Tutorial 6: The parton model of hadrons

Dr. M Flowerdew

February 1, 2013

1 Fermion-fermion scattering: recap

In this tutorial, we will discuss the nature of hadrons, and some of the experimental evidence for partons (quarks and gluons) inside the proton and neutron. These experiments usually focus on interactions of composite hadrons (or nuclei) with elementary leptons, in so-called *deep inelastic scattering*, or DIS. For lack of time, we will only consider electromagnetic interactions, however the same methods have been used to probe the charged- and neutral-current weak interactions of nuclei.

We begin by considering interactions between elementary fermions, before discussing the parton model proper. We finished the last tutorial with the differential cross section for elastic t-channel EM scattering of two dissimilar fermions in the centre of momentum (c.m.) frame. The diagram for this process is redrawn in Figure 1 using symbols conventionally used when describing DIS. The cross section in the c.m. frame (denoted by hats) is

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\hat{\Omega}} = \frac{q_1^2 q_2^2}{32\pi^2 s}\frac{s^2+u^2}{t^2}. \tag{1}$$

The right hand side of the equation is fully Lorentz invariant, however the left side is not. We note that in the c.m. frame, the Mandelstam variable $t = q^2$ may be written as

$$\begin{aligned} t &= (\hat{k}-\hat{k}')^2 \\ &= -2\hat{E}^2(1-\cos\hat{\theta}) \\ &= -\frac{s}{2}(1-\cos\hat{\theta}) \end{aligned} \tag{2}$$

if the angle of deflection is $\hat{\theta}$ and $s = (\hat{k}+\hat{p})^2 = 4\hat{E}^2$. Therefore, the differential change in $-t$ as $\hat{\theta}$ varies with s constant is

$$\mathrm{d}(-t) = \frac{s}{2}\,\mathrm{d}(\cos\theta) = -\frac{s}{4\pi}\int_\phi \mathrm{d}\Omega. \tag{3}$$

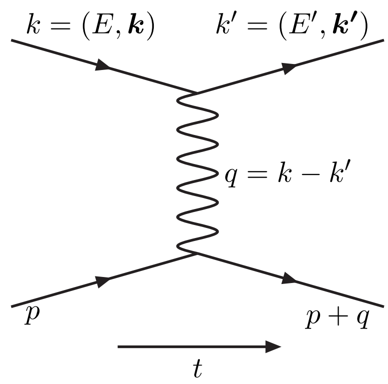

Figure 1: Diagram of first-order electromagnetic scattering of two non-identical fermions.

We can use this to rewrite Equation (1) in a fully Lorentz invariant form:

$$\begin{aligned}\frac{\mathrm{d}\sigma}{\mathrm{d}(-t)} &= \frac{4\pi}{s} \cdot \frac{q_1^2 q_2^2}{32\pi^2 s} \frac{s^2 + u^2}{t^2} \\ &= \frac{q_1^2 q_2^2}{8\pi t^2} \frac{s^2 + u^2}{s^2}. \end{aligned} \tag{4}$$

When considering DIS, Equation (4) is usually rewritten in terms of these new variables:

$$\begin{array}{ll} \text{C.M. energy} & s = 2k \cdot p \\ \text{Momentum transfer} & Q^2 = -q^2 = 2k \cdot k' = -t \\ \text{Inelasticity} & y = \dfrac{2p \cdot q}{s} = 1 + \dfrac{u}{s} \\ \text{Björken scaling variable} & x = \dfrac{k \cdot k'}{k \cdot p} = \dfrac{Q^2}{sy} \end{array} \tag{5}$$

From the last line, it is clear that at most three of these variables are independent. If the fermions in Figure 1 are fundamental, then x is identically 1, so that there are only two independent variables, or just one if s is also considered fixed. In addition, we will normalise the charges to e, writing $q_i^2 = Q_i^2 e^2 = 4\pi Q_i^2 \alpha$. Expressed using these variables, Equation (4) becomes

$$\begin{aligned}\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2} &= Q_1^2 Q_2^2 \frac{2\pi\alpha^2}{Q^4} \left[1 + (1-y)^2\right] \\ &= Q_1^2 Q_2^2 \frac{2\pi\alpha^2}{Q^4} \left[y^2 + 2(1-y)\right] \end{aligned} \tag{6}$$

