Testing the Standard Model of Elementary Particle Physics I

Third lecture

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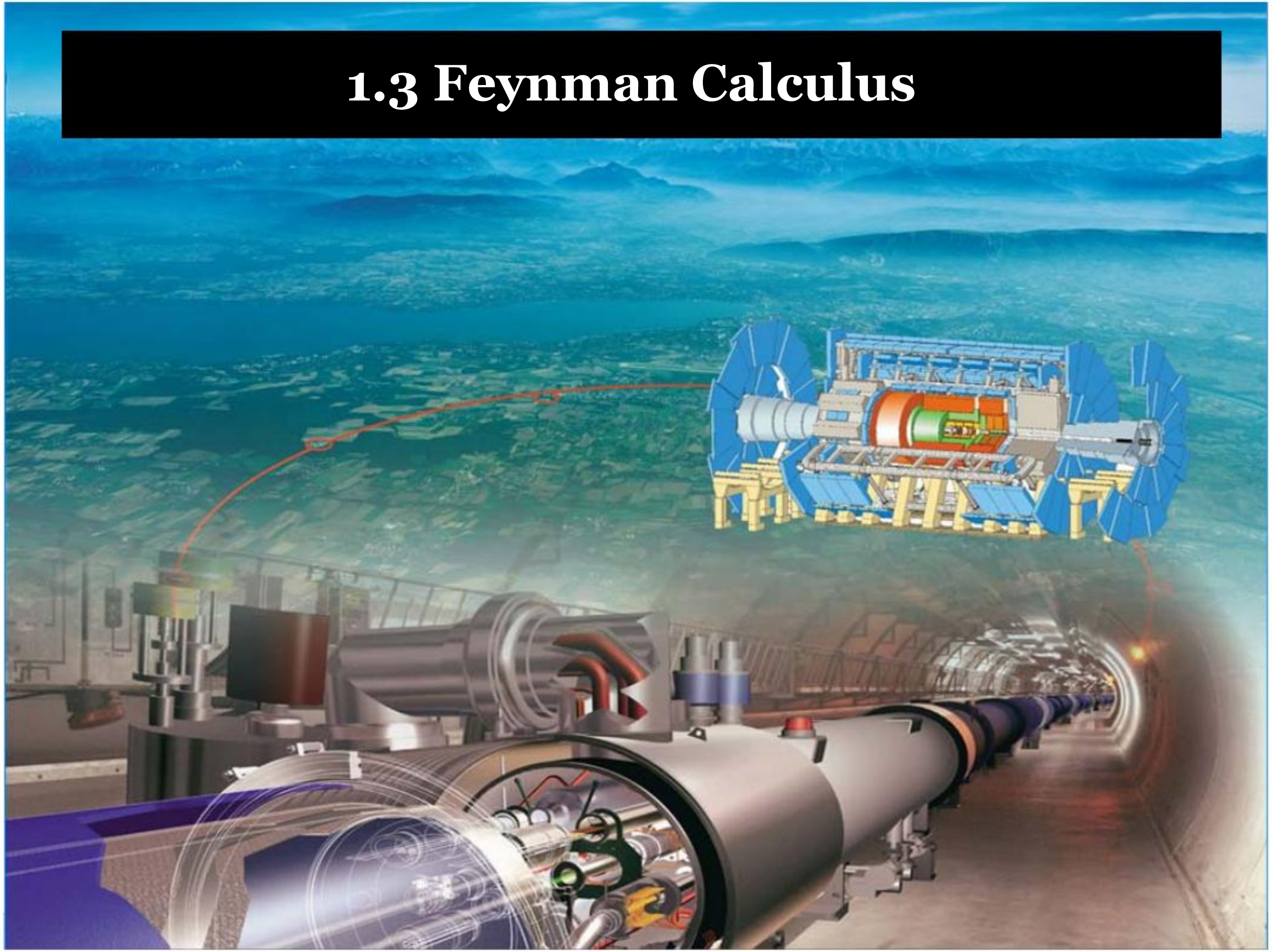
19th November 2020

