Tutorial 14: Two-Higgs doublet models

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October 20, 2015

1 Theoretical overview

In the SM, boson and fermion masses are generated through the action of one complex scalar field ϕ. Recall that this is an SU(2) doublet, and hence has four degrees of freedom. When the field acquires a vacuum expectation value (vev), three degrees of freedom become associated with the longitudinal polarisation modes of the W and Z bosons, leaving the fourth degree of freedom to become the Higgs boson h.

This is a minimal prescription for electroweak symmetry breaking, however it is not unique. The simplest non-minimal approach would be to introduce a second scalar SU(2) doublet, ϕ_2.[1] This is called a *two Higgs doublet model* (2HDM). In the general case, its description is rather complicated, so it is conventional to assume that the Higgs sector conserves CP and that certain terms are absent from the Lagrangian density due to discrete symmetries. In this case, the combined potential of both Higgs doublets can be written as follows[2]:

$$\begin{aligned} V_\phi = {} & m_{11}^2 \phi_1^\dagger \phi_1 + m_{22}^2 \phi_2^\dagger \phi_2 - m_{12}^2 \left(\phi_1^\dagger \phi_2 + \phi_2^\dagger \phi_1 \right) \\ & + \frac{\lambda_1}{2} \left(\phi_1^\dagger \phi_1 \right)^2 + \frac{\lambda_2}{2} \left(\phi_2^\dagger \phi_2 \right)^2 \\ & + \lambda_3 \phi_1^\dagger \phi_1 \phi_2^\dagger \phi_2 + \lambda_4 \phi_1^\dagger \phi_2 \phi_2^\dagger \phi_1 \\ & + \frac{\lambda_5}{2} \left[\left(\phi_1^\dagger \phi_2 \right)^2 + \left(\phi_2^\dagger \phi_1 \right)^2 \right]. \end{aligned} \tag{1}$$

In principle, what we should do next, as in the SM case, is to minimise this potential function, requiring that at least one Higgs doublet obtains a non-zero vev, and study the oscillations around this minimum. In practice, this function can have multiple minima, including ones that produce charge- and/or CP-violating vacua. Exclusing these solution, the position of the

[1] From this point on, we will call the first scalar doublet ϕ_1.

[2] We assume that all coefficients are real.

minimum can be written in this form:

$$\phi_1 = \begin{pmatrix} 0 \\ \frac{v_1}{\sqrt{2}} \end{pmatrix}; \qquad \phi_2 = \begin{pmatrix} 0 \\ \frac{v_2}{\sqrt{2}} \end{pmatrix}. \tag{2}$$

Note that each doublet obtains its own vev[3]. As we will see later, the quadrature sum of these vevs is constrained by electroweak measurements (and must be equal to the SM vev), and so it is useful to introduce a parameter β to describe how the vev is shared between the two doublets:

$$v^2 = v_{\mathrm{SM}}^2 = v_1^2 + v_2^2,$$

$$\tan\beta = \frac{v_2}{v_1} \tag{3}$$

$$\Rightarrow \sin\beta = \frac{v_2}{v}; \quad \cos\beta = \frac{v_1}{v}. \tag{4}$$

At the minimum point, the derivatives of V_ϕ must all be zero, which leads to the following constraints on the model parameters:

$$\begin{aligned} v_1\left(m_{11}^2 + \frac{\lambda_1}{2}v_1^2 + \frac{\lambda_3}{2}v_2^2\right) &= v_2\left(m_{12}^2 - \frac{(\lambda_4+\lambda_5)}{2}v_1v_2\right) \quad \text{from } \frac{\partial V_\phi}{\partial \phi_1^\dagger} = 0. \\ v_2\left(m_{22}^2 + \frac{\lambda_2}{2}v_2^2 + \frac{\lambda_3}{2}v_1^2\right) &= v_1\left(m_{12}^2 - \frac{(\lambda_4+\lambda_5)}{2}v_1v_2\right) \quad \text{from } \frac{\partial V_\phi}{\partial \phi_2^\dagger} = 0. \end{aligned} \tag{5}$$

Naturally, if these equations cannot be satisfied for any values of v_1 and v_2, then such a vacuum is impossible.

1.1 Higgs mass eigenstates

Unlike the BEH theory in the SM, there are a number of possible excitations around the minimum of Equation (2), rather than just one. This means that there are multiple observable Higgs bosons. It turns out that these excitations can be written in the following way:

$$\phi_i = \begin{pmatrix} H_i^+ \\ \frac{v_i + H_i^0 + iA_i^0}{\sqrt{2}} \end{pmatrix} \quad \text{for } i = 1, 2. \tag{6}$$

To determine the masses and properties of these states, we need to compute the Lagrangian terms involving them. We will start by finding the mass eigenstates of the charged Higgs bosons. These can be found by replacing

[3] At this point, there is ambiguity in how the two doublets are defined. In fact, they can be rotated into each other arbitrarily without changing the phenomenology. Later, when we discuss fermion interactions, we will see how this ambiguity might be resolved.

