 34.8d, was chemically extracted and introduced into a low-background proportional chamber every few months.
 - The Auger electrons from electron capture of ³⁷Ar were counted to determine the reaction rate.
- Of course the observed number of neutrinos in the Homestake mine experiment was significantly smaller than the prediction by SSM

 \rightarrow solar neutrino problem

Further information: https://pdg.lbl.gov/2020/reviews/rpp2020-rev-neutrino-mixing.pdf

• Borexino is a liquid scintillator

- Primary goal: Measure the fluxes of solar neutrinos
- Located in the Laboratori Nazionali del Gran Sasso at a depth of 3800m water-equivalent
- Detection of neutrinos via elastic scattering with electrons of the organic scintillator molekules

 $v e^- \rightarrow v e^-$







Fluxes of ⁸B solar neutrinos, ϕ (v_e), and ϕ (v_{µ,τ}), deduced from results provided by SNO and Super-Kamiokande

Accelerator Neutrinos

- Conventional neutrino beams from accelerators are produced by colliding high energy protons onto a target.
 - \circ Producing π and K which then decay into neutrinos
 - Other particles are stopped via a beam dump
 - Magnets are used to concentrate the neutrino beam flux towards the desired direction
 - Neutrino energy is proportional to pion energy

Name	Beamline	Far Detector	L (km)	E_{ν} (GeV)	Year
K2K	KEK-PS	Water Cherenkov	250	1.3	1999-2004
MINOS	NuMI	Iron-scintillator	735	3	2005-2013
MINOS+	NuMI	Iron-scintillator	735	7	2013-2016
OPERA	CNGS	Emulsion	730	17	2008 - 2012
ICARUS	CNGS	Liquid argon TPC	730	17	2010-2012
T2K	J-PARC	Water Cherenkov	295	0.6	2010 -
NOvA	NuMI	Liquid scint. tracking calorimeter	810	2	2014 -

Accelerator Neutrinos



Reactor Neutrino Experiments

- Nuclear reactors are very intense sources of anti-electron neutrinos in the MeV energy region, which are generated in nuclear fission of heavy isotopes (mainly ²³⁵U, ²³⁸U, ²³⁹Pu, or ²⁴¹Pu)
 - The neutrino flux from a reactor can be estimated based on the thermal power output and fuel composition as a function of time

Name	Reactor power (GW_{th})	Baseline (km)	Detector mass (t)	Year
KamLAND	various	180 (ave.)	1,000	2001 -
Double Chooz	$4.25{ imes}2$	1.05	8.3	2011 - 2018
Daya Bay	$2.9{ imes}6$	1.65	20×4	2011 -
RENO	2.8×6	1.38	16	2011 -
JUNO	26.6 (total)	53	20,000	

Reactor Neutrinos

- **KamLAND** (Kamioka Liquid Scintillator Antineutrino Detector)
 - Uses 1,000 t of ultra-pure liquid scintillator contained in a 13m diameter spherical balloon
 - Located in the original Kamiokande cavern, where the neutrino flux is dominated by a few reactors at an average distance of ~180 km
 - Reported first results in 2002 show that the ratio of the observed number of anti-electron neutrinos and the expectations is: 0.611 ± 0.085 ± 0.041



Reactor Neutrinos



Reactor Neutrinos

 The ratio of measured to expected v_e flux from reactor experiments



Neutrinos from cosmic accelerators

- Neutrinos from cosmic accelerators:
 - Galactic sources:
 - Supernova remnants (SNR)
 - Extragalactic sources:
 - Active Galactic Nuclei (i.e. supermassive black holes with 10⁶ –10⁹ solar masses that accrete matter and thus transform huge amounts of gravitational energy into radiation)
 - Gamma Ray Bursts (i.e. events releasing huge amounts of energy in gamma rays with milliseconds to minutes. The favoured explanation for the longer bursts is the collapse of a massive star into a black hole)
 - Starburst galaxies (i.e. galaxies undergoing an episode of large-scale star formation, where the central regions eject a galactic-scale wind driven by the collective effect such as supernova explosions)



Neutrinos from cosmic accelerators

• Neutrinos from cosmic accelerators:

- Neutrino production:
 - Protons from cosmic accelerators generate neutrinos, via charged pion production due to collisions with matter or radiation fields:

$$egin{aligned} & p + ext{nucleus}
ightarrow \pi + X \; (\pi = \pi^{\pm}, \pi^0) \ & p + \gamma
ightarrow \Delta^+
ightarrow \pi^0 + p \end{aligned}$$

$$\pi^0 \to \gamma\gamma, \quad \pi^+ \to \mu^+ \nu_{\nu}, \quad \mu^+ \to e^+ \bar{\nu}_{\mu} \nu_{e}$$

- The neutrino flavour ratio:
 - At the source: $v_e : v_{\mu} : v_{\tau} = 1 : 2 : 0$
 - At earth: $v_e : v_\mu : v_\tau = 1 : 1 : 1$



