Testing the Standard Model of Elementary Particle Physics I

Sixth lecture

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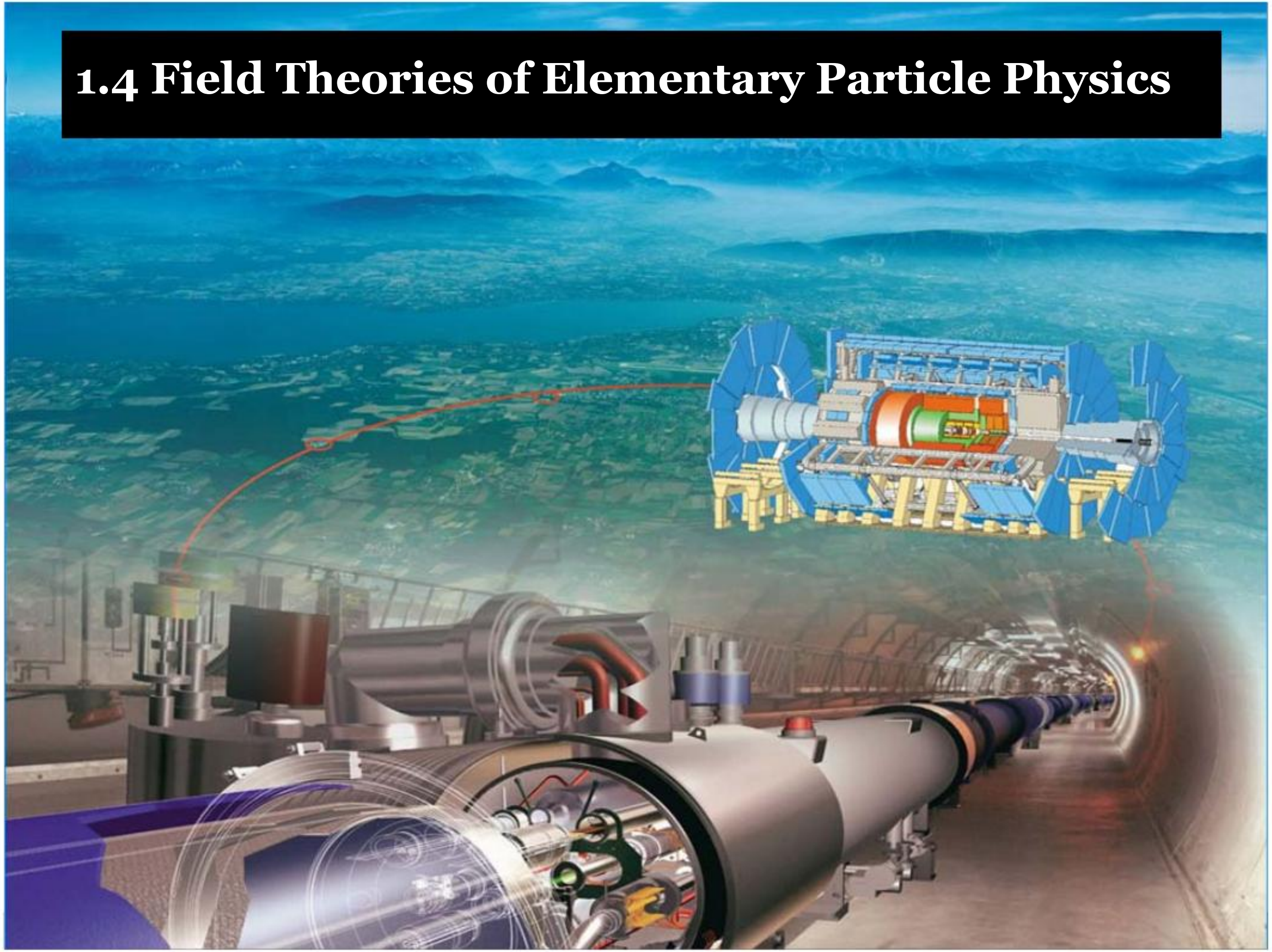
1.4 Field Theories of Elementary Particle Physics



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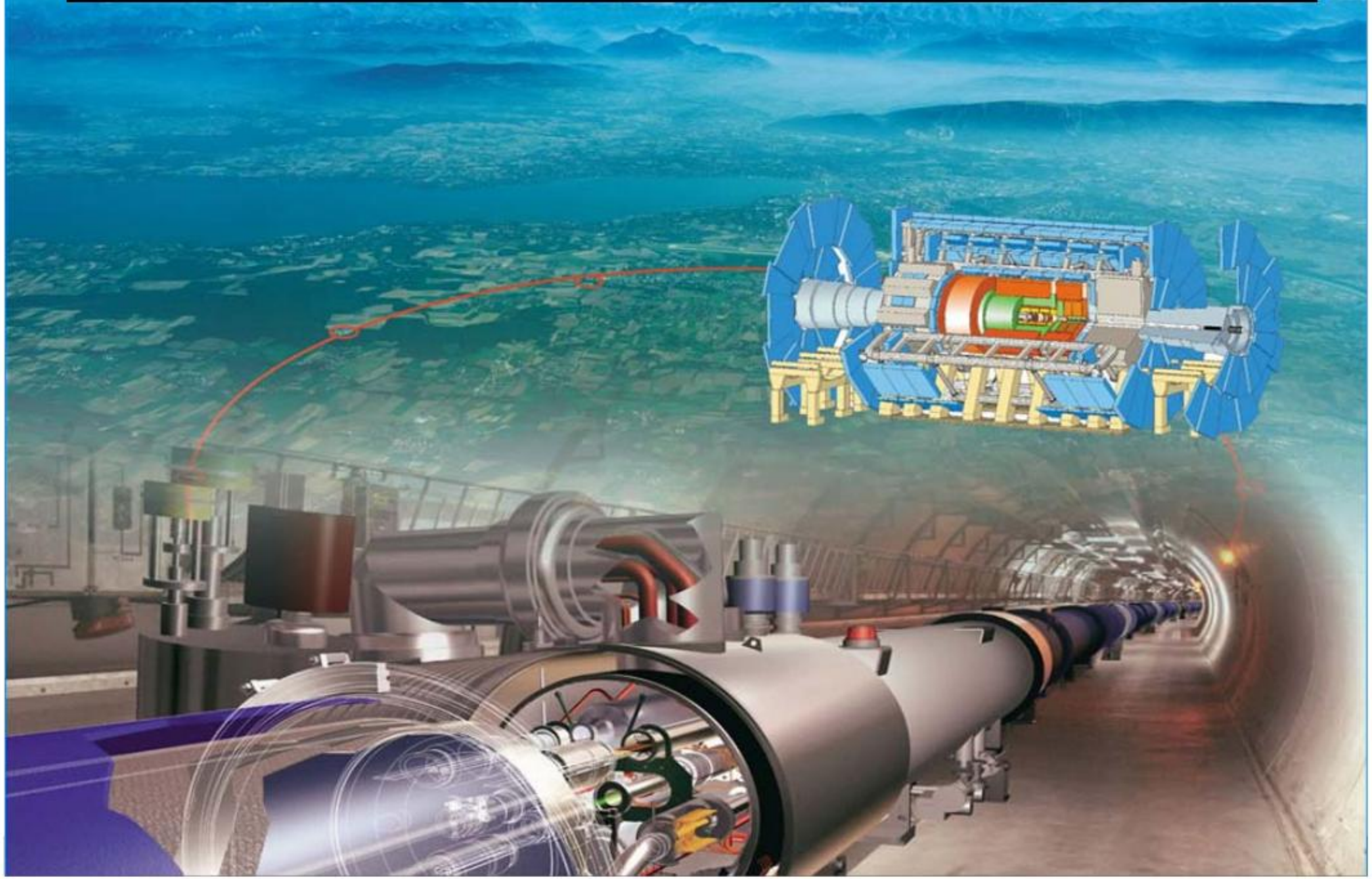
1.4.1 Symmetries and groups



Symmetries in physics

- **Symmetries in nature imply conservation laws.**

Space translation	$\longleftrightarrow$	Momentum
Time translation	$\longleftrightarrow$	Energy
Rotation	$\longleftrightarrow$	Angular momentum
U(1) gauge invariance	$\longleftrightarrow$	Charge conservation

- **Basic principle of QFT: Lagrange functions $\mathscr{L}$ are formulated to be invariant, under global and local phase transformations**

Noether's theorem

- Consider a continuous transform of the field φ, which in infinitesimal form can be written as:

$$\phi(x) \to \phi'(x) = \phi(x) + \alpha\Delta\phi(x)$$

- If $\mathscr{L}(\varphi,\partial_\mu\varphi)$ is invariant under such transform

$$\delta\mathcal{L} = 0$$

- Then there is a current $j_\mu(x)$ conserved

$$j^\mu(x)\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)}\Delta\Phi \quad \to \quad \partial_\mu j^\mu(x) = 0$$

- Conservation law can also be expressed via:

$$Q \equiv \int j^0 d^3x \quad \Longrightarrow \quad \frac{dQ}{dt} = 0$$

Gauge symmetries:

- **Global gauge invariance:**
  - The expectation value of a quantum mechanical observable

$$< \mathcal{O} > = \int \psi^* \mathcal{O} \psi$$

Is invariant with respect to the global phase transformation of the wave function:

$$\psi(x) \longrightarrow \psi'(x) = e^{iQ\alpha}\psi(x)$$

  - The invariance of a Lagrangian with respect to a phase transformation corresponds to a global U(1) symmetry (referred to as gauge symmetry)
    - Leads according to the Noether theorem to a conservation of probability and charge

Gauge symmetries:

- **Global gauge invariance:**
  - The Lagrangian $\mathcal{L}(\phi, \phi^*, \partial_\mu \phi, \partial_\mu \phi^*)$ for a complex scalar is invariant under U(1) gauge transformations:

$$\begin{aligned} \phi(x) &\longrightarrow \phi'(x) = e^{iQ\alpha}\phi(x), \\ \phi^*(x) &\longrightarrow \phi^{*\prime}(x) = e^{-iQ\alpha}\phi^*(x). \end{aligned}$$

    where α does not depend on the space-time (as it is a “global parameter”)
  - The Lagrangian is also invariant under infinitesimal variations of these fields δΦ:

$$\begin{aligned} \phi(x) &\longrightarrow \phi'(x) = \phi(x) + \delta\phi(x) = \phi(x) + iQ(\delta\alpha)\phi(x), \\ \phi^*(x) &\longrightarrow \phi^{*\prime}(x) = \phi^*(x) + \delta\phi^*(x) = \phi^*(x) - iQ(\delta\alpha)\phi^*(x) \end{aligned}$$

  - Such that:

$$\delta(\partial_\mu \phi) = iQ(\delta\alpha)\partial_\mu \phi$$

    follows.

Gauge symmetries:

- **Global gauge invariance:**
  - Exploiting the Euler-Lagrange equation gives:

$$
\begin{aligned}
\delta\mathcal{L} &= \frac{\partial\mathcal{L}}{\partial\phi}\delta\phi + \frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)}\delta(\partial_\mu\phi) + \frac{\partial\mathcal{L}}{\partial\phi^*}\delta\phi^* + \frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi^*)}\delta(\partial_\mu\phi^*) \\
&= \left[\partial_\mu\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)}\right] iQ(\delta\alpha)\phi + \frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)} iQ(\delta\alpha)\partial_\mu\phi + c.c. \\
&= iQ(\delta\alpha)\partial_\mu\left[\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)}\phi\right] - iQ(\delta\alpha)\partial_\mu\left[\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi^*)}\phi^*\right] \equiv 0
\end{aligned}
$$

  - According to the Noether's theorem, we can calculate a current:

$$
j^\mu \equiv -iQ\left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)}\phi - \frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi^*)}\phi^*\right)
$$

    where Q is the conserved charge (since the current fulfills the continuity equation)

    i.e.: $\frac{d}{dt}Q = 0$

Gauge symmetries:

- **Local gauge invariance:**
  - Now we want to ensure that quantum mechanical observables are also invariant under local phase transformations of the wave function:

$$\psi(x) \longrightarrow \psi'(x) = e^{iQ\alpha(x)}\psi(x)$$

  - Space-time dependent phase transformations (local gauge transformations) imply:

$$\partial_\mu\psi(x) \longrightarrow \partial_\mu\psi'(x) = e^{iQ\alpha(x)}[\partial_\mu\psi(x) + iQ(\partial_\mu\alpha(x))\psi(x)]$$

  - Thus need to introduce a covariant derivative like:

Example: **QED**

$$\boxed{\partial_\mu \longrightarrow D_\mu \equiv \partial_\mu + ieQA_\mu(x)} \tag{17}$$

in order to ensure that the Lagrangian becomes invariant under local phase transformations.