Equation (6) into Equation (1) and selecting only those terms proportional to $H_i^- H_j^+$. These terms can be written in the following matrix form:

$$V_\pm = \begin{pmatrix} H_1^- & H_2^- \end{pmatrix} \begin{pmatrix} m_{11}^2 + \frac{1}{2}\lambda_1 v_1^2 + \frac{1}{2}\lambda_3 v_2^2 & -m_{12}^2 + \frac{\lambda_4+\lambda_5}{2} v_1 v_2 \\ -m_{12}^2 + \frac{\lambda_4+\lambda_5}{2} v_1 v_2 & m_{22}^2 + \frac{1}{2}\lambda_2 v_2^2 + \frac{1}{2}\lambda_3 v_1^2 \end{pmatrix} \begin{pmatrix} H_1^+ \\ H_2^+ \end{pmatrix}. \tag{7}$$

This can be simplified considerably by using Equation (5):

$$V_\pm = \left(m_{12}^2 - \frac{\lambda_4 + \lambda_5}{2} v_1 v_2 \right) \begin{pmatrix} H_1^- & H_2^- \end{pmatrix} \begin{pmatrix} \frac{v_2}{v_1} & -1 \\ -1 & \frac{v_1}{v_2} \end{pmatrix} \begin{pmatrix} H_1^+ \\ H_2^+ \end{pmatrix} \tag{8}$$

The two eigenvalues of the mass matrix in Equation (8) are 0 and $\sec\beta \csc\beta = v^2/v_1^2 v_2^2$. The first eigenvalue corresponds to a Goldstone particle (absorbed by the W), while the second is a physical, charged Higgs boson state $H^\pm$. By convention, this state is defined as

$$\begin{aligned} H^\pm &= -H_1^\pm \sin\beta + H_2^\pm \cos\beta \\ \text{with } m_{H^\pm}^2 &= \left[\frac{m_{12}^2}{v_1 v_2} - \frac{\lambda_4 + \lambda_5}{2} \right] v^2. \end{aligned} \tag{9}$$

A similar analysis of the pseudoscalar states A_i^0 yields a similar result – one Goldstone boson that is absorbed by the Z, and one physical eigenstate rotated from the original basis by an angle of β (with a squared mass that depends on $\frac{m_{12}^2}{v_1 v_2} v^2$).

The scalar fields H_i^0 have a more complex behaviour. As in the SM case, these are not absorbed during electroweak symmetry breaking, and thus two physical states are obtained. Conventionally, h and H denote the less massive and more massive states, respectively. They are an admixture of the original H_i^0 states, with a mixing angle α that is in general not equal to β. The neutral Higgs boson states are defined as follows:

$$\begin{aligned} h &= \sqrt{2} \left(H_1^0 \sin\alpha - H_2^0 \cos\alpha \right) \\ H &= -\sqrt{2} \left(H_1^0 \cos\alpha + H_2^0 \sin\alpha \right) \\ A &= \sqrt{2} \left(A_1^0 \sin\beta - A_2^0 \cos\beta \right). \end{aligned} \tag{10}$$

The masses for h and H are difficult to write in closed form, however if m_H is sufficiently larger than m_h, it scales with $m_{H^\pm}$ and m_A forming a near-degenerte set of Higgs states at high mass.

1.2 Couplings to gauge bosons

As in the SM, the gauge boson masses and couplings to the Higgs field are all described by the kinetic terms for the Higgs fields in the Lagrangian.

From tutorial 3, we recall the form of the covariant derivative for the Higgs field:

$$\mathcal{D}_\mu^{\phi_i}\phi_i = \begin{pmatrix} \partial_\mu + ieA_\mu + i\frac{g\cos 2\theta_\mathrm{W}}{2\cos\theta_\mathrm{W}}Z_\mu & i\frac{g}{\sqrt{2}}W_\mu^+ \\ i\frac{g}{\sqrt{2}}W_\mu^- & \partial_\mu - i\frac{g}{2\cos\theta_\mathrm{W}}Z_\mu \end{pmatrix} \begin{pmatrix} H_i^+ \\ \frac{v_i+H_i^0+iA_i^0}{\sqrt{2}} \end{pmatrix}. \tag{11}$$

Upon expansion of $(\mathcal{D}_\mu^{\phi_i}\phi_i)^\dagger\mathcal{D}^{\mu\phi_i}\phi_i$, and re-expression in terms of the physical Higgs boson eigenstates, the interactions of the gauge bosons with the Higgs fields can be determined. We will not attempt a complete overview of these interactions, but simply select a few of the most interesting terms for study.