2 Deep inelastic scattering

Next, we consider inelastic scattering of a fermion (e.g. an electron) from a hadron (e.g. a proton). Hadrons are composite particles, and therefore the interaction does not have the same simple form as Equation (6). Recall that the matrix element for Figure 1 could be written as the product of two fermion tensors, evaluated independently for each interacting particle:

$$|M_{fi}|^2 = \frac{q_1^2 q_2^2}{Q^2} L_1^{\mu\nu} L_{2\mu\nu}. \tag{7}$$

For the hadron case, we simply replace one fermion tensor with an unknown hadron tensor $W_{\mu\nu}$:

$$|M_{fi}|^2 = \frac{q_1^2 q_2^2}{Q^2} L^{\mu\nu} W_{\mu\nu}. \tag{8}$$

The hadron tensor is constrained by the same symmetries as the lepton tensor, but with arbitrary non-perturbative coefficients. The fact that the collision is inelastic also means that the relationship $Q^2 = sy$ no longer holds, and the interaction cross section now in principle depends on the Björken scaling variable x. The differential cross section is therefore parameterised in terms of arbitrary *form factors* $F_i(x, Q^2)$, so that Equation (6) becomes

$$\frac{\mathrm{d}\sigma}{\mathrm{d}x\,\mathrm{d}Q^2} = Q_1^2 Q_2^2 \frac{4\pi\alpha^2}{xQ^4}\left[y^2 x F_1(x, Q^2) + (1-y) F_2(x, Q^2)\right]. \tag{9}$$

Note that some of the coefficient factors have changed with respect to Equation (6), this is purely conventional. Also note that when weak interactions are considered, other terms may arise, not shown here.

The form factors for a hadron must be determined experimentally, however the quark parton model makes specific predictions about their properties. We consider the hadron to be composed of multiple *partons*, only one of which scatters with the incoming fermion. This is illustrated in Figure 2. It is assumed that this parton carries a fraction ξ of the hadron's four-momentum.[1] Any hadron will contain a variety of different parton flavours, which we must sum over to find the total form factor. We must also integrate over all possible values of ξ. The probability of finding a parton a with a momentum ξp in hadron A is described by *parton density functions* (pdfs) $f_A^a(\xi)$. Therefore, we can write the observed double-differential cross section as

$$\frac{\mathrm{d}\sigma(x, Q^2)}{\mathrm{d}x\,\mathrm{d}Q^2} = \int_0^1 \frac{\mathrm{d}\xi}{\xi} \sum_a f_A^a(\xi) \frac{\mathrm{d}\sigma_a(\xi, Q^2)}{\mathrm{d}\xi\,\mathrm{d}Q^2} \tag{10}$$

[1] The parton's momentum perpendicular to the hadron's motion is zero; strictly speaking, this is valid only in the so-called *infinite momentum frame*, where any transverse momentum may be neglected relative to p.

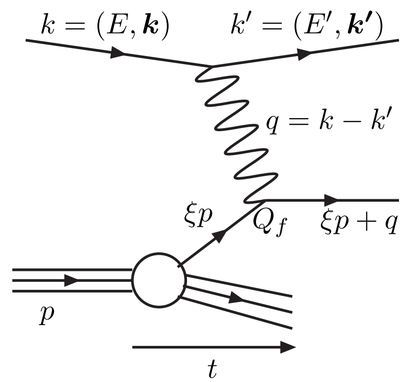

Figure 2: Diagram of deep inelastic scattering of a hadron.

In the quark model, two extra assumptions are made: that the partons are themselves fermionic, and that the scattering is elastic at the parton level. This latter assumption means that we can relate ξ to the DIS variables in Equation (5) by requiring that the parton's mass is unchanged by the collision. Therefore

$$\begin{aligned}(\xi p)^2 &= (\xi p + q)^2 \\ \Rightarrow 0 &= 2\xi p \cdot q + q^2 \\ \Rightarrow \xi &= \frac{-q^2}{2p \cdot q} = \frac{Q^2}{sy} = x. \end{aligned} \tag{11}$$

Thus, the Björken variable x is identified as the momentum fraction carried by the interacting parton. The partonic cross section can then be written down, following Equation (6):

$$\frac{\mathrm{d}\sigma_a(\xi, Q^2)}{\mathrm{d}x\,\mathrm{d}Q^2} = Q_1^2 Q_a^2 \frac{2\pi\alpha^2}{Q^4} \left[y^2 + 2(1-y)\right] \delta(x - \xi). \tag{12}$$

Replacing into Equation (10), we obtain the following:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}x\,\mathrm{d}Q^2} = Q_1^2 \frac{2\pi\alpha^2}{xQ^4} \left[y^2 + 2(1-y)\right] \sum_a Q_a^2 x f_A^a(x). \tag{13}$$

Comparing with Equation (6), we make two important observations:

Björken scaling: F_1 and F_2 depend only on x, and not on the energy scale Q^2. This contrasts strongly with, say, resonance production, where the cross section varies strongly with Q^2.

