Tutorial 2:

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1 Gauge transformations and symmetry groups

1.1 Preamble: Classical electromagnetism

The classical Lagrangian for an otherwise free particle of charge Q in an electromagnetic field is

$$L = \frac{1}{2}m\dot{\boldsymbol{x}}^2 + Q\dot{\boldsymbol{x}} \cdot \boldsymbol{A} - Q\Phi. \tag{1}$$

Using this, one finds that the canonical momentum $\boldsymbol{p} = \partial L/\partial\dot{\boldsymbol{x}}$ is no longer the physical particle's momentum $m\dot{\boldsymbol{x}}$, but has an additional contribution from the electromagnetic field:

$$\boldsymbol{p} = \frac{\partial L}{\partial \dot{\boldsymbol{x}}} = m\dot{\boldsymbol{x}} + Q\boldsymbol{A} \tag{2}$$

Similarly, the classical Hamiltonian has a contribution from the electric potential $Q\Phi$

$$\begin{aligned} H &= \boldsymbol{p} \cdot \dot{\boldsymbol{x}} - L \\ &= (m\dot{\boldsymbol{x}} + Q\boldsymbol{A}) \cdot \dot{\boldsymbol{x}} - \frac{1}{2}m\dot{\boldsymbol{x}}^2 - Q\dot{\boldsymbol{x}} \cdot \boldsymbol{A} + Q\Phi \\ &= \frac{1}{2}m\dot{\boldsymbol{x}}^2 + Q\Phi. \end{aligned} \tag{3}$$

Formally, we can then recover the particle's physical energy and momentum from the canonical variables by subtracting these extra contributions:

$$\begin{aligned} \boldsymbol{p}_{\text{phys.}} &= m\dot{\boldsymbol{x}} = \boldsymbol{p} - Q\boldsymbol{A} \\ E_{\text{phys.}} &= \frac{1}{2}m\dot{\boldsymbol{x}}^2 = H - Q\Phi. \end{aligned} \tag{4}$$

These substitutions have exact analogues in all forms of quantum mechanics, including field theory, when one wishes to incorporate electromagnetic effects on a particle's motion. Naturally, classical variables must be replaced by appropriate quantum mechanical operators, but the form of the substitutions in Equation (4) is identical. As we will later see, the other forces in the

Standard Model can be introduced using similar modifications of canonical variables.

Before moving on to considering quantum mechanical operators, one should remember that the EM potentials Φ and $\boldsymbol{A}$ can be freely modified by a *gauge transformation*, with ultimately no physical effects:

$$\Phi \to \Phi' = \Phi - \frac{\partial \chi}{\partial t}; \qquad \boldsymbol{A} \to \boldsymbol{A'} = \boldsymbol{A} + \nabla \chi. \tag{5}$$

Here, χ is understood to be an arbitrary function of space and time. Note that this will change the canonical variables defined in Equations (1), (2) and (3), but *not* the physical momentum calculated in Equation (4).

1.2 Electromagnetism in quantum mechanics

In the quantum mechanical context, canonical energy and momentum variables are replaced by differential operators in the usual way:

$$\hat{E} = i\frac{\partial}{\partial t} \text{ and } \hat{\boldsymbol{p}} = -i\nabla; \text{ or, in covariant notation } \hat{p}_\mu = i\partial_\mu. \tag{6}$$

Equations of motion inevitably involve these operators acting on wavefunctions or fields, here referred to generically with the symbol ψ. In addition, covariant notation will be used for simplicity; the separation into energy and momentum components (e.g. for application in the Schrödinger equation) is straightforward.

When electromagnetic interactions are included, physical four-momentum operators can be extracted by modifying the canonical operators in a way analagous to Equation (4):

$$\begin{aligned} \hat{p}_\mu \to \hat{p}_\mu &- QA_\mu \\ &= i(\partial_\mu + iQA_\mu) \\ &= i\mathcal{D}_\mu \end{aligned} \tag{7}$$

In the last line, the *covariant derivative* $\mathcal{D}_\mu = \partial_\mu + iQA_\mu$ is introduced, which replaces ∂_μ when calculating physically measurables for interacting particles or fields.