The mass of the Z boson is determined by those terms proportional to $Z_\mu Z^\mu$ and contain no other fields (vevs are allowed). The only such terms arise from the product of the lower right element of $\mathcal{D}_\mu^{\phi_i}$ with v_i, which is then squared to give

$$\mathcal{L}_{m_Z} = \frac{g^2}{8\cos^2\theta_\mathrm{W}}v_1^2 Z_\mu Z^\mu + \frac{g^2}{8\cos^2\theta_\mathrm{W}}v_2^2 Z_\mu Z^\mu = \frac{g^2}{8\cos^2\theta_\mathrm{W}}v^2 Z_\mu Z^\mu. \tag{12}$$

This is the same term that arises in the SM Lagrangian, as anticipated in Equation (4). The same conclusion holds for the W boson, namely that the mass of each boson depends not on v_1 and v_2 individually, but only the combination v (at leading order). As in the SM, the photon remains massless, and only interacts with $H^\pm$ through its electric charge.

Next, we examine the interactions of the Z boson with the scalar fields h and H. Again, only the lower right element of $\mathcal{D}_\mu^{\phi_i}$ is relevant, and (apart from a combinatorial factor of two) we simply replace one vev in each term of Equation (12) with the corresponding scalar field:

$$\begin{aligned}\mathcal{L}_{ZH} &= \frac{g^2}{4\cos^2\theta_\mathrm{W}}\left\{v_1 H_1^0 + v_2 H_2^0\right\} Z_\mu Z^\mu \\ &= -\frac{g^2 v}{4\sqrt{2}\cos^2\theta_\mathrm{W}}\left\{h\sin\beta\cos\alpha + H\sin\beta\sin\alpha\right. \\ &\qquad\qquad \left. -h\cos\beta\sin\alpha + H\cos\beta\cos\alpha\right\} Z_\mu Z^\mu \\ &= -\frac{g^2 v}{4\sqrt{2}\cos^2\theta_\mathrm{W}}\left\{h\sin(\beta-\alpha) + H\cos(\beta-\alpha)\right\} Z_\mu Z^\mu \end{aligned} \tag{13}$$

Again, similar results hold for the W boson. The couplings to bosons thus depend on the relative alignment of the angles α and β. An interesting possibility is the case where $\sin(\beta-\alpha) = 1$. In this case, called the *decoupling limit*, h behaves as the SM Higgs boson, and H does not couple to the W or Z at all. Their roles are reversed if $\cos(\beta-\alpha) = 1$, although this is regarded as a less plausible scenario as it requires the lighter state h to remain unobserved at LEP and the LHC.

1.3 Couplings to fermions

Unlike the bosons, whose masses and couplings are determined by the electroweak symmetry breaking itself, the fermions are assigned couplings to the SM Higgs field in an ad-hoc manner. Nevertheless, with a single Higgs doublet, the assignment is at least unique. In 2HDM models, either or both doublet can be assumed to couple to each fermion. However, if the couplings are completely arbitrary, large flavour-changing neutral currents would be expected, as the mass eigenstates could not be diagonal in the interaction bases of both Higgs doublets simultaneously, in general. These effects have not been observed, and so most models assume that each fermion couples to just one of the doublets.

By convention, it is assumed that the up-type quarks (u, c, t) couple only to ϕ_2. In fact, this defines what we mean by ϕ_2, resolving the ambiguity between the two doublets mentioned in footnote 3. For the down-type quarks and the leptons, a number of different choices can be made, which are classified as follows:

Type I: All fermion fields couple to ϕ_2 only.

Type II: Down-type fermions couple to ϕ_1.

Type III or X: d-quarks couple to ϕ_2, charged leptons to ϕ_1.

Type IV or Y: d-quarks couple to ϕ_1, charged leptons to ϕ_2.