1.3 Feynman Calculus



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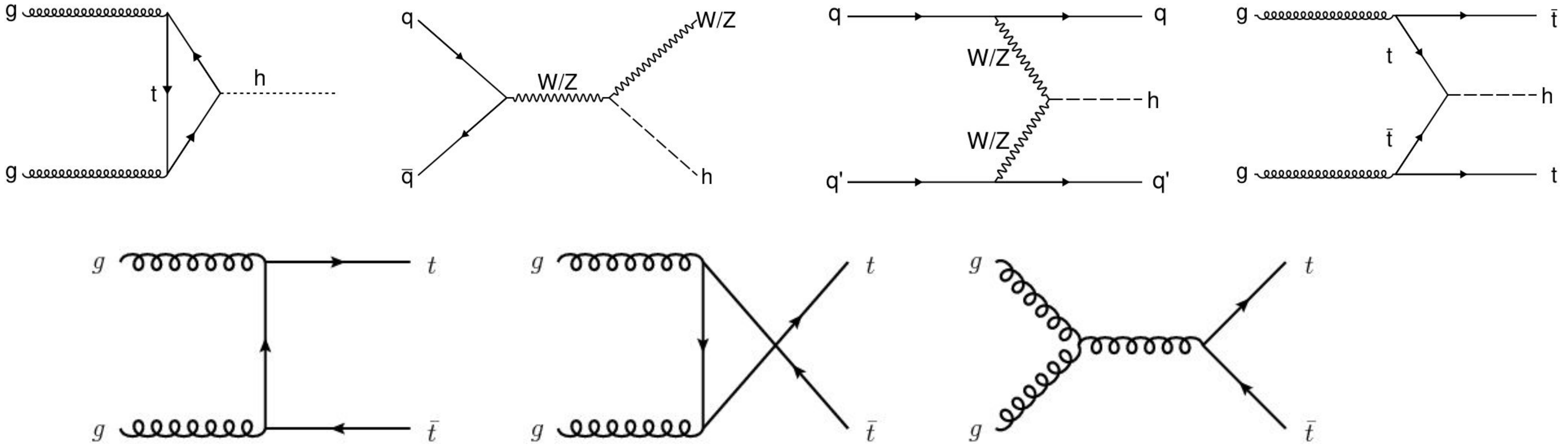
Introduction:

- **Feynman-diagrams:**
  - Schematic representation of the mathematical expressions describing the behavior and interactions of elementary particles.



  - Will introduce methodology of calculating production cross section for any process based on a fixed set of rules, **Feynman rules**.

Introduction:

- **Cross sections:**
    - **The term "cross section" derives from a thought experiment involving the scattering process of hard spheres:**
        - Imagine a hard sphere of radius a, located somewhere within a total area of A
        - A second sphere is thrown towards the first sphere
            - The target sphere shows an area of πa^2
        - The probability that the incoming sphere would scatter from the target sphere is given by:

$$P_S = \frac{\pi a^2}{A}$$

Cross sectional area of the sphere in terms of the beam area

or: $\sigma = P_S A$

        - Imagine we have a parallel beam with the density ρ and the velocity v towards the target.
            - In time t, this beam will fill a volume ρvtA, where A is now the area normal to the beam which fully contains it.
                - Choosing t such that only one particle is contained in this volume, we can write:

$$1 = \rho v t A$$

Introduction:

- **Thus the cross section definition can be rewritten as:**

$$\sigma = \frac{P_S/t}{\rho v}$$

where P_S /t is the transition rate, i.e. the probability of scattering per unit time and ρv is the flux of particles.

A cross sections can be understood as a transition rate per unit of particle flux

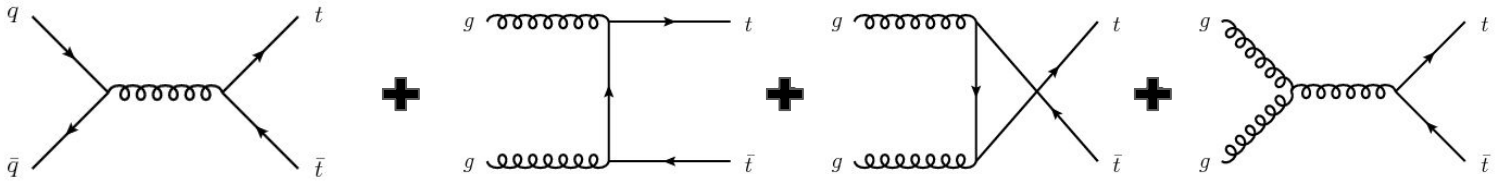
Have to migrate these expressions to quantum mechanical scattering processes

Introduction:

