Tutorial 10

February 1, 2017

"Tests des Standardmodells der Teilchenphysik"

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WS 2016

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Higgs production in e^-e^+ collisions - Bremsstrahlung from the Z^0 boson

At any e^+e^- collider, such as LEP [1] and the future International Linear Collider (ILC) [2], the main production mechanisms of Higgs are the Higgs-strahlung process and the WW fusion mechanism.

1. Draw the corresponding Feynman diagrams.

The integrated cross-section for the Higgs-strahlung process via the Z^0 boson is given by

$$\sigma(e^+e^- \longrightarrow ZH) = \frac{G_F^2 M_Z^4}{96\pi s} \left((g_A^e)^2 + (g_V^e)^2\right) \sqrt{\lambda}\, \frac{\lambda + 12\frac{M_Z^2}{s}}{\left(1 - \frac{M_Z^2}{s}\right)^2} \tag{1}$$

where

- $G_F = 1.16637 \times 10^{-5}$ GeV^{-1} is the Fermi coupling constant,
- $g_A^e = -1$ and $g_V^e = -1 + 4\sin^2\theta_W$ are the $Z^0e^+e^-$ couplings with the electroweak mixing angle being $\sin^2\theta_W = 0.23149$,
- $M_Z = 91.2$ GeV is the Z^0 boson mass,
- M_H is the Higgs boson mass, and
- λ is the two-particle phase-space function given by

$$\lambda = \left(1 - \frac{M_H^2}{s} - \frac{M_Z^2}{s}\right)^2 - \left(\frac{2M_H M_Z}{s}\right)^2 . \tag{2}$$

2. Study the cross-section for the Higgs-strahlung process in e^+e^- collisions for different hypothetical Higgs masses as a function of

   (a) the center-of-mass energy, $\sqrt{s}$;

   (b) the Higgs mass, m_H.

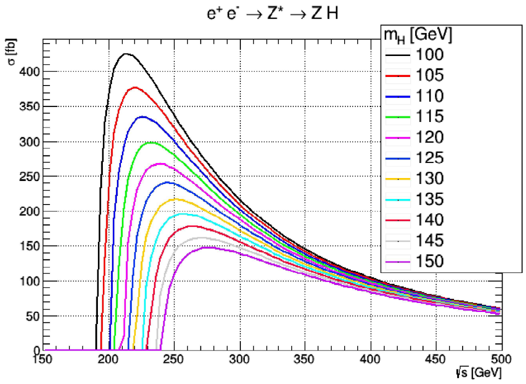
   Discuss the kinematic constrains (kinematic factor) responsible for the sudden drop of the cross-section in the moderate mass range and in particular for the Higgs decaying into fermions ($b\bar{b}$, $\tau^+\tau^-$, ...).

3. With the LEP operating at a center-of-mass energy $\sqrt{s} \leq 209$ GeV, which kind of Higgs searches seem to be more feasible?

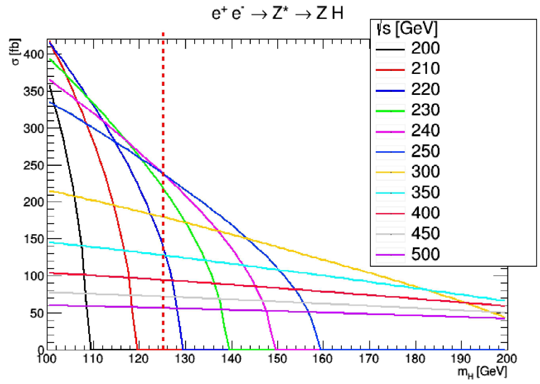
4. List the major search channels considering the possible Z^0 and H^0 decays. Which are the main background processes for each case?

5. Proof the following formula
$$M_H^2 = M_{\text{recoil}}^2 = s\left(1 - \frac{2E_Z\sqrt{s}}{s} + \frac{M_Z^2}{s}\right) \tag{3}$$
which gives the so-called *Higgs Recoil Mass*.

6. Calculate the typical values for the Higgs energy E_H, the Z boson energy E_Z and their momenta $|\boldsymbol{p}_H| \simeq |\boldsymbol{p}_Z|$, for $\sqrt{s} = 250$ GeV and $m_H = 125.5$ GeV.



(a) Production cross section as a function of $\sqrt{s}$



(b) Production cross section as a function of m_H

Figure 1: Higgs production in electron-positron collisions.

Decays of the Higgs Boson

Consider the Higgs boson mass m_h as a free parameter. The coupling of the Higgs boson h^0 to a fermion f, either lepton or quark, is given by
$$\mathcal{L}_f = -\frac{1}{\upsilon} m_f h^0 \bar{f} f \tag{4}$$
and hence at tree level the matrix element is
$$i\mathcal{M} = -i\frac{1}{\upsilon} m_f \; \bar{u}^s(p) \upsilon^r(k) \tag{5}$$
Therefore, the $h^0 \longrightarrow \bar{f}f$ decay involves a single Yukawa vertex only.