One immediate issue is that the covariant derivative varies under a gauge transformation (5) to

$$\mathcal{D}_\mu \to \mathcal{D}'_\mu = \partial_\mu + iQA_\mu - iQ\partial_\mu\chi \tag{8}$$

Thus, it appears that the physical measurable expectation values $\langle\psi|\hat{E}|\psi\rangle$ and $\langle\psi|\hat{\boldsymbol{p}}|\psi\rangle$ change, violating gauge invariance. This apparent problem can be elegantly solved by additionally transforming the wavefunction ψ, which has no classical analogue, according to

$$\psi \to \psi' = e^{iQ\chi}\psi. \tag{9}$$

Then, the combination $\mathcal{D}_\mu\psi$ transforms as follows:

$$\begin{aligned}
\mathcal{D}_\mu\psi \to \mathcal{D}'_\mu\psi' &= \partial_\mu(e^{iQ\chi}\psi) + ie^{iQ\chi}QA_\mu\psi - iQ(\partial^\mu\chi)e^{iQ\chi}\psi \\
&= e^{iQ\chi}\partial_\mu\psi + iQ(\partial_\mu\chi)e^{iQ\chi}\psi + ie^{iQ\chi}QA_\mu\psi - iQ(\partial^\mu\chi)e^{iQ\chi}\psi \\
&= e^{iQ\chi}(\partial_\mu + iQA_\mu)\psi \\
&= e^{iQ\chi}\mathcal{D}_\mu\psi, \qquad (10)
\end{aligned}$$

modified only by an overall (space-time-dependent) phase. Any physical expectation value will be pre-multiplied by the conjugate of a wave function which transforms according to $\psi^* \to \psi^{*\prime} = \psi^* e^{-iQ\chi}$, and will therefore be left unchanged by the *combined* transformation described by Equations (8) and (9).

Exercise: Prove that this also works for the second derivative $\mathcal{D}_\mu\mathcal{D}^\mu\psi$, as in the Klein-Gordan equation, for example.

Exercise: Beginning with a free particle ($A^\mu = 0$), consider the gauge transformation given by $\chi = -\arg(\psi)/Q$. What are the resulting potentials and new wavefunction? Interpret the result in terms of physical quantities.

Exercise: (more tricky) Repeat the previous exercise for $\chi = -\frac{\arg(\psi)}{Q(1+e^{-t})}$.

1.3 Relation to the U(1) symmetry group

It is well known that all observable quantites are invariant under a global phase transformation of a wave function or field

$$\psi \to \psi' = e^{i\phi}\psi, \qquad (11)$$

where the phase angle ϕ does not depend on space-time coordinates. The prefactor $e^{i\phi}$ can be recognised as a member of the U(1) group discussed in the last tutorial. The transformation (11) is referred to as a *global* U(1) *transformation*. Note that it alters an *internal* space of the wavefunction, distinct from external space-time.

When electromagnetic interactions are introduced, we find that now a much more stringent symmetry exists, that of a *local* U(1) transformation, of Equation (9) (recall that χ is an *arbitrary* function of space-time). Thus, it appears that internal symmetries of a wavefunction or field are deeply related to their gauge interactions. If we began with a non-interacting (free) particle, we could "derive" the electromagnetic interaction by requiring that the transformation in Equation (9) has no effect on measurable quantities. We would then be forced to introduce a new field A^μ that simultaneously transforms as Equation (5), and to form a covariant derivative like (7) to represent the particle's physical four-momentum.

In the Standard Model, all gauge interactions are derived in this way, starting from Lie group operators applied to an internal space (called a Hilbert space), and demanding symmetry in the physical equations upon arbitrary local rotations within this space. Unlike electromagnetism, the other gauge groups of the SM are non-Abelian.

2 Non-Abelian groups in gauge theories

When we considered U(1), the analysis of the gauge transformation was considerably simplified by the fact that the group was Abelian. In fact, there is only one U(1) generator, which trivially commutes with itself. The generators of the other symmetries involved in Standard Model forces do *not* commute. In other words, not all structure constants f_{abc} are zero, where f_{abc} is defined by

$$[T_a, T_b] = T_a T_b - T_b T_a = i f_{abc} T_c. \tag{12}$$

For SU(2), the structure constants are equal to the completely antisymmetric Levi-Civita symbol ϵ_{abc}. In the case of SU(3), the constants are more complex, and given in the lecture notes.