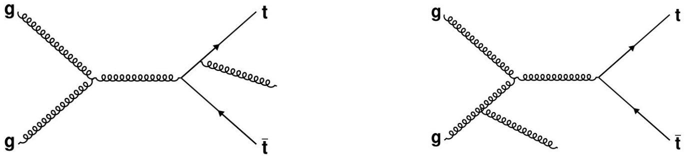
- **Cross sections:**
  - **Calculation of inclusive cross section:**

$$\sigma_{\text{tot}} = \sum_{i}^{N} \sigma_i$$

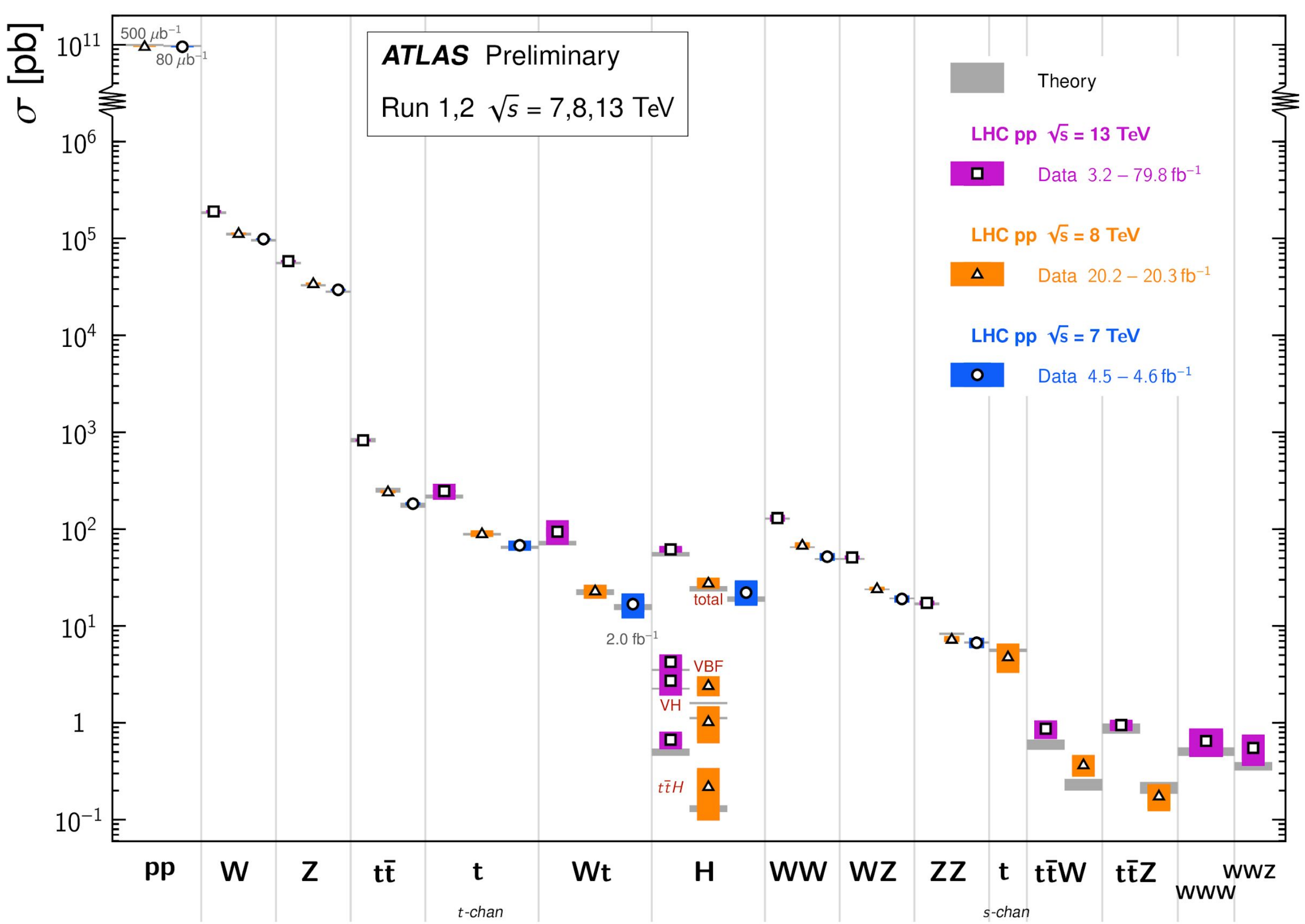
    - Contributions from various production modes.



