1. **Trace Theorems**

   For the calculation of matrix elements so-called trace theorems become useful. Prove the identity of the following relations:

   (a) $\text{Tr}\{\gamma^\mu\gamma^\nu\} = 4g^{\mu\nu}$
   (b) $\text{Tr}\{\gamma^\mu\gamma^\nu\gamma^\rho\} = 0$
   (c) $\text{Tr}\{\gamma^5\gamma^\mu\gamma^\nu\} = 0$.

2. **Completeness Relation**

   Show that the Dirac spinors for particles and anti-particles satisfy the following 'completeness' relations

$$\sum_s^2 u_s\bar{u}_s = (\gamma^\mu p_\mu + m) \quad \text{and} \quad \sum_r^2 v_r\bar{v}_r = (\gamma^\mu p_\mu - m)$$

   where the sums run over the two possible spin states.

3. **Electron-Positron Annihilation I**

   Consider the annihilation of an electron-positron pair into a muon pair: $e^+e^- \to \mu^+\mu^-$.

   (a) Draw the lowest-order Feynman diagram(s) of this process.
   (b) By applying the Feynman rules, derive the expression for the matrix element $\mathcal{M}$.
   (c) Compute the expression for the spin-averaged matrix element squared $\langle|\mathcal{M}|\rangle^2$ by squaring and summing over the spins of the initial state as well as averaging over the spins of the final state.

References

[1] Griffiths, David J. *Introduction to Elementary Particles; 2nd rev. version*. Wiley, 2008.

[2] Thomson, Mark. *Modern Particle Physics*. Cambridge University Press, 2013.