The Callan-Gross relation: $F_2(x) = 2xF_1(x)$, sensitive to the parton spin.

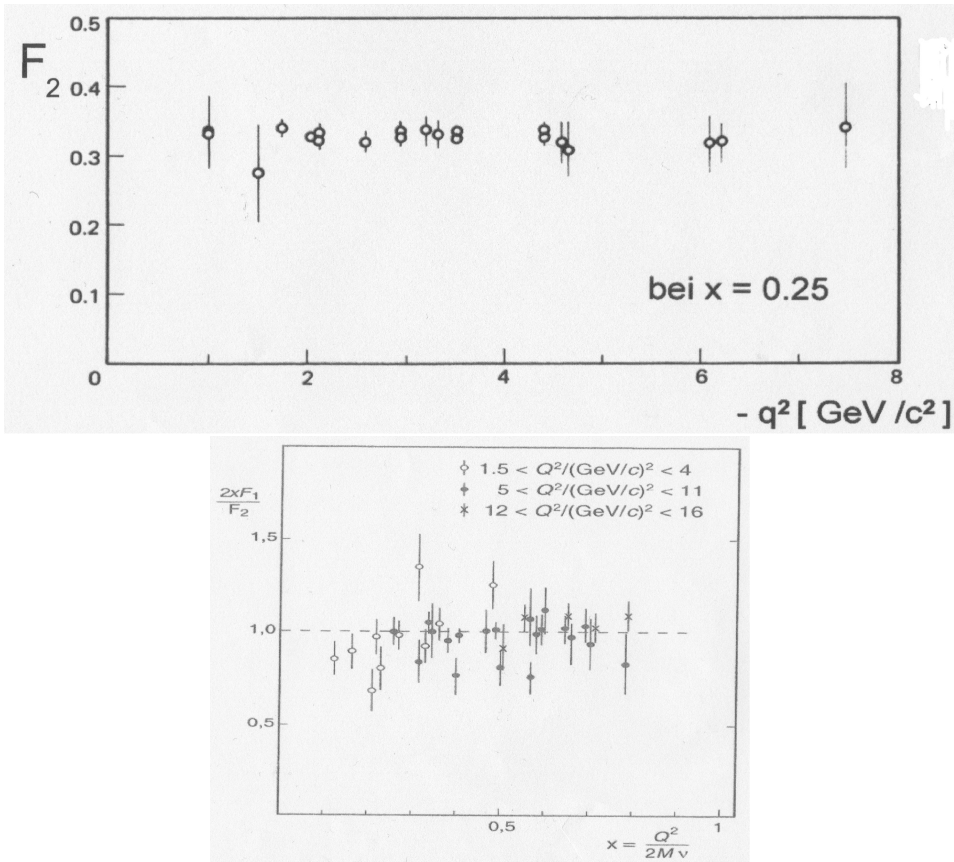

Figure 3: Top: Illustration of Björken scaling, where $F_2(x = 0.25)$ is constant over a wide range of Q^2 values. Bottom: Illustration of the Callan-Gross relation as a function of x for three Q^2 ranges.

Both of these were confirmed at SLAC, as shown in Figure 3. These, and other observations, led to the adoption of the quark parton model.

3 Parton density functions

We finish by taking a closer look at the parton density functions $f_A^a(x)$, specialising now to the case of the proton. In a non-interacting quark model, we would expect the pdfs to be of the form $\delta(x - \frac{1}{N})$ if there are N partons in the proton. Interactions between the quarks will smear this out, leading to a broad peak centred at $x = \frac{1}{N}$.