Gauge symmetries:

- **Local gauge invariance:**
  - In equation (17), we find the electric charge of the fermion field q = eQ, and the vector potential $A_\mu(x)$ of the electromagnetic field (which is the gauge field of the U(1)). The latter field transforms under phase rotation like:

$$A_\mu(x) \longrightarrow A'_\mu(x) = A_\mu(x) - \frac{1}{e}\partial_\mu \alpha(x)$$

  - Thus $\psi^* D_\mu \psi$ is invariant under local phase transformations since:

$$\begin{aligned}
D_\mu \psi(x) &= (\partial_\mu + ieQA_\mu)\psi \\
\longrightarrow D'_\mu \psi'(x) &= (\partial_\mu + ieQA'_\mu(x))e^{iQ\alpha(x)}\psi(x) \\
&= e^{iQ\alpha(x)}[\partial_\mu + iQ\partial_\mu\alpha(x) + ieQA_\mu(x) - iQ\partial_\mu\alpha(x)]\psi(x) \\
&= e^{iQ\alpha(x)}[\partial_\mu + ieQA_\mu(x)]\psi(x) \equiv e^{iQ\alpha(x)} D_\mu \psi(x)
\end{aligned}$$

Gauge symmetries:

- **Local gauge invariance:**
  - The local gauge invariance under U(1) transformations is obtained by introducing an interaction between the fermion field and the electromagnetic field.
  - The global U(1) symmetry of the field equations leads to a conserved charged (which is the source of the electromagnetic field)
  - An interaction (coupling between matter and gauge fields) is unambiguously determined due to the requirement of local phase invariance (local gauge principle) i.e. via the covariant derivative:

$$D_\mu \psi(x) = \partial_\mu \psi(x) + ieQA_\mu(x)\psi(x)$$

Gauge symmetries:

- **Local gauge invariance:**
  - Example: (electromagnetic interaction between the a fermion and the photon field)
    - The Lagrangian of the free Dirac field:

$$\mathcal{L}_{\text{free}} = \overline{\psi}(i\gamma^{\mu}\partial_{\mu} - m)\psi$$

is adjusted to follow local gauge invariance

$$\begin{aligned} \mathcal{L} &= \overline{\psi}(i\gamma^{\mu}D_{\mu} - m)\psi \\ &= \overline{\psi}(i\gamma^{\mu}\partial_{\mu} - m)\psi - eQA_{\mu}\overline{\psi}\gamma^{\mu}\psi \\ &= \mathcal{L}_{\text{free}} - j^{\mu}A_{\mu} \end{aligned}$$

and introducing the coupling to the electromagnetic current:

$$j^{\mu} = eQ\overline{\psi}\gamma^{\mu}\psi$$

Gauge symmetries:

- **Local gauge invariance:**
  - Example: (electromagnetic interaction between the a fermion and the photon field)
    - **Lagrangian of the quantum electrodynamics:**

$$
\begin{aligned}
\mathcal{L}_{QED} &= \mathcal{L}_{\mathrm{free}}^{\mathrm{Photon}} + \mathcal{L}_{\mathrm{free}}^{\mathrm{Fermion}} + \mathcal{L}_{\mathrm{Interaction}} \\
&= -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \overline{\psi}(i\gamma^{\mu}\partial_{\mu} - m)\psi - j^{\mu}A_{\mu} \\
&= -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \overline{\psi}i\gamma^{\mu}\partial_{\mu}\psi - m\overline{\psi}\psi - j^{\mu}A_{\mu} \\
&= \mathcal{L}_{\mathrm{kin}}^{\mathrm{Photon}} + \mathcal{L}_{\mathrm{kin}}^{\mathrm{Fermion}} + \mathcal{L}_{\mathrm{Mass}}^{\mathrm{Fermion}} + \mathcal{L}_{\mathrm{Interaction}}
\end{aligned}
$$

Group theories:

- **Gauge symmetries are described via so-called Lie groups:**
  - i.e. groups of transformations $g(\alpha)$ that are analytical functions of a set of continuous parameters α_a (with a = 1, .., n)
  - The infinitesimal transformation can be written as:

$$\delta g(\alpha_1, \alpha_2, ..., \alpha_n) = \mathbb{1} + i\alpha^a T^a$$

    where T^a (a = 1, ... , n) are the generators of the group.
  - All elements of the group can be written in the form:

$$g(\alpha) = \exp(i\alpha^a T^a)$$

  - These transformations are unitary if the generators are hermitian
  - Unitarity is a condition for symmetry transformations of quantum mechanical states in order to guarantee the conservation of probability
- Lie groups are relevant for the inner symmetries of the SM particles as well as for the gauge symmetry

Group theories:

- **Gauge symmetries are described via so-called Lie groups:**
  - The hermitian generators are quantum mechanical observables and conserved quantities
  - A set of linear independent generators of a Lie group follow the commutation rules:

    $$[T^a, T^b] = T^a T^b - T^b T^a = if^{abc} T^c$$

    with the **structure constants** f^{abc} and the relation:

    $$[T^a, T^b] = -[T^b, T^a]$$

  - The generators and the commutation operation define the Lie Algebra of a group.
  - The structure constants are specific to a Lie Algebra, but they depend also on the choice of independent generators and thus also on the parameters of the group
  - The number of independent parameters/generators gives the order N of a group
  - The maximum number of commuting generators of a Lie group is called its rank R

Group theories:

- **The following Lie groups are relevant for the Standard Model:**
  - **U(1):**
    - abelian (i.e. group is commutative)
    - for unitary 1-dimensional phase transformations
    - Rank 1
  - **SU(N) (N ≥ 2):**
    - non-abelian
    - for special unitary transformations of N-dimensional complex vectors following:
      - $U^{\dagger}U = 1$
      - $Det(U) = 1$
    - The generators of these groups $T = T^{\dagger}$ are hermitian $n \times n$ matrices with $Tr(T) = 0$
      - $N^2 - 1$ independent matrices/generators exists
    - Rank $N-1$

Group theories:

- **Other Lie groups:**
  - **SO(N): (special orthogonal groups)**
    - $O^T O = 1$
    - $\mathrm{Det}(O) = 1$
    - $N(N-1)/2$ generators exist
    - Rank $R = N/2$
  - **Exceptional Lie groups:**
    - $G_2 (N = 14,\ R = 2)$
    - $F_4 (N = 52,\ R = 4)$
    - $E_6 (N = 78,\ R = 6)$
    - $E_7 (N = 133, R = 7)$
    - $E_8 (N = 248, R = 8)$

Used in e.g. string theories

Discrete symmetries

- Symmetries of a free particle (here: fermion) for the electromagnetic and strong interactions but not for the weak interaction:
  - Parity P inverts the direction of the (space) axis: $\vec{x} \longrightarrow -\vec{x}\ , \quad \vec{p} \longrightarrow -\vec{p}$

$$\psi(t,\vec{x}) \longrightarrow \psi^{P}(t,-\vec{x}) = \eta_P \gamma^0 \psi(t,\vec{x})$$

  - Time inversion T: $t \longrightarrow -t$

$$\psi(t,\vec{x}) \longrightarrow \psi^{T}(-t,\vec{x}) = i\gamma^1\gamma^3 \psi^*(t,\vec{x})$$

  - Charge conjugation: particle → antiparticle, $Q_f \longrightarrow -Q_f$

$$\psi(t,\vec{x}) \longrightarrow \psi^{C}(t,\vec{x}) = i\eta_C \gamma^2 \psi^*(t,\vec{x})$$

Discrete symmetries

- CP symmetry:
    - Is experimentally known to be violated by at least the weak interaction
- CPT symmetrie:
    - Is conserved for all local and lorentz invariant field theories with $\mathcal{L}^\dagger = \mathcal{L}$ and a spin–statistics theorem (and thus for all interactions)
        - Some BSM theories actually predict CPT violation (e.g. in string theory)
    - Formulated in the 1950s by Schwinger, Pauli, Lüders
    - Implies that particle/antiparticle have the same mass
    - Experimental effort ongoing to test CPT invariance

CPT Invariance

- **The simplest tests of CPT invariance probe the equality of the masses and lifetimes of a particle and its antiparticle:**
  - The best test currently comes from the limit on the mass difference between K^0 and $\bar{K}^0$
  - Any such difference contributes to the CP violating parameter ε.
  - Assuming CPT invariance, Φ_ε, the phase of ε should be very close to 44°
  - In contrast, if the entire source of CP violation in K^0 decays were a $K^0 - \bar{K}^0$ mass difference, Φ_ε would be 44°+ 90°.
  - Assuming that there is no other sources of CPT violation, it is possible to constrain the mass difference. The current best constrain at a 90%CL is:

$$\left|\frac{(m_{\bar{K}^0} - m_{K^0})}{m_{K^0}}\right| \leq 0.6 \cdot 10^{-18}$$

From particle data group (pdg): https://pdg.lbl.gov/2017/reviews/rpp2017-rev-conservation-laws.pdf

For more information see: CP violation in K decays: results from NA31, prospects in NA48 - INSPIRE

1.4.2 Fundamental Forces and their Unification

Fundamental interactions

- Based on the symmetry principle, local gauge theories (Yang-Mills theory) provide a general description of all known interactions between the fundamental particles (Quarks and Leptons).

- The properties of these interactions are determined by gauge symmetry groups. Fermions (Spin 1/2 particles) build up the multi-pletts used in the formulation of these gauge symmetries.

- The generators of the gauge symmetry groups (Lie-groups) are the generalized charge operators of an interaction. The interactions are mediated via the exchange of vector-bosons (Spin 1 particles).

- The electromagnetic (EM), the weak and the strong interaction can be described via (special) unitary symmetry groups.

Fundamental interactions

Interaction	EM	Weak	Strong
Gauge symmetry	$U(1)$	$SU(2)$	$SU(3)$
Theory	QED	GSW	QCD
Gauge boson	Photon	$W^{\pm}$, Z^0	8 Gluons
Acts on	electric charge	flavour	colour charge
Range	∞	$10^{-18}\mathrm{m}$	$10^{-15}\mathrm{m}$

Gravitation is not described by SM

Fundamental interactions

- The number of independent parameters as well as the number of generators (generalized charges) of a group SU(N) is $N^2 - 1$ (order of a group). Here, N is the number of degrees of freedom of particle states (i.e. the dimension)

- The gauge symmetry group of the Standard Model of particle physics is the product of the individual Lie-groups:

$$U(1) \otimes SU(2) \otimes SU(3)$$

- The free fermion states (f = e, μ, τ, ν_e, ν_μ, ν_τ, u, d, s, c, b, t) of the Standard Model are thus given via:
  - Particles:

$$\psi_f^+(x, E>0) = u(p)e^{-ip_\mu x^\mu} \times e^{iQ_f\alpha} \times \begin{pmatrix} u \\ d \end{pmatrix}_L \times \begin{pmatrix} r \\ g \\ b \end{pmatrix}_q$$

  - Anti-particles:

$$\psi_f^-(x, E<0) = v(p)e^{ip_\mu x^\mu} \times e^{-iQ_f\alpha} \times \begin{pmatrix} u \\ d \end{pmatrix}_R \times \begin{pmatrix} \bar{r} \\ \bar{g} \\ \bar{b} \end{pmatrix}_q$$

Quantum Electrodynamics (QED)

- Gauge theory based on the U(1) symmetry group (electric charge)
- Gauge field (photon field): Potential A_μ
- Gauge boson (photon): Spin-1, massless (due to gauge symmetry requirement)
- Lagrangian density describing the coupling between a photon and fermion:

$$\mathcal{L}_{QED} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}_f(i\gamma^\mu D_\mu - m_f)\psi_f \tag{18}$$

with the covariant derivative:

$$D_\mu = \partial_\mu + ieQA_\mu$$

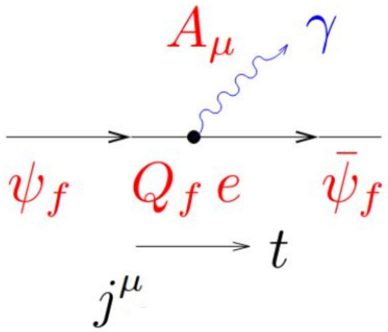
and the field-strength tensor:

$$F_{\mu\nu} = \partial_\nu A_\mu - \partial_\mu A_\nu$$

Quantum Electrodynamics (QED)