These assignments affect the fermion-Higgs couplings, and hence the phenomenology observed in Higgs boson production and decay, as given in Table 1. To see how these are computed, consider the mass term for a fermion f in a Type I model (or up-type quarks in any model):

$$\mathcal{L}_{m_f} = -\frac{y_f v_2}{\sqrt{2}} \bar{f} f = -m_f \bar{f} f. \tag{14}$$

In terms of the SM Higgs sector parameters, $m_f = y_f^{\mathrm{SM}} v/\sqrt{2}$. Therefore, $y_f = y_f^{\mathrm{SM}} v/v_2 = y_f^{\mathrm{SM}}/\sin\beta$. Now, the coupling of the fermion to the neutral, scalar Higgs sector is given by

$$\mathcal{L}_{m_f} = -\frac{y_f}{\sqrt{2}} H_2^0 \bar{f} f. \tag{15}$$

Using Equation (10) to express H_2^0 in terms of mass eigenstates, it is evident then that the couping to the lighter field h is proportional to $\cos\alpha/\sin\beta$, and the coupling to the heavier field H is proportional to $\sin\alpha/\sin\beta$. The fermion couplings in the other models (and to A) arise in a similar way, via modifications of the Yukawa couplings from the SM values and the mixing between the different Higgs fields. Note that in the case of a Type I model, it is possible for the h (or H) to be fermiphobic, i.e. to have zero or negligible couplings to fermions. This is strongly disfavoured by the LHC observations, as the gluon fusion production channel would be absent.

<table>
<tr><th></th><th></th><th>u</th><th>d</th><th>ℓ</th><th>W/Z</th></tr>
<tr><td rowspan="3">Type I</td><td>h</td><td colspan="3">$\cos\alpha/\sin\beta$</td><td>$\sin(\beta-\alpha)$</td></tr>
<tr><td>H</td><td colspan="3">$\sin\alpha/\sin\beta$</td><td>$\cos(\beta-\alpha)$</td></tr>
<tr><td>A</td><td>$\cot\beta$</td><td colspan="2">$-\cot\beta$</td><td>0</td></tr>
<tr><td rowspan="3">Type II</td><td>h</td><td>$\cos\alpha/\sin\beta$</td><td colspan="2">$-\sin\alpha/\cos\beta$</td><td>$\sin(\beta-\alpha)$</td></tr>
<tr><td>H</td><td>$\sin\alpha/\sin\beta$</td><td colspan="2">$\cos\alpha/\cos\beta$</td><td>$\cos(\beta-\alpha)$</td></tr>
<tr><td>A</td><td>$\cot\beta$</td><td colspan="2">$\tan\beta$</td><td>0</td></tr>
<tr><td rowspan="3">Type III(X)</td><td>h</td><td colspan="2">$\cos\alpha/\sin\beta$</td><td>$-\sin\alpha/\cos\beta$</td><td>$\sin(\beta-\alpha)$</td></tr>
<tr><td>H</td><td colspan="2">$\sin\alpha/\sin\beta$</td><td>$\cos\alpha/\cos\beta$</td><td>$\cos(\beta-\alpha)$</td></tr>
<tr><td>A</td><td>$\cot\beta$</td><td>$-\cot\beta$</td><td>$\tan\beta$</td><td>0</td></tr>
<tr><td rowspan="3">Type IV(Y)</td><td>h</td><td>$\cos\alpha/\sin\beta$</td><td>$-\sin\alpha/\cos\beta$</td><td>$\cos\alpha/\sin\beta$</td><td>$\sin(\beta-\alpha)$</td></tr>
<tr><td>H</td><td>$\sin\alpha/\sin\beta$</td><td>$\cos\alpha/\cos\beta$</td><td>$\sin\alpha/\sin\beta$</td><td>$\cos(\beta-\alpha)$</td></tr>
<tr><td>A</td><td>$\cot\beta$</td><td>$\tan\beta$</td><td>$-\cot\beta$</td><td>0</td></tr>
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Table 1: Couplings to the neutral Higgs boson fields in 2HDM models that forbid FCNCs at tree level. The couplings for gauge bosons and up-type quarks are independent of the model variety, but are included for completeness.

Figure 1: 95% CL limits on additional Higgs bosons in the MSSM, from the $H/A \to \tau^+\tau^-$ channel.

2 LHC constraints on 2HDMs

The predictions of 2HDM models fall into two main categories. First, the couplings of the known Higgs boson (usually assumed to be h) can deviate from those predicted by the SM, by the amounts listed in Table 1. The strongest constraints come from measurements of couplings to bosons, which limit the allowed range of $(\beta-\alpha)$. Constraints from measurements of the couplings to fermions constrain different combinations of β and α, in a way that depends on the assumed coupling scheme. Current constraints from the ATLAS experiment are shown in Figure 2, where the constraints from leptonic modes are expressed in terms of $\tan\beta$. In all cases, SM-like couplings are preferred, corresponding to $\cos(\beta-\alpha)\approx 0$ and $\tan\beta\sim 1$ (this corresponds to $\beta\approx-\alpha\approx\frac{\pi}{4}$).