1. Show that the decay width is
$$\Gamma\left(h^0 \longrightarrow \bar{f}f\right) = N_c^f \times \frac{G_F}{4\pi\sqrt{2}} m_h m_f^2 \left[1 - \left(\frac{2m_f}{m_h}\right)^2\right]^{3/2} \;, \quad \text{where} \quad N_c^f = \begin{cases} 1 & \text{for leptons} \\ 3 & \text{for quarks} \end{cases} . \tag{6}$$

The Yukawa couplings of the Higgs bosons to the massive gauge bosons $V = W^{\pm}$, Z^0 are
$$\mathcal{L}_V = \frac{1}{\upsilon}\left(2M_W^2 \; h^0 W_\mu^+ W^{-\mu} + \; M_Z^2 \; h^0 Z_\mu^0 Z^{0\mu}\right) \tag{7}$$
and thus the amplitude for the $h^0 \to W^+W^-$ process is
$$i\mathcal{M} = \frac{1}{2}\upsilon g^2 g^{\mu\nu} \epsilon_\mu^*(p,\, \lambda) \epsilon_\nu^*(k,\, \rho) \; . \tag{8}$$

2. Show that in the on-shell approximation the decay width is
$$\Gamma(h^0 \longrightarrow W^+W^-) = \frac{G_F}{8\pi\sqrt{2}} m_h^3 (1 - 4\lambda_W)^{\frac{1}{2}} \left(1 - 4\lambda_W + 12\lambda_W^2\right) \;, \tag{9}$$
where $\lambda_W = (M_W/m_h)^2$. Similarly, show that decay rate for the process $h^0 \longrightarrow Z^0Z^0$ is
$$\Gamma(h^0 \longrightarrow Z^0Z^0) = \frac{G_F}{16\pi\sqrt{2}} m_h^3 (1 - 4\lambda_Z)^{\frac{1}{2}} \left(1 - 4\lambda_Z + 12\lambda_Z^2\right) \;, \tag{10}$$
where $\lambda_Z = (M_Z/m_h)^2$.

3. Calculate the ratios
$$\frac{\Gamma(h^0 \to b\bar{b})}{\Gamma(h^0 \to c\bar{c})}, \quad \frac{\Gamma(h^0 \to b\bar{b})}{\Gamma(h^0 \to \tau^+\tau^-)}, \quad \frac{\Gamma(h^0 \to b\bar{b})}{\Gamma(h^0 \to \mu^+\mu^-)} \quad \text{and} \tag{11}$$
using the data provided below to provide arithmetic results. Make a plot of the ratio
$$\frac{\Gamma(h^0 \to W^+W^-)}{\Gamma(h^0 \to Z^0Z^0)} \tag{12}$$
as a function of the Higgs mass, m_h.

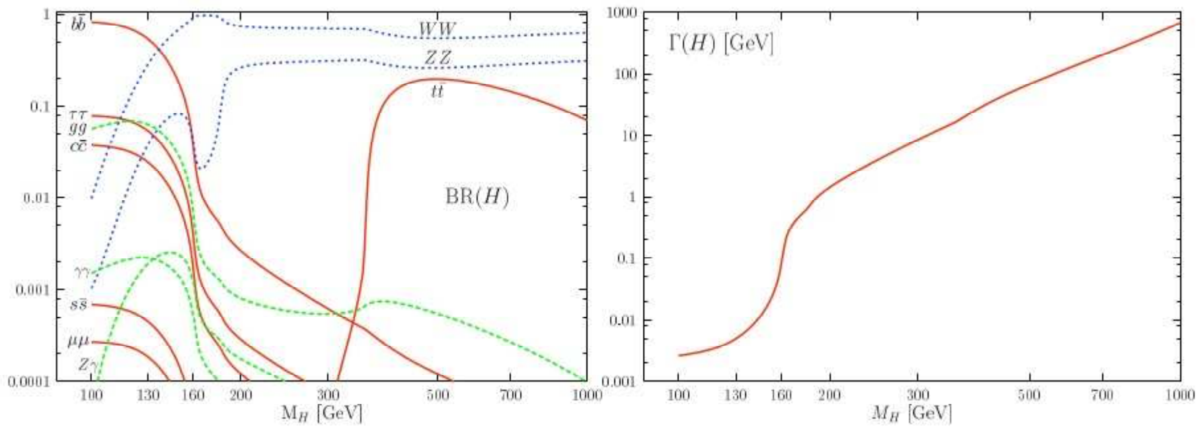


Figure 2: The branching ratios (left) and the total decay width (right) of SM Higgs boson as a function of its mass(See Refs. [3] and [4]).

Data

$m_h = 125.09$ GeV	$m_\tau = 1.77$ GeV	$m_b = 4.18$ GeV	$m_c = 1.27$ GeV
$m_\mu = 0.106$ GeV	$m_Z = 91.19$ GeV	$m_W = 80.2$ GeV	$\sin^2\theta_W = 0.23$
$\upsilon = \frac{1}{\sqrt{\sqrt{2}G_F}} = 246$ GeV	$(1\ \text{GeV})^{-2} = 0.389$ mb		

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