Tutorial 9

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"Tests des Standardmodells der Teilchenphysik"

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Z^0 Boson Decay

The mediators of weak interactions are the massive intermediate vector bosons $W^{\pm}$ and Z^0.

1. Before the direct observation of the Z^0 boson at CERN in 1983, and the precise measurement of its mass, it was believed to weigh at least $m_Z \gtrsim 80$ GeV. Given that the weak and electromagnetic interactions have approximately similar intrinsic strength, as manifested in unified gauge theories, and that charged and neutral currents are of about comparable strength, show that this particular lower bound on the Z^0 is a very reasonable and valid assumption. See also plot in Figure 2.

2. Let us consider the general decay rate of the Z^0 bosons into fermions,

$$Z^0 \to f\bar{f} \tag{1}$$

where the boson is at rest. The Feynman diagram of this decay process is shown in Figure 1. Write down

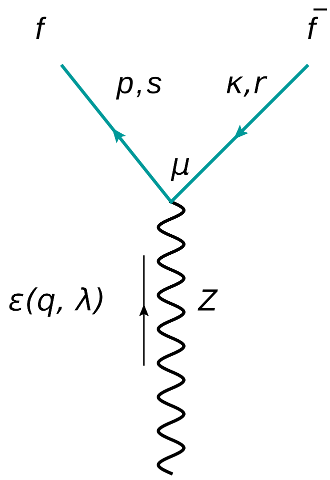

Figure 1: Z boson decay into a pair of fermions.

the expression for the matrix element corresponding to this two-body decay process. The Feynman rules for the "V-A" theory are summarized in the Addendum.

3. The next step is to calculate the squared amplitude. To sum over the spins of the emitted fermions and average over the three polarization states of the Z^0 boson we need to make use of the relations of completeness as shown in the Addendum. Finally, to work out the traces we will need the properties of the γ^5 matrices. Show that the squared amplitude takes the form

$$\langle|\mathcal{M}|^2\rangle = \frac{g_Z^2}{3}\left[(c_V^2 + c_A^2)\left(\kappa \cdot p + \frac{2}{M^Z}(\kappa \cdot q)(p \cdot q)\right) + 3m_f^2(c_V^2 - c_A^2)\right] . \tag{2}$$

and in the case of "massless" fermions it simplifies to

$$\langle|\mathcal{M}|^2\rangle \simeq \frac{g_Z^2 M_Z^2}{3}\left(c_V^2 + c_A^2\right) \ . \tag{3}$$

4. Show that its total decay width is

$$\Gamma = \sum_f N_c^f \, N_g^f \, \Gamma_f \quad \text{with} \quad \Gamma_f = \frac{g_Z^2 M_Z}{48\pi}\left[\left(c_V^f\right)^2 + \left(c_A^f\right)^2\right] \quad \text{and} \quad m_f < \frac{M_Z}{2} \tag{4}$$

assuming that decay products have negligible mass with respect to the parent particle, $m_f \ll m_Z$. N_g^f is the number of generations in which the fermion pair $f\bar{f}$ (neutrinos, charged leptons, up-type quarks and down-type quarks) belongs to. N_c^f stands for the number of colors in the case of quarks.

5. How much sensitive is the Z^0 lifetime to a possible additional (fourth) generation of light fermions and how much it would change in such scenario?

6. Calculate the total decay width (in GeV) and the mean lifetime (in seconds) of the Z^0 boson.

7. Make a list the various decay widths and branching ratios for each species of quarks and leptons, taking also into account the color charge of the quarks. The (anti)top quark is of course much heavier than the Z^0 boson with $m_t > m_Z/2$. Employ the following value of the weak mixing angle, $\sin^2\theta_w = 0.23$, in your final arithmetic calculations to derive a numeric result for each branching ratio.

8. Write down also the general formula for the Z^0 decay width into hadrons considering the number of fermionic generations, N_g^f, to be a free parameter. If the cross-section of the Z^0 boson around its resonance region ($\sqrt{s} \simeq M_Z$) can be precisely measured in hadronic and muonic final states according to the formula

$$\sigma(s) = \frac{12\pi}{M_Z^2}\frac{s\,\Gamma_{ee}\,\Gamma_{f\bar{f}}}{(s - M_Z^2)^2 + \frac{s^2\Gamma_Z^2}{M_Z^2}} \ , \tag{5}$$

how one can deduce the number of leptonic generations N_g^l (and thus the number of generations of quarks N_g^q, since according to the Standard Model the number of lepton and quark generations correspond) knowing $\Gamma_{\mu\mu}$ and Γ_{hadrons}? (Hint: *find the ratio* $\sigma_{\mu\mu}/\sigma_{\text{hadrons}}$ *around the Z resonance. What is currently the number* N_g^f *known to be?*)