We will start by generalising the transformations derived in the previous section, without assuming that the group elements commute. In general, a state ψ will transforms as follows:

$$\psi \to \psi' = G\psi = e^{i\boldsymbol{\alpha}\cdot\boldsymbol{T}}\psi, \tag{13}$$

where G is a local Hilbert space transformation. We introduce a covariant derivative to absorb changes to the Lagrangian resulting from this transformation:

$$\mathcal{D}_\mu = \partial_\mu + igW_\mu = \partial_\mu + ig\boldsymbol{T}\cdot\boldsymbol{W}_\mu. \tag{14}$$

$\boldsymbol{W}_\mu$ is a vector of three new gauge fields, corresponding to the number of group generators, while g is an associated coupling strength, analagous to the electric charge. Under the gauge transformation G, the field W_μ transforms as follows:

$$W_\mu \to W'_\mu = GW_\mu G^{-1} + \frac{i}{g}(\partial_\mu G)G^{-1}. \tag{15}$$

Exercise: Show that under this transformation $\mathcal{D}'_\mu\psi' = G\mathcal{D}_\mu\psi$.

Equation (15) is the general form for the transformation of a gauge field derived through symmetry principles. For Abelian groups, Equation (15) reduces to the same form as given in Equation (5) for the U(1) transformation. For non-Abelian groups, G and W_μ do not commute, which complicates further analysis but also yields a rich structure for the gauge fields.

It is useful to consider Equation (15) written in terms of the component fields

$$\boldsymbol{T}\cdot\boldsymbol{W}'_\mu = e^{i\boldsymbol{\alpha}\cdot\boldsymbol{T}}\boldsymbol{T}\cdot\boldsymbol{W}_\mu e^{-i\boldsymbol{\alpha}\cdot\boldsymbol{T}} + \frac{i}{g}(\partial_\mu e^{i\boldsymbol{\alpha}\cdot\boldsymbol{T}})e^{-i\boldsymbol{\alpha}\cdot\boldsymbol{T}}$$
$$\text{or } T_a W'_{\mu,a} = e^{i\alpha_b T_b} W_{\mu,a} T_a e^{-i\alpha_c T_c} + \frac{i}{g}(\partial_\mu e^{i\alpha_a T_a})e^{-i\alpha_b T_b} \tag{16}$$

In the second version, the Einstein summation convention for indices has been assumed. This describes a general transformation, but often we will be interested in the perturbative regime, when the transformation G is close to the identity. In this case, $|\boldsymbol{\alpha}|$ is small, and $e^{i\boldsymbol{\alpha}\cdot\boldsymbol{T}} \approx 1 + i\boldsymbol{\alpha}\cdot\boldsymbol{T}$. Under this infinitesimal transformation, Equation (16) becomes the following (dropping any second-order terms in $\boldsymbol{\alpha}$ and $\partial_\mu\boldsymbol{\alpha}$):

$$\begin{aligned} T_a W'_{\mu,a} &\simeq (1 + i\alpha_b T_b) W_{\mu,a} T_a (1 - i\alpha_c T_c) - \frac{1}{g}(\partial_\mu \alpha_a) T_a (1 - i\alpha_b T_b) \\ &= W_{\mu,a} T_a - i W_{\mu,a}(\alpha_c T_a T_c - \alpha_b T_b T_a) - \frac{1}{g}(\partial_\mu\alpha_a)T_a + \mathcal{O}(\boldsymbol{\alpha}^2) \end{aligned} \tag{17}$$

Here, we note that the indices b and c on the right hand side are arbitrary, so we can rewrite $\alpha_c T_a T_c$ as $\alpha_b T_a T_b$ to obtain

$$\begin{aligned} T_a W'_{\mu,a} &= W_{\mu,a} T_a - i\alpha_b W_{\mu,a}[T_a, T_b] - \frac{1}{g}(\partial_\mu\alpha_a)T_a \\ &= W_{\mu,a} T_a + f_{abc}\alpha_b W_{\mu,a} T_c - \frac{1}{g}(\partial_\mu\alpha_a)T_a \end{aligned} \tag{18}$$

Exercise: Find an expression for $W'_{\mu,a}$ that will always satisfy Equation (18). Make sure it reduces to the electromagnetic gauge transformation in the case of a U(1) symmetry.