Figure 4 shows measured parton density functions for three values of Q^2, spanning six orders of magnitude. For values of Q^2 at or below m_p^2, the up and down quark pdfs do peak at values of $x \sim 0.2 - 0.3$, and furthermore

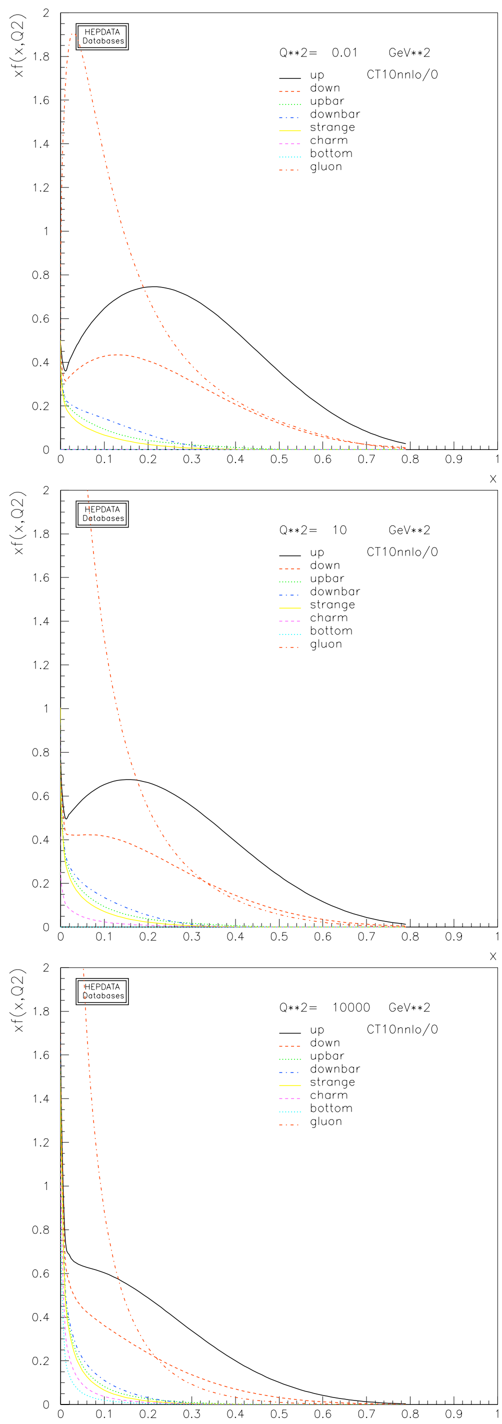

Figure 4: Parton density functions from a recent next-to-next-to-leading order fit to data from multiple experiments at three different Q^2 values. Top: $Q^2 = 0.01$ GeV^2, well below m_p^2. Middle: $Q^2 = 10$ GeV^2, larger than m_p^2. Bottom: $Q^2 = 10^4$ GeV^2, comparable to m_Z^2.

the pdfs are more or less (but not entirely) independent of Q^2. These peaks correspond to the *valence quarks* of the proton.

The Figure also shows behaviour not predicted by the simple model outlined in Section 2. When the proton structure functions are integrated, it turns out that the quarks only carry about 55% of the proton's momentum. The remainder can be attributed to the presence of gluons, which cannot be directly measured in electroweak scattering experiments. These gluons constantly split into quark-antiquark pairs (which quickly annihilate), creating non-zero *sea quark* pdfs. Both of these contributions to the proton can also be seen in Figure 4. The gluon splitting rate is, however, highly dependent on both x and Q^2, and this breaks Björken scaling. This is seen most easily in the lowest panel of Figure 4, where the proton is now dominated by gluons and sea quarks at low x. To balance this out, the valence pdfs at high x decrease. Scaling violation can also be seen, to a lesser extent, in the upper two panels. Even at low Q^2, the proton has a non-negligible antiquark component.

The existence of sea quarks has important implications for hadron collider design. On-shell W and Z boson production involves Q^2 values of around $m^2_{W/Z}$, comparable to the lower panel of Figure 4. These particles are produced by quark-antiquark annihilation. The Tevatron has a (anti)proton beam energy of about 1 TeV, so that these bosons can be produced by partons with $x \sim \frac{80}{1000} = 0.08$. Production of more massive particles, such as a Higgs boson, would require higher values of x. In this region, the sea quark pdfs are still relatively small, and it is necessary to collide protons with antiprotons to obtain quark-antiquark collisions at sufficiently high rates for physics studies. The LHC, by contrast, was designed to collide 7 TeV proton beams. Now, a typical value of x for electroweak boson production is $\frac{80}{7000} \approx 0.01$. Here, the sea quark (including antiquark) pdfs are large, and so it is not necessary to collide antiprotons to produce these particles at appreciable rates. Therefore, the LHC can benefit from the relative ease of making high-density proton beams, achieving very high collision luminosities.