Addendum

Z mass dependence on the Weinberg Angle

Gamma matrices and Traceology

$$
\begin{aligned}
\gamma^5 &= \frac{i}{4!}\varepsilon_{\mu\nu\alpha\beta}\gamma^\mu\gamma^\nu\gamma^\alpha\gamma^\beta && (6)\\
(\gamma^5)^\dagger &= \gamma^5 && (7)\\
(\gamma^5)^2 &= 1 && (8)\\
\left\{\gamma^5,\gamma^\mu\right\} &= \gamma^5\gamma^\mu + \gamma^\mu\gamma^5 = 0 && (9)\\
\mathrm{tr}(\gamma^\mu) &= 0 && (10)\\
\mathrm{tr}(\gamma^\mu\gamma^\nu) &= 4g^{\mu\nu} && (11)\\
\mathrm{tr}(\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma) &= 4(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\rho}) && (12)\\
\mathrm{tr}(\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma^5) &= -4i\epsilon^{\mu\nu\rho\sigma} && (13)\\
\mathrm{tr}(\gamma^5\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma) &= +4i\epsilon^{\mu\nu\rho\sigma} && (14)\\
\mathrm{tr}(\gamma^5) &= \mathrm{tr}(\gamma^\mu\gamma^\nu\gamma^5) = 0 && (15)\\
\mathrm{tr}(\text{odd number of } \gamma\text{'s}) &= 0 && (16)\\
\varepsilon_{ijkl} &= \begin{cases} +1 & \text{if } (i,j,k,l) \text{ is an even permutation of } (1,2,3,4)\\ -1 & \text{if } (i,j,k,l) \text{ is an odd permutation of } (1,2,3,4)\\ 0 & \text{otherwise}\end{cases} && (17)\\
\varepsilon^{ijkl} &= \varepsilon_{ijkl} && (18)\\
\varepsilon_{\mu\nu\rho\sigma}\varepsilon^{\mu\nu\kappa\lambda} &= -2\left(\delta_\sigma{}^\kappa\delta_\rho{}^\lambda - \delta_\rho{}^\kappa\delta_\sigma{}^\lambda\right) && (19)\\
& && (20)
\end{aligned}
$$

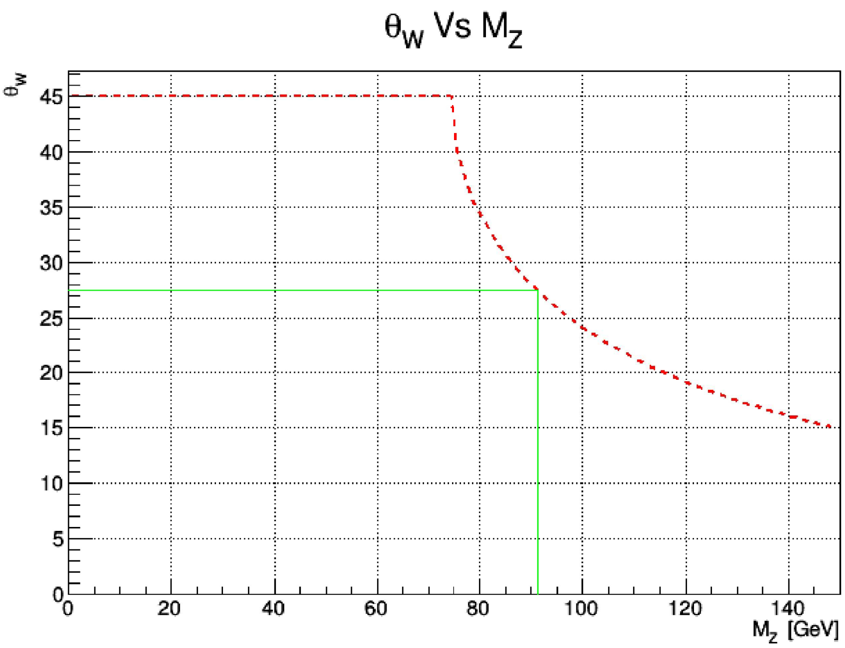

Figure 2: Mass of the Z boson as a function of the Weinberg angle θ_W.

Vector and axial-vector weak couplings for neutral currents

f	Q_f	c_A^f	c_V^f
$\nu_e,\ \nu_\mu,\ \nu_\tau$	0	$+1/2$	$+1/2$
$e^-,\ \mu^-,\ \tau^-$	-1	$-1/2$	$-\frac{1}{2}+2\sin^2\theta_W \simeq -0.03$
$u,\ c,\ t$	$+2/3$	$+1/2$	$+\frac{1}{2}-\frac{4}{3}\sin^2\theta_W \simeq +0.19$
$d,\ s,\ b$	$-1/3$	$-1/2$	$-\frac{1}{2}+\frac{2}{3}\sin^2\theta_W \simeq -0.34$

Table 1: Neutral vector and axial-vector $Z^0 f\bar{f}$ couplings in the GWS model with $\sin^2\theta_W = 0.234$.