- Gauge field couples to current:

$$\mathcal{L}_{\text{Interaction}} = -j^\mu A_\mu$$



- Here the conserved (electromagnetic) current is defined by:

$$j^\mu = eQ_f \bar{\psi} \gamma^\mu \psi$$

- The couplings strength is determined via the elementary charge e, while Q_f is the eigenvalue of the charge operator (generator of the U(1) gauge symmetry group).
- Local U(1) gauge transformations defined by:

$$\psi_f(x) \to \psi'_f(x) = e^{-ieQ_f a(x)} \psi_f(x)$$

$$A_\mu(x) \to A'_\mu(x) = A_\mu(x) + \partial_\mu a(x)$$

keep the Lagrangian from equation (18) invariant

Quantum Electrodynamics (QED)

- Global U(1) gauge symmetry → Conservation of electric charge (Noether's theorem)
- Continuity equation:

$$\partial_\mu j^\mu = \frac{\partial \rho}{\partial t} + \vec{\nabla}\vec{j} = 0$$

$$\rightarrow \frac{dQ}{dt} = \int d^3x \frac{\partial \rho}{\partial t} = -\int d^3x \vec{\nabla}\vec{j} = -\oint d\vec{\sigma}\vec{j} = 0$$

- Gauge symmetry requires $m_\gamma = 0$
    - Experimental limits:
        - Measurement of Jupiter's magnetic field by Pioneer 10-probe: $m_\gamma < 4.5 \cdot 10^{-16}$ eV
        - Measurement of galactic magnetic field: $m_\gamma < 3.5 \cdot 10^{-27}$ eV
- **Gauge interactions must have infinite range !**
    - In contradiction with what has been said before (more on this later !!!)

Quantum Electrodynamics (QED)

- Additional symmetries of the QED:
  - Continuous symmetries:
    - Lorentz invariants
    - Invariants under space-time shifts
    - Rotation

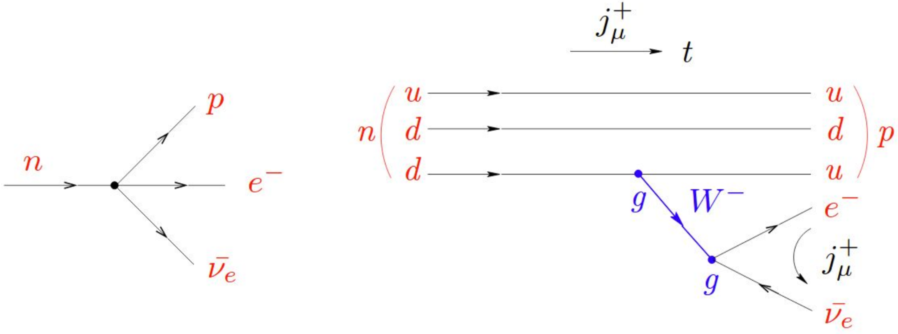
→ **Conservation of energy, momentum and angular momentum (Noether's theorem)**

  - Discrete symmetries:
    - Lepton- and quark-flavour conservation
    - Parity P transformation: $\vec{x} \to -\vec{x}$ and $\vec{p} \to -\vec{p}$
    - Time T transformation: $t \to -t$
    - Charge conjugation C : $Q_f \to -Q_f$

Weak interaction

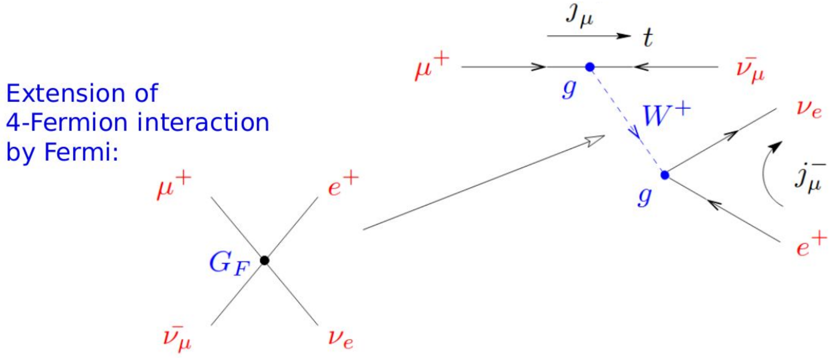
- Description is analogous to QED with coupling of weak currents to an electric charged gauge boson: $j^{\mu-} W_\mu^+ + j^{\mu+} W_\mu^-$
- Short ranged interaction changing lepton and quark flavours.
  - Nuclear β-decay:

$$n \rightarrow p e^- \bar{\nu}_e$$



  - Muon decay:

$$\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu$$



Weak interaction

- Weak interactions are described using dubletts of a fundamental SU(2) group (analogous to Spin and Isospin).

- Left-handed particles are sorted into SU(2)-Dubletts:

electric charge

hyper charge

3rd component of weak Isospin

$$\begin{array}{lccccccc}
 & & & & & Q_f & Y_f & I_f^3 \\
L_\ell: & \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L & \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L & \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L & & \begin{array}{c} 0 \\ -1 \end{array} & \begin{array}{c} -1 \\ -1 \end{array} & \begin{array}{c} +\frac{1}{2} \\ -\frac{1}{2} \end{array} \\
L_q: & \begin{pmatrix} u \\ d' \end{pmatrix}_L & \begin{pmatrix} c \\ s' \end{pmatrix}_L & \begin{pmatrix} t \\ b' \end{pmatrix}_L & & \begin{array}{c} +\frac{2}{3} \\ -\frac{1}{3} \end{array} & \begin{array}{c} +\frac{1}{3} \\ +\frac{1}{3} \end{array} & \begin{array}{c} +\frac{1}{2} \\ -\frac{1}{2} \end{array}
\end{array}$$

Weak interaction

- Right-handed particles are sorted into SU(2)-Singuletts:

				Q_f	Y_f	I_f^3
L_ℓ:	ν_{eR}	$\nu_{\mu R}$	$\nu_{\tau R}$	0	0	0
	e_R^-	μ_R^-	τ_R^-	-1	-2	0
L_q :	u_R	c_R	t_R	$+\frac{2}{3}$	$+\frac{4}{3}$	0
	d_R	s_R	b_R	$-\frac{1}{3}$	$-\frac{2}{3}$	0

- Relationship between electric charge and hypercharge described by Gell-Mann-Nishijijima formula:

$$Q_f = I_f^3 + Y_f/2$$

- The weak interaction induces flavour-changing transitions within the fermion-dubletts L_ℓ (for leptons) and L_q (for quarks).

Weak interaction

- Only left-handed fermions ψ_L and right-handed anti-fermions $\overline{\psi}_R$ participate in the weak interaction.
- Due to the V−A structure of the weak currents, the weak interaction is maximal parity violating.
- Proof for parity violation in the weak interaction:
  - $K^+ \rightarrow \pi^+ \pi^0$ and $K^+ \rightarrow \pi^+ \pi^- \pi^+$ (Lee & Yang in 1956)
  - Polarisation of electrons from nuclear β decay (Wu in 1957)
- Projection of the chiral fermion states:

$$\psi_L = P_L\psi = \left(\frac{1-\gamma_5}{2}\right)\psi$$

$$\psi_R = P_R\psi = \left(\frac{1+\gamma_5}{2}\right)\psi$$

with the chirality: $\gamma_5 = \gamma_5^\dagger$

$$\gamma_5\psi_L = -\psi_L$$

$$\gamma_5\psi_R = \psi_R$$

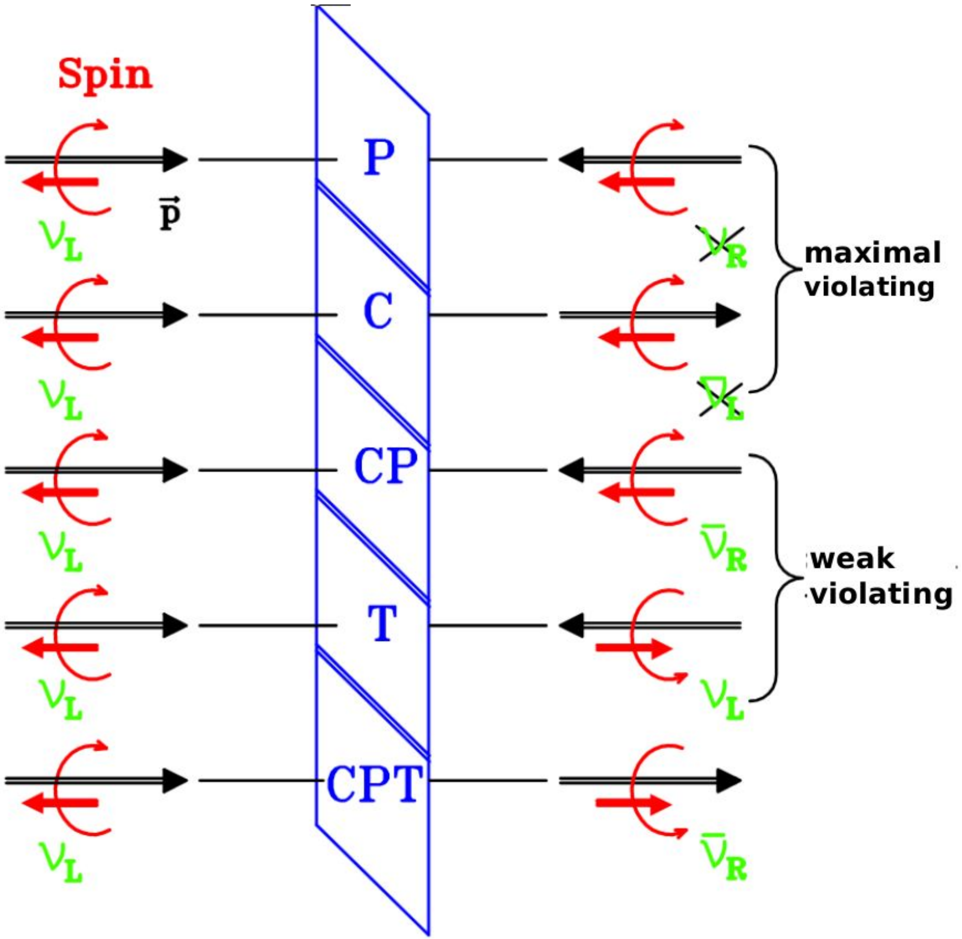
Weak interaction

- Thus the weak fermion currents are defined via:

$$
\begin{aligned}
j^{\mu}_{\mathrm{weak}} &= \underline{\overline{\psi}'_L \gamma^{\mu} \psi_L} \\
&= (P_L \psi')^{\dagger} \gamma^0 \gamma^{\mu} P_L \psi = \psi'^{\dagger} P_L \gamma^0 \gamma^{\mu} P_L \psi \\
&= \overline{\psi}' \gamma^{\mu} P_L \psi = \underline{\frac{1}{2} \overline{\psi}' \gamma^{\mu} (1 - \gamma_5) \psi}
\end{aligned}
$$

- Thus we speak about vector (γ^{μ}) - axial vector ($\gamma^{\mu} \gamma_5$) or V − A current

Weak interaction



Weak interaction

- Use local weak Isospin gauge symmetry $SU(2)_L$ to describe the weak interaction:
  - $SU(2)_L$-Dubletts (L): $L_f \rightarrow e^{i\vec{I}\vec{\beta}(x)} L_f$
  - $SU(2)_L$-Dubletts (R): $\psi_R \rightarrow \psi_R$
  - **3 Generator (charges)**
    - Isospin vector: $\vec{I} = \begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix}$
    - Lie-algebra: $[I_i, I_j] = i\varepsilon_{ijk} I_k$ (i.e. weak interaction is non-abelian)
    - Creation and annihilation operators: $I^{\pm} = \frac{1}{2}(I_1 \pm iI_2)$
    - Fermion-dubletts with: $|\vec{I}| = \frac{1}{2},\ I_z = \pm\frac{1}{2}$
    - $\vec{I} = \frac{\vec{\sigma}}{2}$ with Pauli's spin matrices σ_i (with i = 1, 2, 3)