Equation (18) shows how W_μ transforms under an infinitesimal gauge transformation, but we have not yet shown that physical observables are unchanged. By analogy with the U(1) case, it is enough to show that $\mathcal{D}'_\mu\psi' = G\mathcal{D}_\mu\psi$, as the conjugate part of any expectation value will always cancel the G on the right hand side. This condition can be easily verified, again to $\mathcal{O}(\boldsymbol{\alpha})$:

$$\begin{aligned} \mathcal{D}'_\mu\psi' &= (\partial_\mu + igW_{\mu,a}T_a - i(\partial_\mu\alpha_a)T_a + igf_{abc}\alpha_b W_{\mu,a}T_c)(1 + i\alpha_t T_t)\psi \\ &= i(\partial_\mu\alpha_t)T_t\psi + (1 + i\alpha_t T_t)(\partial_\mu\psi) \\ &\quad + (1 + i\alpha_t T_t)(igW_{\mu,a}T_a) - gW_{\mu,a}\alpha_t(if_{atb}T_b) \\ &\quad - i(\partial_\mu\alpha_a)T_a\psi + igf_{abc}\alpha_b W_{\mu,a}T_c + \mathcal{O}(\boldsymbol{\alpha}^2) \\ &\simeq (1 + i\alpha_t T_t)(\partial_\mu + igW_{\mu,a}T_a)\psi, \end{aligned} \tag{19}$$

where the commutator (12) has been used to obtain the third and fourth term on the second line.

Thus, the principal equations and expectation values will be invariant under the gauge transformations being discussed.

3 Building an interacting Lagrangian

The Dirac equation for a free fermion field ψ can be obtained from the following Lagrangian density

$$\mathcal{L} = \bar{\psi}(i\gamma^{\mu}\partial_{\mu} - m)\psi. \tag{20}$$

A gauge interaction for this fermion may be introduced by replacing ∂_μ by the covariant derivative from Equation (14):

$$\begin{aligned}\mathcal{L} &= \bar{\psi}(i\gamma^{\mu}\mathcal{D}_{\mu} - m)\psi \\ &= \bar{\psi}(i\gamma^{\mu}\partial_{\mu} - m)\psi - g\bar{\psi}\gamma^{\mu}\boldsymbol{T}\cdot\boldsymbol{W}_{\mu}\psi.\end{aligned} \tag{21}$$

The final term of Equation (21) represents interactions between the fermion and the gauge field. These modify the propagation of the free fermion field, described in the first two terms. However, there is another class of terms involving just the fields $\boldsymbol{W}_\mu$. It turns out that the only Lorentz covariant object allowed by gauge symmetry that we can form from the gauge field alone is the commutator of the covariant derivative, $[\mathcal{D}_\mu, \mathcal{D}_\nu]$. This can be evaluated as follows:

$$\begin{aligned}[\mathcal{D}_{\mu}, \mathcal{D}_{\nu}] &= [\partial_{\mu} + ig\boldsymbol{T}\cdot\boldsymbol{W}_{\mu}, \partial_{\nu} + ig\boldsymbol{T}\cdot\boldsymbol{W}_{\nu}] \\ &= [\partial_{\mu}, \partial_{\nu}] + ig[\partial_{\mu}, \boldsymbol{T}\cdot\boldsymbol{W}_{\nu}] + ig[\boldsymbol{T}\cdot\boldsymbol{W}_{\mu}, \partial_{\nu}] \\ &\quad - g^2[\boldsymbol{T}\cdot\boldsymbol{W}_{\mu}, \boldsymbol{T}\cdot\boldsymbol{W}_{\nu}].\end{aligned} \tag{22}$$

The first commutator is evidently zero. The commutator in the second term can be found by considering what happens when this operates on a wavefunction:

$$\begin{aligned}[\partial_{\mu}, \boldsymbol{T}\cdot\boldsymbol{W}_{\nu}]\psi &= \partial_{\mu}(\boldsymbol{T}\cdot\boldsymbol{W}_{\nu}\psi) - \boldsymbol{T}\cdot\boldsymbol{W}_{\nu}(\partial_{\mu}\psi) \\ &= (\partial_{\mu}\boldsymbol{T}\cdot\boldsymbol{W}_{\nu})\psi.\end{aligned} \tag{23}$$