The second main set of predictions is that there should be additional Higgs bosons, the H, A and $H^{\pm}$. Figure 1 shows an example search for $H/A\to\tau^+\tau^-$ in the minimal supersymmetric SM (which is a kind of Type II 2HDM). The parameter m_A corresponds to the general scale of the new bosons, although substantial mixing occurs between the H and h at the lower end of the scale. Comparing with Table 1, we see that the couplings for down-type quarks and charged leptons to the (pseudo-)scalar bosons all increase as $\tan\beta$ increases (and therefore $\cos\beta$ decreases), and this is the reason that the constraint is best for large values of $\tan\beta$. Searches have also been carried out for the charged Higgs boson, in the channel $t \to H^+b \to \tau^+\nu_\tau b$, with negative results. This allows nearly the full range of $m_{H^\pm} < 160$ GeV to be excluded, except for in a narrow range of $\tan\beta \sim 10$.

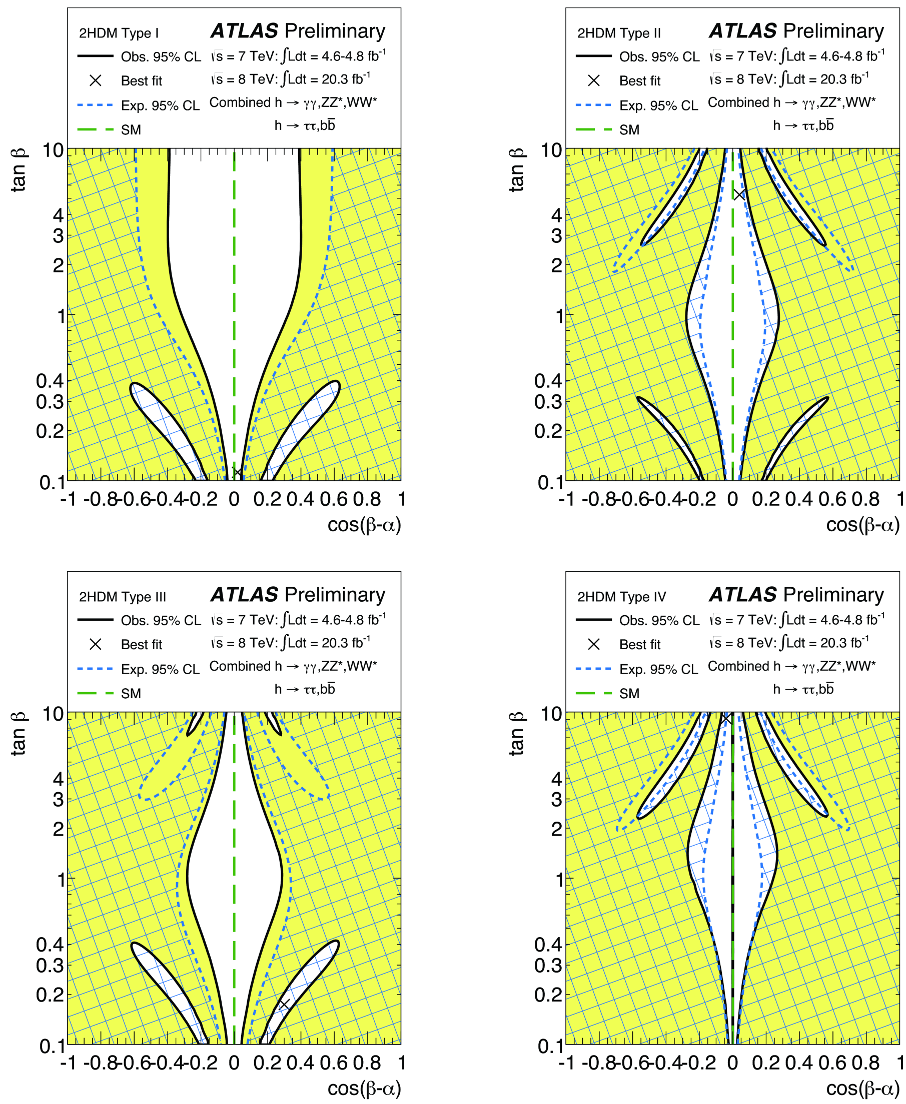

Figure 2: Constraints on 2HDMs from measurements of the Higgs boson coupling strengths by ATLAS. A value of $\cos(\beta-\alpha)=0$ corresponds to the decoupling limit, where the coupling of h to bosons is the same as in the SM.