Creation and Annihilation operator:

$$\sigma^{\pm} = \frac{1}{2}(\sigma_1 \pm \sigma_2)$$

Electroweak unification

- Common approach using local $SU(2)_L$ and $U(1)_Y$ gauge symmetry (Gashow 1961, Salam 1968, Weinberg 1967) → (GSW theory)
  - Electromagnetic interaction has to be considered as well due to the electric charge of the weak gauge bosons $W^{\pm}$

- Y is weak hypercharge: $[I_i, Y] = 0$ with i = 1, 2, 3
- Therefore Y_f is the same for both components of a $SU(2)_L$-doublet wherase $Q_f \neq Y_f$
- The electric charge within a multiplet is derived from $Q_f = I_3 + Y/2$
- A unified gauge theory of a combined weak and electromagnetic interaction is described via:

$$\mathcal{L}_{SU(2)\times U(1)} = -\frac{1}{4} f_{\mu\nu} f^{\mu\nu} - \frac{1}{4} F^i_{\mu\nu} F^{i\mu\nu} + \sum_f (\overline{\psi}_{fR} i\gamma^\mu D_{\mu R}) \psi_{fR} + \sum_L (\overline{L}_f i\gamma^\mu D_{\mu L}) L_f$$

Electroweak unification

where L_f is a left-handed SU(2)-doublets and ψ_f is a SU(2)-singulett, while the covariant derivatives are defined via:

$$
\begin{aligned}
D_{\mu L} &= \partial_\mu \cdot \mathbb{1} + ig' \frac{Y_{fL}}{2} B_\mu(x) \cdot \mathbb{1} + ig \vec{I} \cdot \vec{W}_\mu(x) \\
&= \partial_\mu \cdot \mathbb{1} + i\frac{g'}{2} Y_{fL} B_\mu(x) \cdot \mathbb{1} + i\frac{g}{2}\vec{\sigma} \cdot \vec{W}_\mu(x) \\
D_{\mu R} &= \partial_\mu + ig' \frac{Y_{fR}}{2} B_\mu(x)
\end{aligned}
$$

Here, the coupling constants g and g’ are the weak Isospin and the weak hypercharge, respectively.

⇒ minimal gauge invariant coupling to 4 massless gauge bosons from a $U(1)_Y$ and $SU(2)_L$:

$$
B_\mu(x) \ \text{and} \ \vec{W}_\mu(x) = \begin{pmatrix} W_\mu^x(x) \\ W_\mu^y(x) \\ W_\mu^z(x) \end{pmatrix}
$$

Electroweak unification

- The field-strength tensors of the $SU(2)_L \times U(1)_Y$ are defined via:

$$\begin{aligned} f_{\mu\nu} &= \partial_\nu B_\mu(x) - \partial_\mu B_\nu(x) \\ F^i_{\mu\nu} &= \partial_\nu W^i_\mu(x) - \partial_\mu W^i_\nu(x) + g\varepsilon^{ijk} W^j_\mu(x) W^k_\nu(x) \end{aligned}$$

  i.e. the formulation of the Lagrangian for free gauge fields is done analogous to that of the QED.

- All fermion masses need to be = 0, due to global $SU(2)_L$ invariance
  - Different masses in the fermion doublets violate the SU(2) symmetry

- A mass term for dirac particles

$$\bar{\psi} P_L = \bar{\psi}_R$$

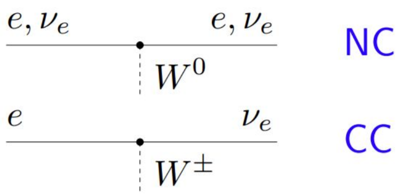
$$m\overline{\psi}\psi = m\overline{\psi}(P_L^2 + P_R^2)\psi = m(\overline{\psi}_L\psi_R + \overline{\psi}_R\psi_L)$$

$$\bar{\psi} P_R = \bar{\psi}_L$$

  is not invariant

⇒ masses of electroweak gauge bosons (except for γ) and fermions are generated by a spontaneous symmetry breaking of the local $SU(2)_L \times U(1)_Y$ gauge symmetry (i.e. Higgs-mechanism).

Electroweak unification



- Local SU(2) gauge transformations U(x) are defined via:

$$L \longrightarrow L' = U(x) \cdot L = e^{ig\frac{\vec{\sigma}}{2}\vec{\beta}(x)} L$$

- The interaction term of the electroweak Lagrangian can also be written as:

$$\begin{aligned}
\mathcal{L}_{\text{Interaction}} &= -g' \underbrace{\sum_f \left(\overline{\psi}_f \gamma^\mu \frac{Y_f}{2} \psi_f \right)}_{j_Y^\mu} B_\mu - g \underbrace{\sum_L \left(\overline{L}_f \gamma^\mu \frac{\vec{\sigma}}{2} \cdot L_f \right)}_{\vec{j}_I^\mu} \vec{W}_\mu \\
&= -g' j_Y^\mu B_\mu - g\, j_I^{\mu\, 0} W_\mu^0 - \frac{g}{\sqrt{2}} \left(j_I^{\mu\, -} W_\mu^+ + j_I^{\mu\, +} W_\mu^- \right) \\
&= \mathcal{L}_{\text{NC}} + \mathcal{L}_{CC}
\end{aligned}$$

with the definition of the flavour-changing charged currents and the charged gauge bosons

$$j_I^{\mu\pm} = \sum_L \overline{L}_f \gamma^\mu \tau^\pm L_f = j_I^{\mu 1} \pm i j_i^{\mu 2} \qquad \text{and} \qquad W_\mu^\pm = \frac{1}{\sqrt{2}} (W_\mu^1 \pm i W_\mu^2)$$

Electroweak unification

- **Neutral currents** of neutrinos can only be mediated by weak interaction.
  - Discovered in 1973 at CERN via $\nu_\mu\, p \rightarrow \nu_\mu\, p$.
- Z and γ boson result from spontaneous symmetry breaking (rotated by angle θ_W wrt original W^0 and B^0 vector boson plane):

$$\begin{pmatrix} B_\mu \\ W^0_\mu \end{pmatrix} \longrightarrow \begin{pmatrix} A_\mu \\ Z^0_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & \sin\theta_W \\ -\sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} B_\mu \\ W^0_\mu \end{pmatrix}$$

with:

$$\cos\theta_W = \frac{g}{\sqrt{g^2 + g'^2 Y_L^2}}; \qquad \sin\theta_W = \frac{g' Y_L}{\sqrt{g^2 + g'^2 Y_L^2}}$$

Electroweak unification

- The photon field is defined via:

$$A_\mu = \frac{gB_\mu - g' Y_L W^0_\mu}{\sqrt{g^2 + g'^2 Y_L^2}}$$

while the Z -field is defined via:

$$Z^0_\mu = \frac{gW^0_\mu + g' Y_L B_\mu}{\sqrt{g^2 + g'^2 Y_L^2}}$$

- Both, the Z^0 as well as the $W^\pm$ bosons were discovered in 1983 at CERN in $p\bar{p}$ collisions.
- Neutral weak currents (coupling of neutral fermion currents to the Z^0 gauge boson) was already predicted by the GSW theory and finally observed in 1973 in neutrino scattering experiments at CERN (using bubble chambers).

Electroweak unification

- Inverting the transformations gives:

$$B_\mu = \frac{gA_\mu + g' Y_L Z^0_\mu}{\sqrt{g^2 + g'^2 Y_L^2}} \quad \text{and} \quad W^0_\mu = \frac{gZ^0_\mu - g' Y_L A_\mu}{\sqrt{g^2 + g'^2 Y_L^2}}$$

- Including this into the Lagrangian gives:

$$\begin{aligned}
\mathcal{L}_{NC} \quad = \quad & -\frac{\sqrt{g^2 + g'^2 Y_L^2}}{2} Z^0_\mu (\overline{\nu}_{eL} \gamma^\mu \nu_{eL}) \\
& -\frac{gg' Y_L}{\sqrt{g^2 + g'^2 Y_L^2}} A_\mu (\overline{e}_L \gamma^\mu e_L) - \frac{gg' Y_R}{2\sqrt{g^2 + g'^2 Y_L^2}} A_\mu (\overline{e}_R \gamma^\mu e_R) \\
& -\frac{g'^2 Y_L^2 - g^2}{2\sqrt{g^2 + g'^2 Y_L^2}} Z^0_\mu (\overline{e}_L \gamma^\mu e_L) - \frac{g'^2 Y_L Y_R}{2\sqrt{g^2 + g'^2 Y_L^2}} Z^0_\mu (\overline{e}_R \gamma^\mu e_R)
\end{aligned}$$

⇒ Neutrinos do not couple to the electromagnetic field A_μ

- Only left-handed neutrinos couple to Z_μ, while right-handed neutrinos do not interact since their weak charge $Y(\nu_R) = 0$.

Electroweak unification

- For the electromagnetic interaction we define:

$$Y_L = e\frac{\sqrt{g^2 + g'^2 Y_L^2}}{gg'} = -1 \quad \text{and} \quad Y_R = 2Y_L$$

and thus:

$$\sqrt{g^2 + g'^2} = \frac{e}{\cos\theta_W \sin\theta_W} \qquad e = \frac{-gg'}{\sqrt{g^2 + g'^2}} = g'\cos\theta_W = g\sin\theta_W$$

$$\frac{g'^2 - g^2}{2\sqrt{g^2 + g'^2}} = \frac{e}{\cos\theta_W \sin\theta_W}\left(-\frac{1}{2} + \sin^2\theta_W\right)$$

$$-\frac{g'^2}{\sqrt{g^2 + g'^2}} = \frac{e}{\cos\theta_W \sin\theta_W}\left(-\sin^2\theta_W\right)$$

Electroweak unification

- With these expressions, the Lagrangian for interactions with neutral currents changes to:

$$\mathcal{L}_{NC} = -\frac{e}{\cos\theta_W \sin\theta_W}\left[\left(-\frac{1}{2}+\sin^2\theta_W\right)(\overline{e}_L\gamma^\mu e_L)+(-\sin^2\theta_W)(\overline{e}_R\gamma^\mu e_R)\right]$$
$$-\frac{g}{2\cos\theta_W}(\overline{\nu}_{eL}\gamma^\mu\nu_{eL})Z^0_\mu - e\sum_f(Q_f\overline{\psi}_f\gamma^\mu\psi_f)A_\mu$$

- In a more general representation:

$$\mathcal{L}_{NC} = -\frac{e}{\cos\theta_W \sin\theta_W}\cdot\sum_{f_R,f_L}\left[(I^3_{fL,R}-Q_{fL,R}\sin^2\theta_W)(\overline{\psi}_{fL,R}\gamma^\mu\psi_{fL,R})\right]Z^0_\mu$$
$$-e\sum_f(Q_f\overline{\psi}_f\gamma^\mu\psi_f)A_\mu$$

- The weak neutral coupling of all left- and right-handed fermion states to the Z boson is described via:

$$\frac{e}{\cos\theta_W \sin\theta_W}(I^3_f - Q_f\sin^2\theta_W)$$

Quantum chromodynamics (QCD)

- SU(3) gauge theory of the strong interaction between quarks and 8 charges (generators) λ^a (a = 1, ..., 8):

$$[\lambda^a, \lambda^b] = 2if^{abc}\lambda^c$$

with the structure constants of the SU(3)-Lie algebra f^{abc}.