Note that the end result does not depend in any way on the wavefunction we temporarily introduced. Similarly, $[\boldsymbol{T}\cdot\boldsymbol{W}_\mu, \partial_\nu] = -\partial_\nu\boldsymbol{T}\cdot\boldsymbol{W}_\mu$ in the third term of Equation (22). The final term is evaluated using the group algebra, giving the following result

$$\begin{aligned}[\mathcal{D}_{\mu}, \mathcal{D}_{\nu}] &= ig\,\boldsymbol{T}\cdot(\partial_{\mu}\boldsymbol{W}_{\nu} - \partial_{\nu}\boldsymbol{W}_{\mu}) - ig^2 f_{abc}W_{\mu,a}W_{\nu,b}T_c \\ &= [ig(\partial_{\mu}W_{\nu,c} - \partial_{\nu}W_{\mu,c}) - ig^2 f_{abc}W_{\mu,a}W_{\nu,b}]T_c.\end{aligned} \tag{24}$$

In the U(1) case, the structure constants vanish, and Equation (24) is proportional to the field tensor $F_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu$, familiar from electromagnetism. In the non-Abelian case, the first terms of Equation (24) also describe free fields that propagate in the vacuum much like the photon, but the final term will not vanish.

The only Lorentz scalar field propagation term that we can construct using $F_{\mu\nu} \propto [\mathcal{D}_\mu, \mathcal{D}_\nu]$ is $F_{\mu\nu}F^{\mu\nu}$. The final term of Equation (24) will yield terms proportional to $(\partial_\mu W_\nu)W^\mu W^\nu$ and $W_\mu W_\nu W^\mu W^\nu$, where the group generator indices have been suppressed. These terms ultimately correspond to interactions between the various components of the $\boldsymbol{W}_\mu$ field, something that will be explored further later in the course.

Finally, note that Equation (24) (and therefore the Lagrangian density) does not contain any terms proportional to $\boldsymbol{W}^\mu \cdot \boldsymbol{W}_\mu$. These would give the $\boldsymbol{W}_\mu$ field a mass (c.f. the Klein-Gordan equation for a massive boson), but such terms are forbidden as they break the gauge symmetry. Thus, gauge boson fields produced via the symmetry principle are massless by construction.

4 A first look at the Higgs field

Massless gauge fields describe the electromagnetic and strong forces well, but are insufficient for the weak nuclear force. The W and Z gauge bosons that mediate the weak force have substantial masses of about 80 and 90 GeV, respectively, which has the effect of setting a short range $\sim 1/M_{W/Z}$ for weak interactions in the low energy limit.

What is perhaps less obvious is that, due to the peculiar nature of the weak interaction, all weakly interacting fermions must also be massless. It might naively be thought that a fermionic mass term $m\bar{\psi}\psi$ (c.f. Equation (20)) would always remain invariant under any transformation $\psi \to G\psi$. It turns out that this is not the case for the weak nuclear force, which acts differently on the left- and right-handed components of ψ. We can always rewrite ψ as a sum of these components:

$$\begin{aligned} \psi &= \psi_{\mathrm{L}} + \psi_{\mathrm{R}} \\ &= \frac{1}{2}(1-\gamma^5)\psi + \frac{1}{2}(1+\gamma^5)\psi. \end{aligned} \tag{25}$$

The mass term $m\bar{\psi}\psi$ therefore has four components, as follows:

$$m\bar{\psi}\psi = m(\bar{\psi}_{\mathrm{L}}\psi_{\mathrm{L}} + \bar{\psi}_{\mathrm{L}}\psi_{\mathrm{R}} + \bar{\psi}_{\mathrm{R}}\psi_{\mathrm{L}} + \bar{\psi}_{\mathrm{R}}\psi_{\mathrm{R}}). \tag{26}$$

The matrix γ^5 is Hermitian, which allows us to evaluate $\bar{\psi}_{\mathrm{L}}$ as $\psi^\dagger(1-\gamma^5)\gamma^0 = \bar{\psi}(1+\gamma^5)$, and similarly $\bar{\psi}_{\mathrm{R}} = \bar{\psi}(1-\gamma^5)$. Recalling that $(\gamma^5)^2 = 1$, the straight terms $m\bar{\psi}_{\mathrm{L}}\psi_{\mathrm{L}}$ and $m\bar{\psi}_{\mathrm{R}}\psi_{\mathrm{R}}$ are seen to vanish, leaving just the cross terms:

$$m\bar{\psi}\psi = m(\bar{\psi}_{\mathrm{L}}\psi_{\mathrm{R}} + \bar{\psi}_{\mathrm{R}}\psi_{\mathrm{L}}). \tag{27}$$

This is clearly variant under an SU(2) symmetry operation, as ψ_{L} is an SU(2) doublet, while ψ_{R} is a singlet. Therefore, the terms in Equation (27) are not allowed in the Standard Model Lagrangian.