Taken from: https://arxiv.org/abs/1111.0507

Neutrinos from cosmic accelerators



The IceCube Detector



ANTARES

ANTARES consists of a sparse array of PMTs deployed under the Mediterranean Sea at a depth of about 2.5 km to instrument a 10⁵m³ volume







Types of Neutrinos

• Dirac-Neutrinos:

• Neutrino and antineutrino are different

• Majorana-Neutrino:

- Neutrinos with definite masses are Majorana particles
- \circ Majorana mass term \rightarrow See-saw mechanism
 - Would explain why neutrino masses are so much smaller than masses of other SM particles
 - See-saw mechanism for neutrino masses are described in e.g. SO(10) Grand Unified Theories
 - Require heavy neutrinos beyond the scale of electroweak symmetry breaking
 - Dark matter candidates

• Sterile Neutrinos:

 Hypothetical particles which are assumed to interact only via gravity and not via any other the fundamental interactions of the SM

Neutrinoless double beta decay

- Neutrinoless double beta decays (0vββ) are the most sensitive probes to whether neutrinos are Dirac or Majorana states
 - \circ In order to induce the $0\nu\beta\beta$ decay, neutrinos must be Majorana particles
 - The $0\nu\beta\beta$ decay leads to a lepton number violation
- The half-life of the decay is to be determined by experiments (assuming that the Majorana neutrino mass is the only source of lepton number violation at low energies):



phase space integral taking into account the final atomic state

 ν_M

Neutrinoless double beta decay

- Experimental Search for Neutrinoless Double-beta Decay:
 - The $0\nu\beta\beta$ signature: sum of energy of two electrons is equal to the Q-value of the nuclear transition.
 - The sensitivity to the half-life is proportional to:
 - Detection efficiency of the signal
 - Source mass
 - The background rate
 - The energy resolution
 - There are 35 candidate nuclei for double-beta decay
 - Recent experiments use: ⁷⁶Ge or ¹³⁶Xe
 - The energy from the outgoing electrons is measured with either ionization or scintillation
 - Recent Experiments:
 - GERDA
 - AMoRE
 - NEMO-3
 - COBRA (more information can be found <u>here</u>)

Neutrinoless double beta decay



Taken from: https://link.springer.com/content/pdf/10.1007/s12648-019-01569-6.pdf

GERDA (GERmanium Detector Array)





Neutrino mass measurements

- Limits on masses of v_{μ} and v_{τ} are set based on studies of:
 - Pion decays into muons and v_{μ} (at PSI)
 - Investigation of τ -decays into 5 pions and v_{τ} (at LEP)
- Upper limits:
 - \circ m(v_u) < 190 keV at 90 % CL
 - m(v_⊥) < 18.2 MeV at 95 % CL
- Both limits are much larger than the interesting range of neutrino mass values
- However, experiments investigating the mass of the electron neutrino v_e by analysing β decays with emission of electrons are providing a sensitivity in the interesting energy range (< eV)

Neutrino mass measurements

• Tritium β decay:

• The most sensitive direct studies of the electron neutrino mass (up to now) are based on the investigation of the electron spectrum of tritium β decays:

$${}^{3}\mathrm{H} \rightarrow {}^{3}\mathrm{He}^{+} + \mathrm{e}^{-} + \bar{\nu}_{e}$$

• Energy spectrum of a neutrino with mass m_v is given by:



- Neutrino mass leads to a "distortion" of the energy spectrum of the neutrino
- independent of whether the electron neutrino is a Majorana or a Dirac particle.

Neutrino mass measurements



The electron energy spectrum of tritium β decays for (a) the complete and (b) the narrow region around the endpoint energy E₀. The β spectrum is shown for neutrino masses of 0 and 1 eV.

Taken from: https://www.katrin.kit.edu/publikationen/DesignReport2004-12Jan2005.pdf

KATRIN

- Karlsruhe Tritium Neutrino Experiment (KATRIN) aims to directly measure the mass of the electron neutrino
 - Taking data since 11. Juni 2018

• Why use tritium ?

- Tritium has a particular low endpoint energy of $E_0 = 18.6$ keV
- Tritium has a rather short half life $t_{1/2}$ = 12.3 a
- Simple electron shell configuration (i.e. simple correction terms due to interaction of β-electrons with the tritium source)
- $\circ \quad \mbox{inelastic scattering of out-going} \\ \beta\mbox{-electrons within the } \beta\mbox{ source are small.} \\$



KATRIN



The 70 m long KATRIN reference setup with its major components:

- a) the gaseous tritium source
- b) the transport elements (i.e. an active pumping part and a passive cryotrapping section)
- c) the two electrostatic spectrometers
- d) the detector for β -counting

Taken from: https://www.katrin.kit.edu/publikationen/DesignReport2004-12Jan2005.pdf

KATRIN



- The two electrostatic spectrometers of KATRIN act basically as filter:
 - Electrons with less energy than a retarding potential are reflected
 - Only want to analyse electrons with energies close to E₀

Taken from: https://www.katrin.kit.edu/publikationen/DesignReport2004-12Jan2005.pdf