- Local gauge transformations are given via:

$$\Psi_q(x) \longrightarrow \Psi'_q(x) = U(x)\Psi_q(x) = e^{ig_s\frac{\lambda^a}{2}\gamma^a(x)}\Psi_q(x)$$

while the Lagrangian is defined as:

$$\mathcal{L}_{QCD} = -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + \sum_q \overline{\Psi}_q(i\gamma^\mu D_\mu - m_q)\Psi_q$$

with the covariant derivative (with $D_\mu\psi = U(D_\mu\psi)$):

$$D_\mu = \partial_\mu + ig_s\frac{\lambda^a}{2}G^a_\mu$$

Gauge fields of the QCD (with a = 1, ..., 8)

Quantum chromodynamics (QCD)

- The field strength tensor is given via:

$$F^a_{\mu\nu} = \partial_\nu G^a_\mu - \partial_\mu G^a_\nu + g_s f^{abc} G^b_\mu G^c_\nu$$

- Within the fundamental SU(3) representation, the quark-fields are sorted into tripletts using the quantum number: Colour ("red","green","blue"):

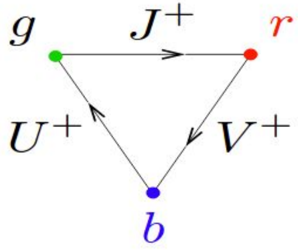
$$\chi_C = \begin{pmatrix} q_\mathrm{r} \\ q_\mathrm{g} \\ q_\mathrm{b} \end{pmatrix} ; \quad \chi_r = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} ; \quad \chi_g = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} ; \quad \chi_b = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

whereas antiquarks carry anticolour

$$(\bar{r}, \bar{g}, \bar{b})$$

- Introduction of creation and annihilation operators within the SU(3)$_C$ colour-tripletts:

$$\begin{aligned} I^\pm_C &= \frac{1}{2}(\lambda_1 \pm i\lambda_2); \quad (\mathrm{T}\mathit{ransformation}\ \ g \longleftrightarrow r), \\ V^\pm_C &= \frac{1}{2}(\lambda_4 \mp i\lambda_5); \quad (\mathrm{T}\mathit{ransformation}\ \ r \longleftrightarrow b), \\ U^\pm_C &= \frac{1}{2}(\lambda_6 \pm i\lambda_7); \quad (\mathrm{T}\mathit{ransformation}\ \ b \longleftrightarrow g). \end{aligned}$$



Quantum chromodynamics (QCD)

- Here, the three dimensional Gell-Mann matrices are used (which are defined analogously to the Pauli matrices of the SU(2)):

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \ \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \ (g \longleftrightarrow r)$$

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}; \ \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \ (r \longleftrightarrow b)$$

$$\lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}; \ \lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \ (b \longleftrightarrow g)$$

Quantum chromodynamics (QCD)

$$\lambda_3 \quad = \quad \begin{array}{cc} \begin{array}{ccc} r & g & b \end{array} & \\ \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} & \begin{array}{c} \bar{r} \\ \bar{g} \\ \bar{b} \end{array} \end{array} \quad (\mathrm{c}ouples \quad r\bar{r}, -g\bar{g})$$

$$\lambda_8 \quad = \quad \frac{1}{\sqrt{3}} \begin{array}{cc} \begin{array}{ccc} r & g & b \end{array} & \\ \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix} & \begin{array}{c} \bar{r} \\ \bar{g} \\ \bar{b} \end{array} \end{array} \quad (\mathrm{c}ouples \quad r\bar{r}, g\bar{g}, -2b\bar{b}).$$

Quantum chromodynamics (QCD)

- **Interaction part of the QCD-Lagrangian:**

conserved colour current

$$
\begin{aligned}
\mathcal{L}_{WW}(SU(3)_C) &= \boxed{-g_s(\overline{\Psi}\gamma^\mu \frac{\lambda^a}{2}\Psi)G^a_\mu} \\
&= -\frac{g_s}{\sqrt{2}} \Big[\overline{\Psi}\gamma^\mu I^+_C \Psi(g\bar{r})_\mu + \overline{\Psi}\gamma^\mu I^-_C \Psi(r\bar{g})_\mu \\
&\quad + \overline{\Psi}\gamma^\mu V^+_C \Psi(r\bar{b})_\mu + \overline{\Psi}\gamma^\mu V^-_C \Psi(b\bar{r})_\mu \\
&\quad + \overline{\Psi}\gamma^\mu U^+_C \Psi(b\bar{g})_\mu + \overline{\Psi}\gamma^\mu U^-_C \Psi(g\bar{b})_\mu \\
&\quad + \frac{1}{\sqrt{2}}\overline{\Psi}\gamma^\mu \lambda^3 \Psi G^3_\mu + \frac{1}{\sqrt{2}}\overline{\Psi}\gamma^\mu \lambda^8 \Psi G^8_\mu \Big]
\end{aligned}
$$

Quantum chromodynamics (QCD)

with the eight gluon fields:

$$
\begin{aligned}
g_{\mu 1} &= (g\bar{r})_\mu = \frac{1}{\sqrt{2}}(G^1_\mu - iG^2_\mu), \\
g_{\mu 2} &= (r\bar{g})_\mu = \frac{1}{\sqrt{2}}(G^1_\mu + iG^2_\mu) = \overline{g}_{\mu 1}, \\
g_{\mu 4} &= (r\bar{b})_\mu = \frac{1}{\sqrt{2}}(G^4_\mu + iG^5_\mu), \\
g_{\mu 5} &= (b\bar{r})_\mu = \frac{1}{\sqrt{2}}(G^4_\mu - iG^5_\mu) = \overline{g}_{\mu 4}, \\
g_{\mu 6} &= (b\bar{g})_\mu = \frac{1}{\sqrt{2}}(G^6_\mu - iG^7_\mu), \\
g_{\mu 7} &= (g\bar{b})_\mu = \frac{1}{\sqrt{2}}(G^6_\mu + iG^7_\mu) = \overline{g}_{\mu 6}, \\
g_{\mu 3} &= \frac{1}{\sqrt{2}}(r\bar{r} - g\bar{g}) = G^3_\mu \quad (\textit{colour}\ \text{neutral}), \\
g_{\mu 8} &= \frac{1}{\sqrt{2}}(r\bar{r} + g\bar{g} - 2b\bar{b}) = G^8_\mu \quad (\textit{colour}\ \text{neutral}).
\end{aligned}
$$

Quantum chromodynamics (QCD)

- Obtain eight colour charge operators from the $3_C \otimes \bar{3}_C = 1_C + 8_C$ of the SU(3)
- Exchange of a gluon changes die Colour quantum numbers not the flavour quantum numbers of quarks.
- Gluons do not exists as colour-singlets in the SU(3) (in contrast to U(3)).
  - Such states would couple to colourless states, Mesons ($q\bar{q}$) and Baryons (qqq), and would induce strong and far ranged nuclear forces similar to the electromagnetic force
- Coloured particles (quarks and gluons) are bound to colour-singlet states (Mesonen and Baryons) and do not appear as free states (**confinement**)
  - Example:

$$\pi^+ = \frac{1}{\sqrt{3}}(u_r\bar{d}_{\bar{r}} + u_g\bar{d}_{\bar{g}} + u_b\bar{d}_{\bar{b}})$$

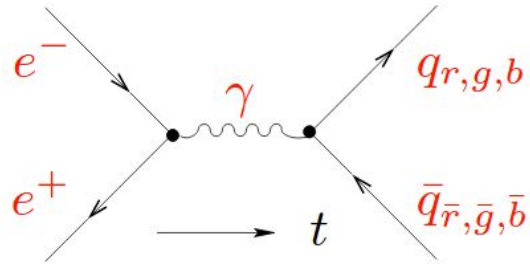
Quantum chromodynamics (QCD)

- **Motivation of colour quantum numbers:**
  - The new inner degrees of freedom of the $SU(3)_C$ colour symmetry allow the construction of a antisymmetric wave function for the $\Delta^{++} = (u\uparrow u\uparrow u\uparrow)$ baryon ($J^P = 3/2$ and $L = 0$):

$$\chi_C(\Delta^{++}) = \frac{1}{\sqrt{6}}\varepsilon_{ijk} u_i\, u_j\, u_k$$

    - Forbidden without colour charge due to Pauli's principle
  - Hadronic cross section in e^+e^- annihilation:

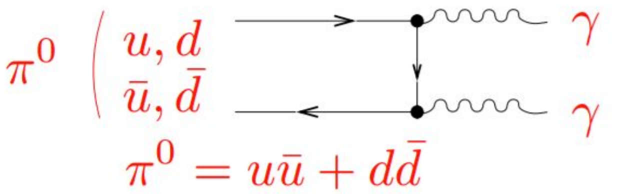
$$\begin{aligned} R &= \sigma(e^+e^- \to \sum_{q(E_{CM}>2m_q)} q\bar{q})/\sigma(e^+e^- \to \mu^+\mu^-) \\ &= N_C \cdot \sum_{q(E_{CM}>2m_q)} Q_q^2 \end{aligned}$$



    - Is thus $N_C = 3$ (number of colour charges) times higher than the leptonic cross section.

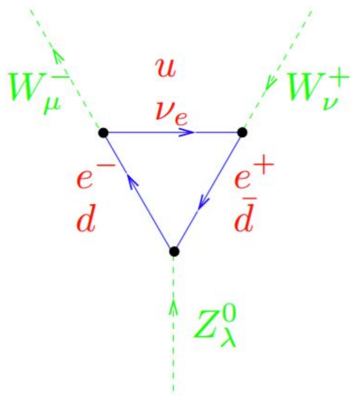
Quantum chromodynamics (QCD)

- **Motivation of colour quantum numbers:**
  - Decay of the π^0 into two photons:



$$\Gamma\left(\pi^0 \to \gamma\gamma\right) \sim N_C^2$$

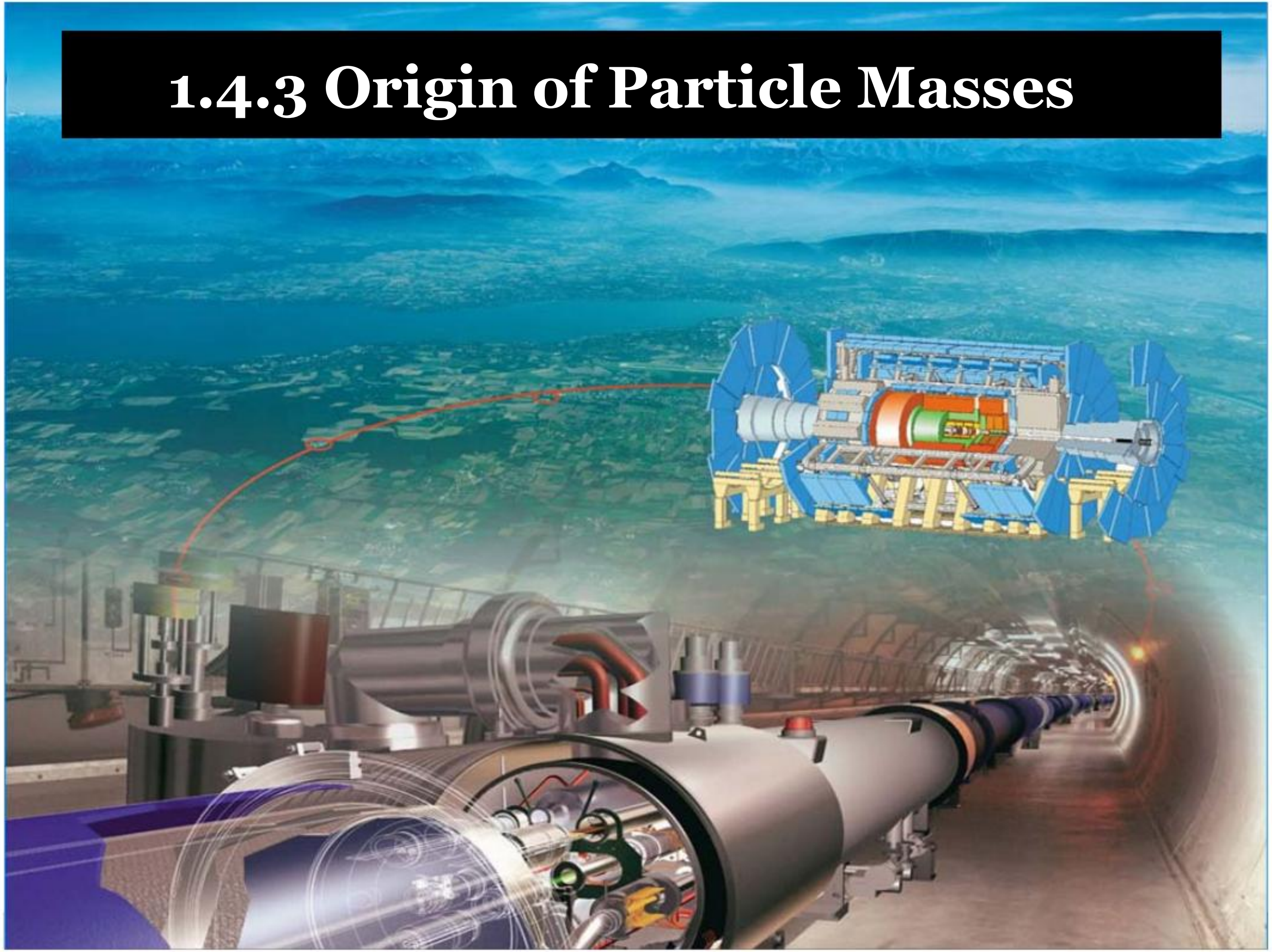
  - Renormalizability of the electroweak interactions:
    - Divergent terms (appearing at higher orders perturbation) for interactions between two vector currents and one axial vector current are canceled if



$$\sum_f Q_f = \sum_\ell Q_\ell + N_C \cdot \sum_q Q_q = 0$$

Holds true (i.e. if quark and lepton contribution cancel each other).

