Testing the Standard Model I
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Tutorial Set 3
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1. Symmetry Transformations

A symmetry can be expressed by requiring that all physical predictions are invariant under the wavefunction transformation

$$\psi \to \psi' = \hat{U}\psi$$

with operator $\hat{U}$ corresponding for example to a finite rotation of the coordinate axes. Finite continuous transformations can be built up from a series of infinitesimal transformations of the form

$$\hat{U} = I + i\epsilon\hat{G},$$

where ϵ is an infinitesimal small parameter and $\hat{G}$ is called the generator of the transformation.

(a) Show that $\hat{U}$ needs to be unitary by requiring that the normalization of the wavefunction remains unchanged by the transformation.

(b) Argue why the Hamiltonian must also possess the symmetry in question, i.e. $\hat{H} \to \hat{H}' = \hat{H}$. Show that this also implies that $\hat{U}$ commutes with $\hat{H}$.

(c) Show that the unitary of $\hat{U}$ implies that $\hat{G}$ is Hermitian.

(d) Verify that finite transformations α can be expressed as a series of infinitesimal transformations via

$$\hat{U}(\alpha) = \lim_{n\to\infty}\left(1 + i\frac{1}{n}\alpha\cdot\hat{G}\right)^n = \mathrm{e}^{i\alpha\cdot\hat{G}}$$

by writing down the Binominal expansion.

(e) For an infinitesimal rotation about the z-axis through an angle ϵ show that

$$\hat{U} = 1 - i\epsilon\hat{J}_z$$

where $\hat{J}_z$ is the angular momentum operator $\hat{J}_z = x\hat{p}_y - y\hat{p}_x$.

2. Spin of a Two-Nucleon System

The isopin states $|I, I_3\rangle$ of a two-nucleon system can be obtained as a superposition of single particle states involving the proton (p) with $|I_p, I_{3p}\rangle = |\frac{1}{2}, +\frac{1}{2}\rangle$ and the neutron (n) with $|I_n, I_{3n}\rangle = |\frac{1}{2}, -\frac{1}{2}\rangle$.

(a) Write down the four isopin states of a two-nucleon system.

(b) Which of these states is the deuteron?

(c) From measurements, it is known the the relative orbital angular momentum of p and n is $L = 0$ with an admixture of $L = 2$ (i.e., the spatial part of the wavefunction is even under the exchange of p and n). Give an argument why the spin of the deuteron is $S = 1$.

3. The Delta Resonance

The $\Delta^0(1232)$ is a baryon resonance. Possible strong decays are

$$\Delta^0 \to \pi^- p \quad \text{and} \quad \Delta^0 \to \pi^0 n\,.$$

One measures for the ratio of partial decay widths:

$$\Gamma(\Delta^0 \to \pi^- p)/\Gamma(\Delta^0 \to \pi^0 n) = 0.50 \pm 0.01$$

What do you conclude from that for the isospin state $|I, I_3\rangle$ of the Δ^0?

Hint: write the isospin state of the Δ^0 as superposition of the possible two-particle isospin states. The partial decay widths Γ are proportional to the square of the Clebsch-Gordan coefficients.

References

[1] Griffiths, David J. *Introduction to Elementary Particles; 2nd rev. version*. Wiley, 2008.

[2] Thomson, Mark. *Modern Particle Physics*. Cambridge University Press, 2013.