Useful Formulæ

$$Z^0 ff \text{ vertex factor} = -ig_z\gamma^\mu \frac{1}{2}\left(c_V^f - c_A^f\gamma^5\right),\ f = \text{any fermion} \tag{21a}$$

$$Z^0,\ W^\pm \text{ propagator} = -i\frac{g_{\mu\nu} - \frac{q_\mu q_\nu}{M^2_{Z/W}}}{q^2 - M^2_{Z/W}} \tag{21b}$$

$$g_W = \sqrt{4\pi\alpha_W} = \text{weak coupling constant for CC} \tag{21c}$$

$$\alpha_W = \frac{g_W^2}{4\pi} = \frac{1}{29} = \text{weak fine structure constant} \tag{21d}$$

$$g_W = \frac{g_e}{\sin\theta_W} \tag{21e}$$

$$g_Z = \frac{g_e}{\sin\theta_w\cos\theta_W} = \text{weak coupling constant for NC} \tag{21f}$$

$$\frac{g_Z}{M_Z} = \frac{g_W}{M_W} \tag{21g}$$

$$g_e = \sqrt{4\pi\alpha} = \text{electromagnetic coupling constant} \tag{21h}$$

$$\alpha = \frac{e^2}{4\pi} = \text{fine structure constant} \tag{21i}$$

$$\frac{G_F}{\sqrt{2}} = \frac{g_W^2}{8m_W^2} = \text{Fermi's coupling constant} \tag{21j}$$

$$m_W = m_Z\cos\theta_W \tag{21k}$$

$$\sin^2\theta_W = 0.23120 \pm 0.00015 = \text{weak mixing angle} \tag{21l}$$

$$c_V^f = T_f^3 - 2\sin^2\theta_W.Q_f = \text{vector weak coupling} \tag{21m}$$

$$c_A^f = T_f^3 = \text{axial-vector weak coupling} \tag{21n}$$

$$T_f^3 = \text{3rd component of the weak isospin of fermion } f \tag{21o}$$

$$Q_f = \text{ charge of interacting fermion } f \tag{21p}$$

$$\tag{21q}$$

The general formula of the differential decay rate of an unstable particle A to a given final state of n particles with momenta $\{p_f\}$ is

$$d\Gamma = \frac{1}{2m_A}\ \langle|\mathcal{M}(p_A \to \{p_f\})|^2\rangle\ d\Pi_n \tag{22}$$

where the Lorentz invariant phase-space for the n-body system is defined as

$$\int d\Pi_n = \left(\prod_f \int \frac{d^3p_f}{(2\pi)^3}\frac{1}{2E_f}\right)(2\pi)^4\ \delta^{(4)}\left(\sum_i p_i - \sum_f p_f\right)\ . \tag{23}$$

This integral is manifestly Lorentz invariant, since it is built up from invariant 3-momentum integrals constrained by a 4-momentum delta function. It is known as *relativistically invariant n-body phase-space*. For the special case of a two-particle decay, this phase-space takes the simple form

$$\int \frac{d\Omega_{\rm CM}}{4\pi}\frac{1}{8\pi}\left(\frac{2|\boldsymbol{p}|}{E_{\rm CM}}\right)\ , \tag{24}$$

where $|\boldsymbol{p}|$ is the magnitude of the 3-momentum vector of either particle in the center-of-mass frame.

Polarization of massive spin-1 bosons

The intermediate vector bosons of the weak interactions, as massive particles of spin 1, have *three* allowed physical polarization states, $m_s = -1,\ 0,\ +1$. They satisfy the relation of completeness

$$\sum_{\lambda=1,2,3}\epsilon_\mu^{(\lambda)}(q).\epsilon_\nu^{*(\lambda)}(q) = -\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{M^2}\right)$$

for the three four-component polarization vectors: $\epsilon_\mu^{(1)}(q)$, $\epsilon_\mu^{(2)}(q)$ and $\epsilon_\mu^{(3)}(q)$, that are orthogonal

$$\epsilon_\mu^{*(1)}\epsilon^{(2)\mu} = 0\,,$$

normalized according to

$$\epsilon_\mu^* \epsilon^\mu = -1$$

and they satisfy the condition

$$q^\mu \epsilon_\mu = 0\,.$$

Propagator for massive spin-1 bosons

The propagator for the massive gauge bosons Z and W is given by

$$-i \frac{g_{\mu\nu} - \frac{q_\mu q_\nu}{M_{Z/W}^2}}{q^2 - M_{Z/W}^2} \tag{25}$$

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