1.4.3 Origin of Particle Masses



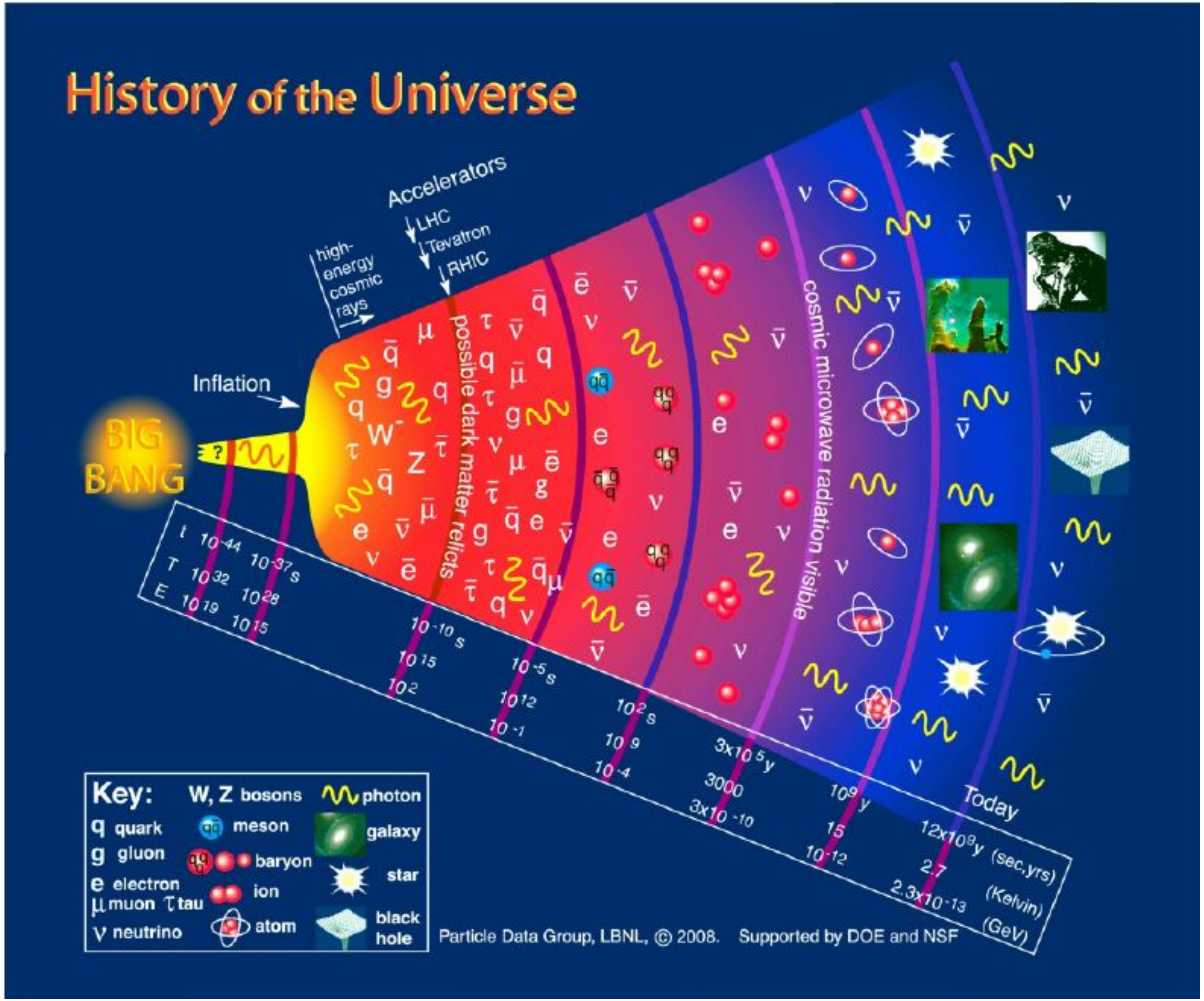
Origin of particle masses

- While the electromagnetic interaction has an infinite range, the weak interaction is short ranged
  - i.e. the weak interaction must be mediated by **massive gauge bosons**
- Explicite mass terms of gauge bosons (described by Proca equation) violate the local gauge symmetry of the Lagrangian
- Explicite mass terms of fermions (described by Dirac equation) violate the global $SU(2)_L$ gauge symmetry
- However, gauge symmetry is necessary to cancel divergences in every order of perturbation theory i.e. renormalizability of the electroweak theory (similar to QED)
- Solution: Introduce spontaneous symmetry breaking (SSB) of the vacuum expectation value of the field theory
  - **However, the gauge symmetry of the full Lagrangian is conserved**

Origin of particle masses

- **Higgs mechanism:**
  - Constructed analogously to 2nd order phase transitions in solid matter state physics: SSB below a critical temperature T_C
    - **In particle physics:**
      - Full symmetry of vacuum recovered at high energies/temperatures
        - I.e. phase transition, SSB, happened during the cooling of the expanding early universe

- **Goldstone's theorem:**
  - The spontaneous breaking of a continuous symmetry leads to the existence of a massless scalar ("**Nambu-Goldstone boson**")
    - Examples:
      - The longitudinal polarisation components of the W- and Z- bosons correspond to the Goldstone bosons of the spontaneously broken part of the electroweak symmetry $SU(2) \otimes U(1)$


History of the Universe
Accelerators
LHC
Tevatron
RHIC
high-energy cosmic rays
Inflation
BIG BANG
possible dark matter relicts
cosmic microwave radiation visible
t 10^-44 10^-37 s
T 10^32 10^28
E 10^19 10^15
10^-10 s
10^15
10^2
10^-5 s
10^12
10^-1
10^2 s
10^9
10^-4
3x10^5 y
3000
3x10^-10
10^9 y
15
10^-12
Today
12x10^9 y (sec,yrs)
2.7 (Kelvin)
2.3x10^-13 (GeV)
Key:
q quark
g gluon
e electron
μ muon τ tau
ν neutrino
W, Z bosons
meson
baryon
ion
atom
photon
galaxy
star
black hole
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Higgs mechanism

- Goldstone bosons are excitations of the field (in the direction of the broken symmetry)
  - **Example:**
    - Quasiparticles in solid state physics (such as phonons)

- In case of a spontaneously broken local gauge symmetry, the goldstone bosons are “eaten-up” by the gauge fields (unitary transformation)
  - This process provides the longitudinal polarisation states to the gauge bosons and a mass term

- Higgs mechanism is introduced analogously to the Meißner-Ochsenfeld effect:
  - Local U(1) phase symmetry is spontaneously broken in the ground state:
    - **Photon field obtains an effective mass as it is dampened due to interactions with Cooper pairs**
  - Independently proposed by research teams around:
    - **Peter Higgs**
    - **François Englert & Robert Brout**

Higgs mechanism

- Introduce an additional complex scalar field within a $SU(2)_L$- doublet:

$$\begin{array}{cccc} & Q & I_3 & Y = 2(Q - I_3) \\ \Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} & \begin{array}{c} +1 \\ 0 \end{array} & \begin{array}{c} +\frac{1}{2} \\ -\frac{1}{2} \end{array} & \begin{array}{c} +1 \\ +1 \end{array} \end{array}$$

which fulfills the Klein-Gordon equation and is described by a *SU(2) × U(1) gauge invariant Lagrangian of the form:*

$$\mathcal{L}_{\text{Higgs}} = (D_\mu \Phi)^\dagger (D^\mu \Phi) - V(\Phi^\dagger \Phi)$$

with the covariant derivative

$$D_\mu = \partial_\mu \cdot \mathbb{1} + i g' Y B_\mu \cdot \mathbb{1} + i \frac{g}{2} \vec{\sigma} \cdot \vec{W}_\mu$$

and the potential ($\lambda > 0$):

Self coupling parameter

$$V(\Phi^\dagger \Phi) = \mu^2 (\Phi^\dagger \Phi) + \lambda (\Phi^\dagger \Phi)^2$$

Higgs mechanism

- For $\mu^2 < 0$, the ground state (kinetic energy T = 0 and $V = V_{min}$) is at a non-zero value of the scalar field

$$\frac{\partial V}{\partial|\Phi|} = 2\mu^2|\Phi_0| + 4\lambda|\Phi_0|^3 = 0$$

$$\Longrightarrow |\Phi_0| = \sqrt{\frac{-\mu^2}{2\lambda}} =: \frac{v}{\sqrt{2}}$$

where the vacuum expectation value of the Higgs field is:

$$v = \frac{|\mu|}{\sqrt{\lambda}} \approx 246\,\mathrm{GeV}$$

- The ground state

$$\Phi_0 = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$$

spontaneously breaks the $SU(2)_L \times U(1)_Y$ symmetry.

Higgs mechanism

- The weak isospin doublet of the Higgs field can also be written as:

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \Phi_1 + i\Phi_2 \\ \Phi_3 + i\Phi_4 \end{pmatrix}$$

- A fluctuation around the minimum v is written as:

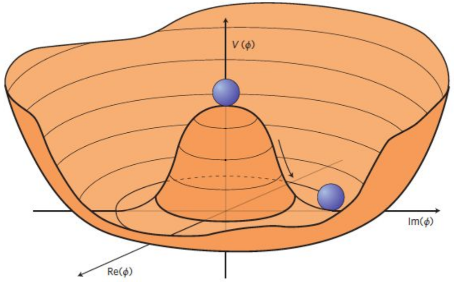
$$\frac{1}{\sqrt{2}} \begin{pmatrix} \Phi_1 + i\Phi_2 \\ \Phi_3 + i\Phi_4 \end{pmatrix} \Rightarrow \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h(x) + i\zeta(x) \end{pmatrix} \tag{19}$$

- The scalar field h(x) describes a physical Higgs boson, while ζ is an unphysical massless state (Goldstone boson)
- Rewriting the original Lagrangian in terms of the quantum fields h and ζ yields:

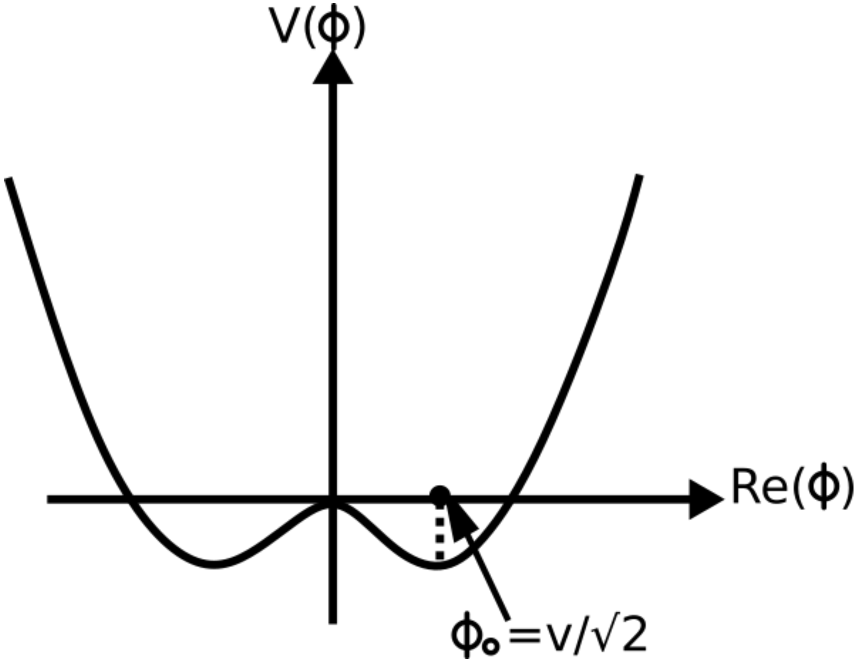
$$\mathcal{L} = \frac{1}{2}(\partial^\mu h \partial_\mu h) + \frac{1}{2}(\partial^\mu \zeta \partial_\mu \zeta) - \lambda v^2 h^2 - \lambda v h \left(h^2 + \zeta^2\right) - \frac{\lambda}{4}(h^2 + \zeta^2)^2$$

