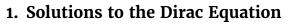


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Consider once again the Dirac equation $(i\gamma^{\mu}\partial_{\mu} - m)\psi = 0$.

- (a) By looking for free-particle plane wave solutions of the form $\psi = u(p)e^{-ipx}$ derive the Dirac equation for the spinor u(p).
- (b) How do the solutions for a particle at rest with $\vec{p} = 0$ look like? What are the negative energy solutions?
- (c) How are these negative energy solutions interpreted in the Feynman–Stückelberg interpretation? Consider the $e^+e^- \rightarrow \gamma e^+e^-$ annihilation process and discuss energy and charge conservation for the two cases where
 - i. the negative solutions of the Dirac equation are interpreted as negative energy particles propagating backwards in time;
 - ii. the negative solutions of the Dirac equation are interpreted as positive energy anti-particles propagating forwards in time.
- (d) Using this picture, how does the Dirac equation for antiparticle spinors look like?
- (e) Starting from your result from (a) show that the corresponding result for the adjoint spinor $\bar{u} = u^{\dagger}\gamma^{0}$ is

$$\bar{u}(\gamma^{\mu}p_{\mu}-m)=0\,.$$

From that, without using the explicit form of the u spinors, show that the normalization condition $u^{\dagger}u=2E$ leads to

$$\bar{u}u = 2m$$

and that

$$\bar{u}\gamma^{\mu}u = 2p^{\mu}$$
 .

(f) Derive the identity

$$\bar{u}(p')\gamma^{\mu}u(p) = \frac{1}{2m}\bar{u}(p')(p+p')^{\mu}u(p) + \frac{i}{m}\bar{u}(p')\Sigma^{\mu\nu}q_{\nu}u(p)\,,$$

where q = p' - p and $\Sigma^{\mu\nu} = \frac{i}{4}[\gamma^{\mu}, \gamma^{\nu}]$ by starting from the dirac equations for spinors and adjoint spinor.

2. Chirality Operators

(a) Show that the chirality operators $P_R = \frac{1}{2}(1+\gamma^5)$ and $P_L = \frac{1}{2}(1-\gamma^5)$ have the following properties of a projection operator:

i.
$$P_L^2 = P_L, P_R^2 = P_R$$

- ii. $P_L + P_R = 1$,
- **iii.** $P_R P_L = P_L P_R = 0$.
- (b) Apply P_R and P_L to the four Dirac spinors for massless fermions (see sheet 2 with m = 0 and choose $\vec{p} = (0, 0, p_z)$) and interpret the result.
- (c) Are the spinors still eigenvectors of the chirality operators if $m \neq 0$?

3. Chirality and Helicity

(a) Show that QED currents $\bar{\psi}^{(f)}\gamma^{\mu}\psi^{(i)}$ conserve chirality.



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- (b) Are helicity-flip reactions allowed in QED?
- (c) Show that the current $\bar{\psi}^{(f)}\gamma^{\mu}\gamma^{5}\psi^{(i)}$ conserves chirality.
- (d) Are helicity-flip reactions allowed in weak interactions?
- (e) Show that the V-A form of the antifermion current $\bar{v}\gamma^{\mu}\frac{1}{2}(1-\gamma^5)v$ is equivalent to the statement that only the right-handed chirality components of antifermions participate in weak interactions.

References

- [1] Griffiths, David J. Introduction to Elementary Particles; 2nd rev. version. Wiley, 2008.
- [2] Thomson, Mark. Modern Particle Physics. Cambridge University Press, 2013.