The Higgs field was postulated to overcome these apparent difficulties. Instead of changing the basic symmetry principles or the particle content of the Standard Model, the nature of the stable vacuum state is altered instead. In all the fields considered so far, it has been implicitly assumed that the vacuum state corresponds to $\langle\psi\rangle = 0$, up to zero-point fluctuations. This arises naturally if the potential for a particle has a minumum at zero, as in the left-hand image in Figure 1. The x-axis here corresponds to the field strength $|\psi|$, and the potential to a regular mass term $m\bar{\psi}\psi$. Adding more energy to the field[1] increases the maximum possible value for $|\psi|$.

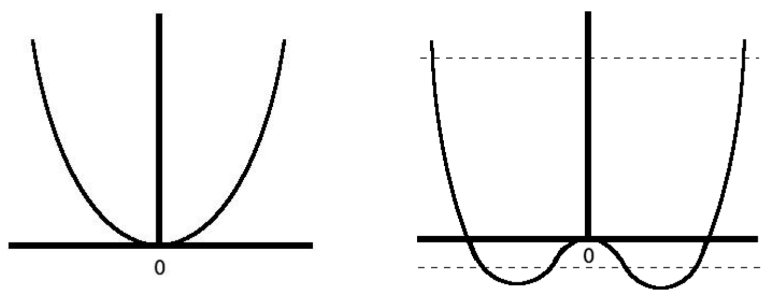

Figure 1: Left: A quadratic potential function for a massive particle. Right: The Higgs field potential, with quadratic and quartic terms.

The Higgs field ϕ, on the other hand, has a quartic potential, illustrated on the right-hand side of Figure 1. At very high energies (the upper dotted line), this is difficult to distinguish from the quadratic case, but at low energies (lower dotted line), it is clear that the "bump" at $\psi \sim 0$ will affect the ground state significantly. In the simple one-dimensional case, there will be two degenerate vacuum states, each centered on one of the two minima illustrated, with the same average magnitude $\langle|\phi|\rangle = v$.

The physical Higgs field is a complex SU(2) doublet, with four real components. Despite this, the vacuum expectation value, v, and real fluctuations around it, $H(x)$, can be written with full generality as

$$\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}, \tag{28}$$

where $v > 0$ is a real constant. This amounts to a specific choice of

[1] Strictly speaking, this means adding quanta to the particular mode described by $\psi(E, \boldsymbol{p})$.

(spacetime-dependent) SU(2) gauge, fixing three of the four free parameters of ϕ.

In this new vacuum, previously massless particles now appear to have mass. To see how this might work, at least for fermions, consider the Lagrangian interaction term $\lambda\phi^{\mathrm{T}}\bar{\psi}_{\mathrm{R}}\psi_{\mathrm{L}}$, where λ is an (at this point) arbitrary coupling constant between ψ and ϕ. This, unlike Equation (27), is a gauge-invariant scalar quantity, and is thus allowed in the Lagrangian. With the specific choice of Equation (28), and supposing that we are concerned with electron-like fields $\psi_{\mathrm{L}} = \begin{pmatrix} \nu \\ e_{\mathrm{L}} \end{pmatrix}$, we have

$$\begin{aligned} \lambda\phi^{\mathrm{T}}\bar{\psi}_{\mathrm{R}}\psi_{\mathrm{L}} &= \frac{\lambda}{\sqrt{2}} \begin{pmatrix} 0 & v + H(x) \end{pmatrix} \bar{e}_{\mathrm{R}} \begin{pmatrix} \nu \\ e_{\mathrm{L}} \end{pmatrix} \\ &= \frac{\lambda}{\sqrt{2}} v \bar{e}_{\mathrm{R}} e_{\mathrm{L}} + \lambda H(x) \bar{e}_{\mathrm{R}} e_{\mathrm{L}}. \end{aligned} \tag{29}$$

Now, the first term has an identical form to the second term of Equation (27), while the second term describes an interaction between the electron field and excitations of the Higgs field $H(x)$. Adding on the Hermitian conjugate $\lambda^* \bar{\psi}_{\mathrm{L}}\psi_{\mathrm{R}}\phi$ gives a term proportional to $\bar{e}_{\mathrm{L}} e_{\mathrm{R}}$. The two together therefore give the appearance that the fermion field e has mass.