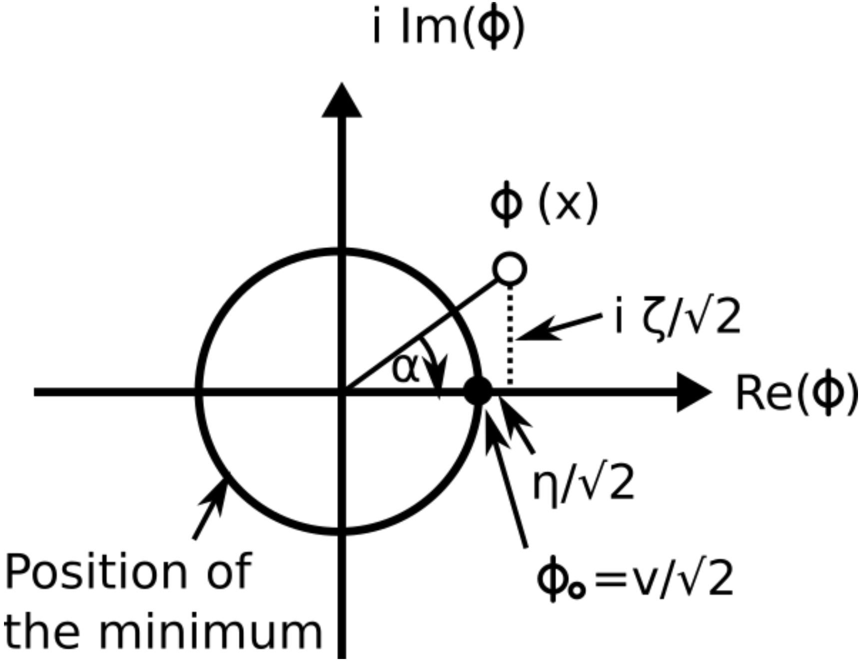
where the h field obtained a mass $m = \sqrt{2\lambda v^2}$

Higgs potential



- The Higgs potential has the shape of a “mexican hat”
- Higgs boson is a radial excitation of the field





Gauge boson masses

- The gauge bosons obtain their masses via coupling to the Higgs field:
  - Inserting the vev component from equation **(19)** into the covariant derivative:

$$D_\mu \Phi = \left(\partial_\mu \cdot \mathbb{1} + i\frac{g'}{2} B_\mu \cdot \mathbb{1} + i\frac{g}{2}\vec{\sigma} \cdot \vec{W}_\mu \right) \Phi$$

$$D_\mu \Phi = \partial_\mu \begin{pmatrix} \Phi^+ \\ \Phi_0 \end{pmatrix} + \frac{i}{2} \begin{pmatrix} gW_\mu^3 + g' B_\mu & \sqrt{2} g W_\mu^+ \\ \sqrt{2} g W_\mu^- & -gW_\mu^3 + g' B_\mu \end{pmatrix} \begin{pmatrix} \Phi^+ \\ \Phi_0 \end{pmatrix}$$

leads to a term

$$+\frac{i}{2} \begin{pmatrix} gW_\mu^3 + g' B_\mu & \sqrt{2} g W_\mu^+ \\ \sqrt{2} g W_\mu^- & -gW_\mu^3 + g' B_\mu \end{pmatrix} \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}$$

which results in:

$$\mathcal{L}_{\text{partial}} = \frac{1}{4} g^2 v^2 W_\mu^+ W_\mu^- + \frac{1}{8} v^2 \left(-gW_\mu^3 + g' B_\mu \right)^2 \qquad \textbf{(20)}$$

Gauge boson masses

- Studying equation **(20),** we can identify**:**
  - The W boson mass term:

$$m_W = \frac{1}{2} g v$$

  - The Z boson mass term:

$$m_Z = \frac{1}{2}\left(g^2 + g'^2\right)^{\frac{1}{2}} v = \frac{m_W}{\cos\theta_W}$$

  - A^μ remains massless
- Before the interaction with the Higgs field:
  - 8 degrees of freedom (2 polarisation states for each W, B)
  - 4 degrees of freedom from the Higgs field
- After the interaction with the Higgs field:
  - 9 degrees of freedom (3 polarisation states for each W^+, W^-, Z)
  - 2 degrees of freedom (2 polarisation states for A^μ)
  - 1 degrees of freedom for the Higgs boson h

With:

$$Z_\mu = \cos\theta_W W^3_\mu - \sin\theta_W B_\mu$$
$$A_\mu = \sin\theta_W W^3_\mu + \cos\theta_W B_\mu$$
$$\tan\theta_W = \frac{g'}{g}$$

and the general expressions for the mass terms of a charged spin-1 and a real spin-1 field, respectively:

$$m^2 V^\dagger_\mu V^\mu \quad \text{and} \quad \frac{1}{2} m^2 V_\mu V^\mu$$

Yukawa coupling

- The Dirac equation is only invariant under SU(2) transformations of the left-handed fermion doublets, if the two constituents of a given doublet have the same mass (i.e. $m_e = m_\nu$)
  - To keep gauge invariance, Higgs-Mechanism is also used to generate fermions masses

- The Yukawa interaction describes the coupling between the Higgs field and the fermion fields:

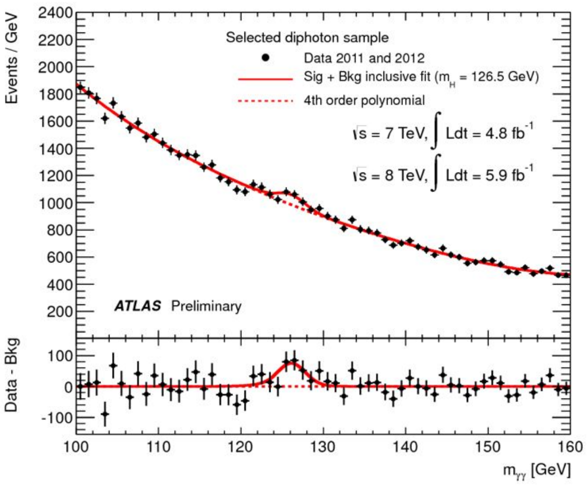
$$\mathcal{L}_{\text{Yukawa}} = -\sum_f g_f \left[\left(\overline{L}_f \Phi \right) \psi_{fR} + \overline{\psi}_{fR} \left(\Phi^\dagger L_f \right) \right]$$

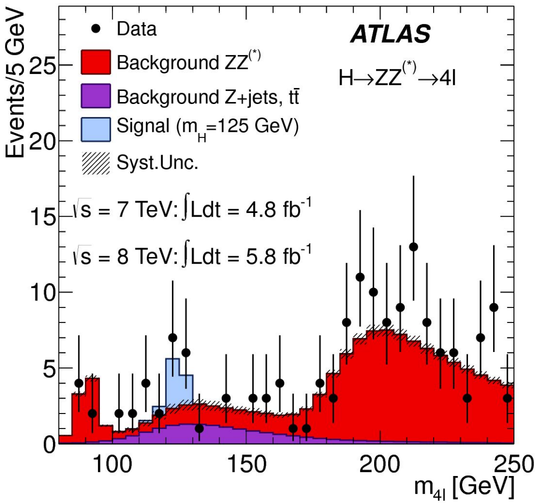
where the strength of the Higgs-fermion coupling g_f is proportional to the mass of the fermion:

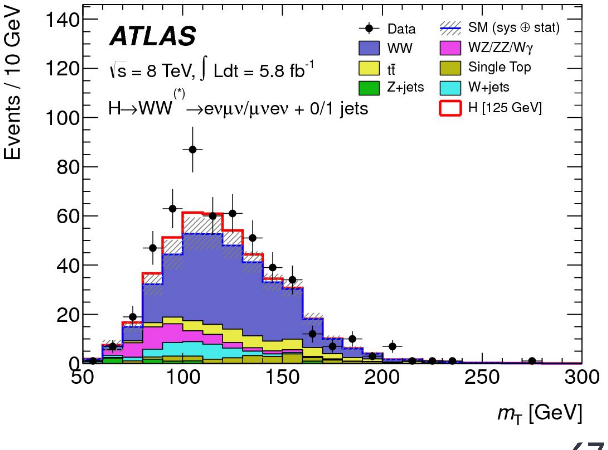
$$g_f = \frac{\sqrt{2} \cdot m_f}{v}$$

Higgs boson discovery

- Discovery of a new particle compatible with the SM Higgs boson was published by the ATLAS and CMS collaborations in the Summer of 2012
  - Considering the following decay modes:
    - $h \rightarrow \gamma\gamma$ (ATLAS & CMS)
    - $h \rightarrow ZZ^* \rightarrow \ell\ell\ell\ell$ (ATLAS & CMS)
    - $h \rightarrow WW^* \rightarrow \ell\nu\ell\nu$ (ATLAS)







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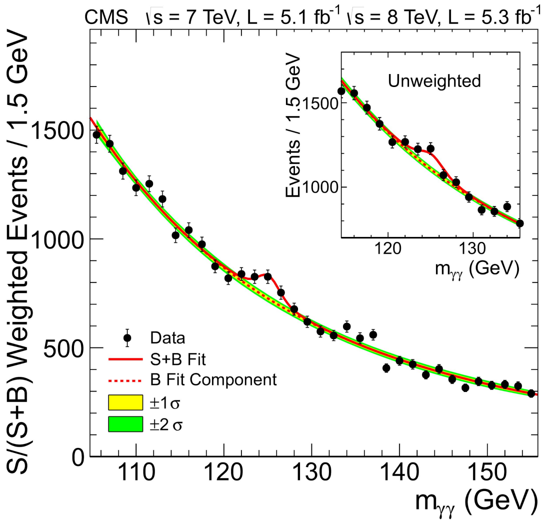
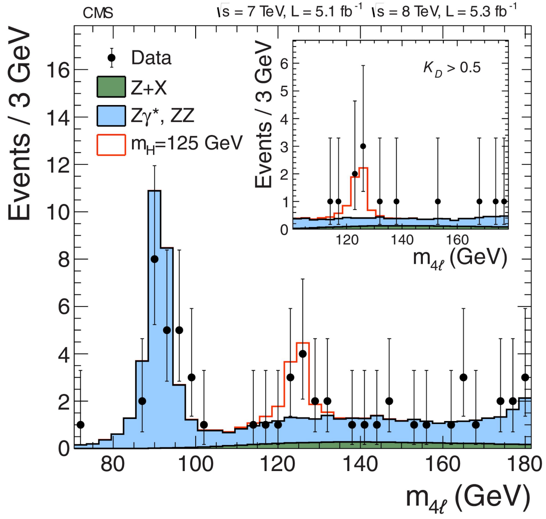
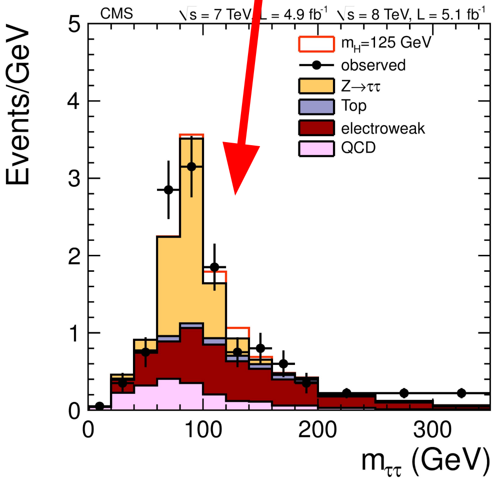
Other channels such as $h \to \tau\tau$ were not sensitive enough back then

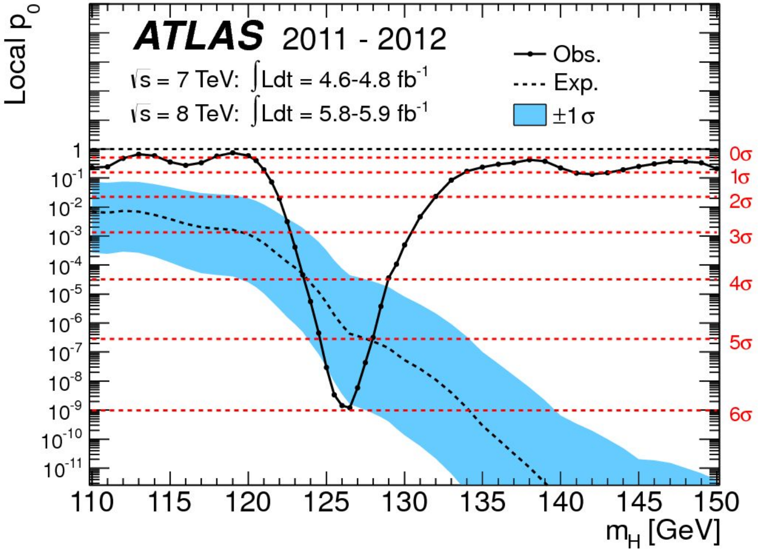


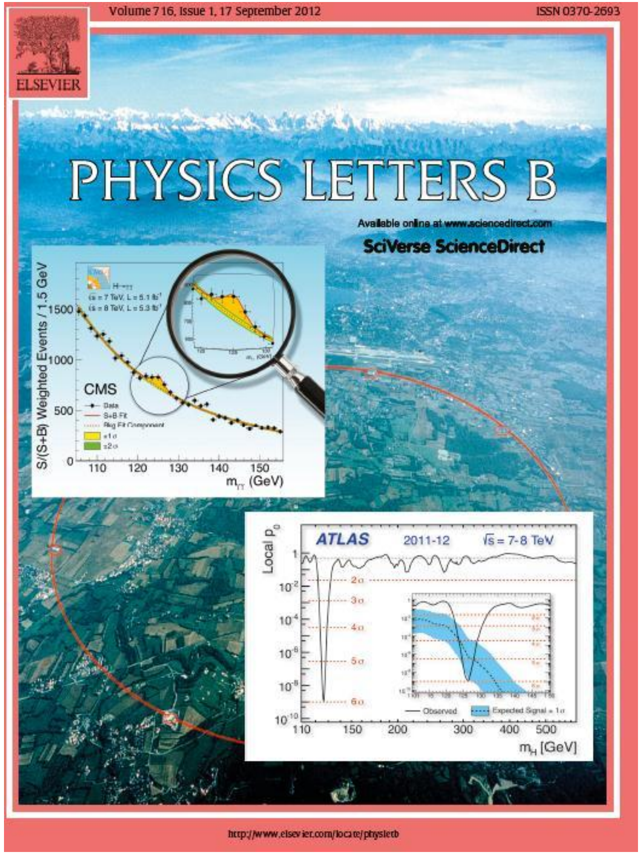




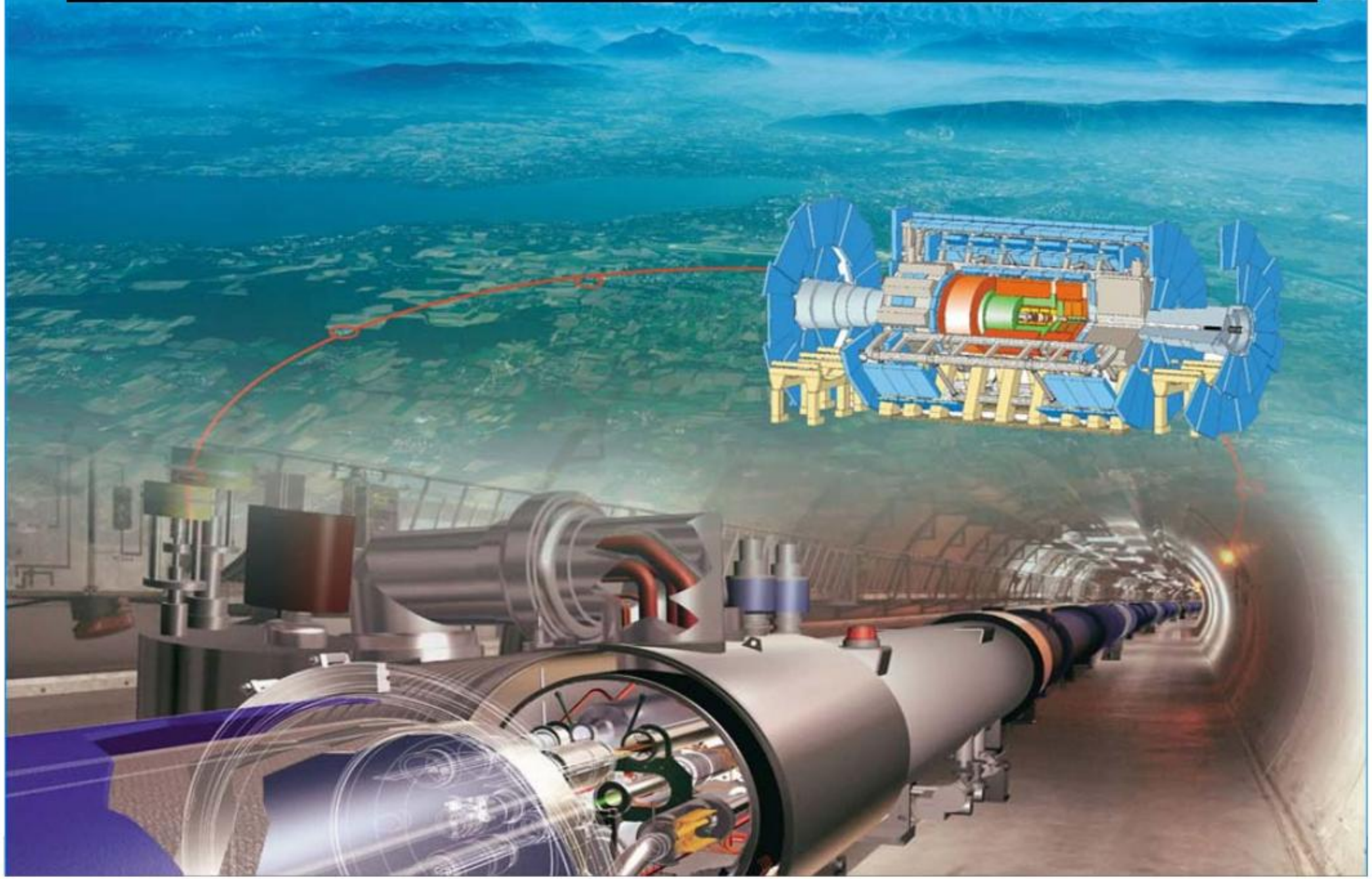
Higgs boson discovery

- The discovery of the Higgs boson concluded a search lasting several decades
  - Previous inconclusive searches at LEP and TeVatron
- Resulted in Nobel prizes for: François Englert and Peter W. Higgs (for the prediction of the Higgs boson)





1.4.4 Quark-flavour mixing



Quark-flavour mixing

- Mass eigenstates and flavour eigenstates of down-type quarks are different to the weak interaction
  - The mixing between the down-type quarks is described by an unitary matrix i.e. the so-called **Cabibbo-Kobayashi-Maskawa (CKM)-matrix:**

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = V_{CKM} \begin{pmatrix} d \\ s \\ b \end{pmatrix} \qquad \text{with:} \quad V_{\mathrm{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{tc} & V_{tb} \end{pmatrix}$$

- The matrix elements weight the transition probability for decays (via a weak charged current) of an up-type quark into a down-type quark (or vise versa)

$$u \leftrightarrow d' = V_{ud} d + V_{us} s + V_{ub} b$$

- The masses and mixings of quarks have a common origin in the Standard Model as they arise from the Yukawa interactions with the Higgs field
- Elements of the CKM matrix are not predicted by the SM, but have to be determined experimentally

Quark-flavour mixing

$\|V_{ud}\| = 0.97446 \pm 0.00010$	via nuclear β-decays
$\|V_{us}\| = 0.22452 \pm 0.00044$	via semileptonic kaon decays (e.g. $K \to \pi e \nu_e$)
$\|V_{ub}\| = 0.00365 \pm 0.00012$	via semileptonic B decays (e.g. $B \to X_u \ell \nu_\ell$)
$\|V_{cd}\| = 0.22438 \pm 0.00044$	extracted from semileptonic charm decays
$\|V_{cs}\| = 0.97359^{+0.00010}_{-0.00011}$	from semileptonic D or leptonic D_s decays
$\|V_{cb}\| = 0.04214 \pm 0.00076$	semileptonic decays of B mesons to charm
$\|V_{td}\| = 0.00896^{+0.00024}_{-0.00023}$	via B-$\bar{B}$ oscillations
$\|V_{ts}\| = 0.04133 \pm 0.00074$	via B-$\bar{B}$ oscillations
$\|V_{tb}\| = 0.999105 \pm 0.000032$	from top quark decays

Latest measurements of CKM matrix elements taken from: https://pdg.lbl.gov/2019/reviews/rpp2019-rev-ckm-matrix.pdf

Quark-flavour mixing

- The CKM matrix can be parameterized by three mixing angles and the CP-violating complex phase. Of the many possible conventions, a standard choice has become:

$$V_{CKM} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \cdot \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \cdot \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -s_{23}c_{12} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

where $s_{ij} = \sin\theta_{ij}$, $c_{ij} = \cos\theta_{ij}$, and δ is the phase responsible for all CP-violating phenomena in flavor-changing processes in the SM. The angles θ_{ij} can be chosen to lie in the first quadrant, so $s_{ij}, c_{ij} \geq 0$.

Quark-flavour mixing

- An alternative representation of the CKM matrix is the Wolfenstein parameterisation:

$$V_{CKM} = \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

- The Wolfenstein parameters can be translated via:

$$s_{12} = \lambda = \frac{|V_{us}|}{\sqrt{|V_{ud}|^2 + |V_{us}|^2}} \qquad s_{23} = A\lambda^2 = \lambda\left|\frac{V_{cb}}{V_{us}}\right| \qquad s_{13}e^{i\delta} = V_{ub}^* = A\lambda^3\left(\rho + i\eta\right)$$

Quark-flavour mixing

- The unitarity of the CKM matrix imposes:

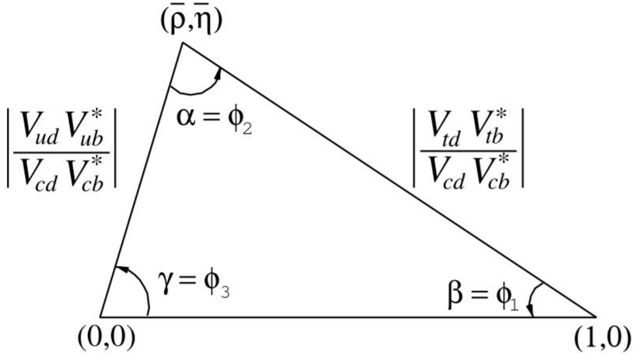
$$\sum_i V_{ij} V_{ik}^* = \partial_{jk}$$

$$\sum_j V_{ij} V_{kj}^* = \partial_{ik}$$

- The six vanishing combinations can be represented as triangles (i.e. by the **unitarity triangles**) in a complex plane
- The most commonly used unitarity triangle arises from

$$V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^* = 0$$

by dividing each side by the best-known one, $V_{cd} V_{cb}^*$



- **Phases of CKM elements:**

$$\beta = \Phi_1 = \arg\left(-\frac{V_{cd} V_{cb}^*}{V_{td} V_{tb}^*}\right)$$

$$\alpha = \Phi_2 = \arg\left(-\frac{V_{td} V_{tb}^*}{V_{ud} V_{ub}^*}\right)$$

$$\gamma = \Phi_3 = \arg\left(-\frac{V_{ud} V_{ub}^*}{V_{cd} V_{cb}^*}\right)$$

Quark-flavour mixing

- **Some last remarks:**
  - The main consequences of the quark-flavour mixing in weak charged interactions are:

    1) Quark-flavour oscillations
    2) Violation of the CP symmetry

    → Will come back to studies on quark-flavour oscillation and CP violation during the lectures next semester

  - Flavour changing neutral currents (i.e. FCNC) do not appear in tree level processes within the SM