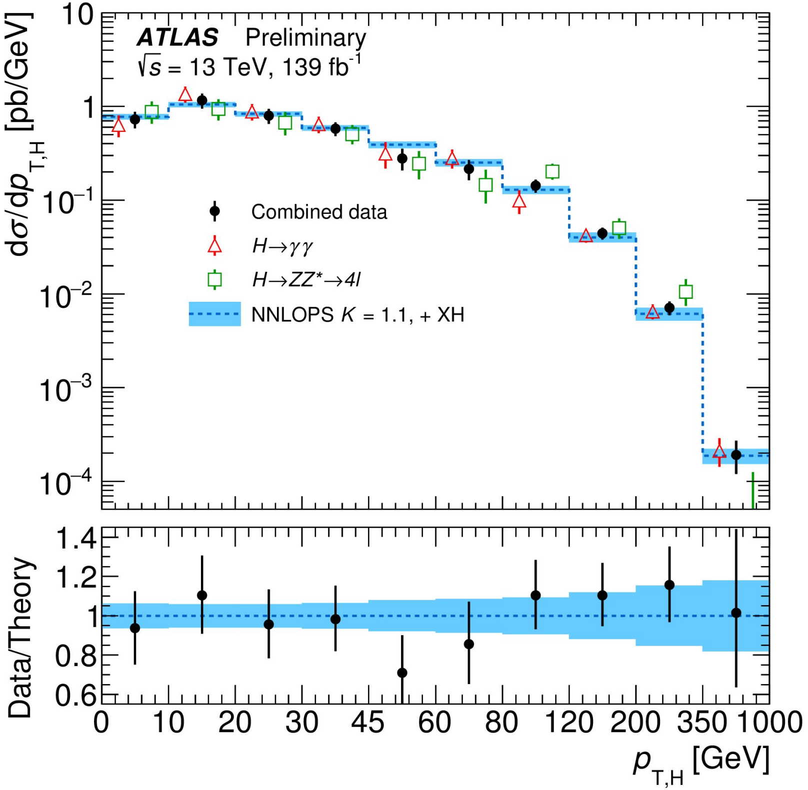
    - Tree level diagrams and ideally higher order diagrams:

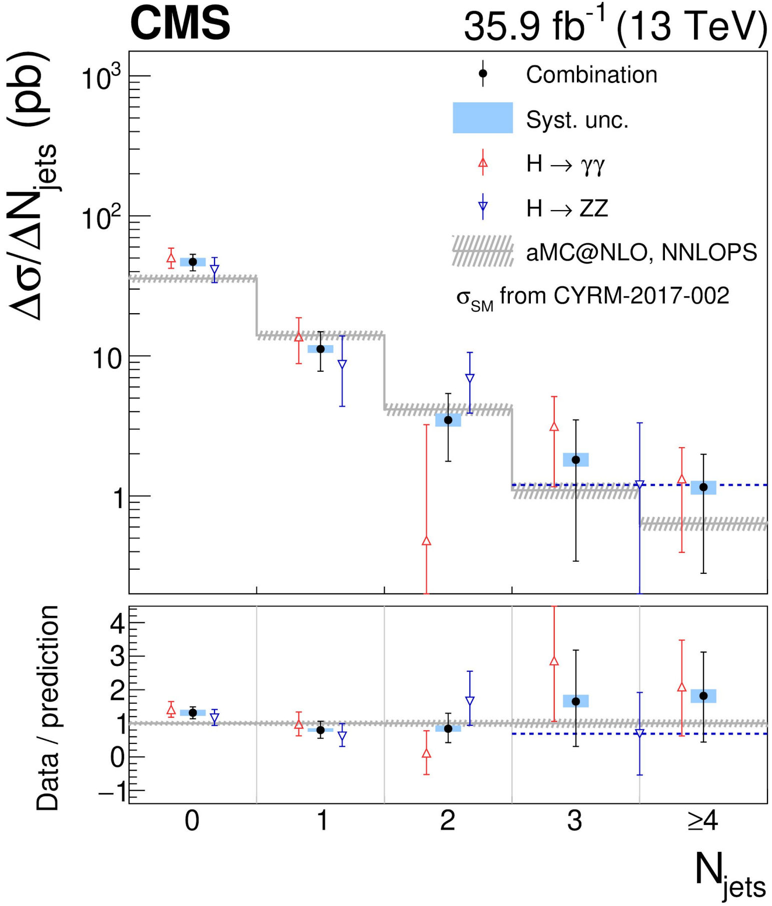


Standard Model Total Production Cross Section Measurements



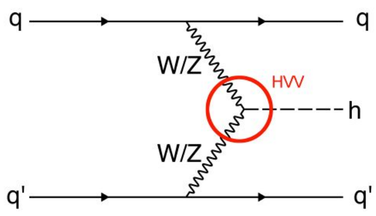
Differential cross sections:

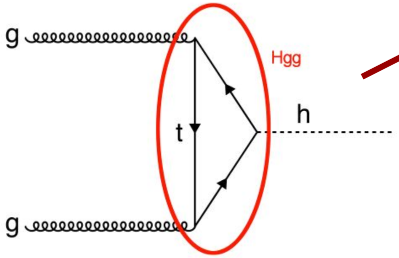


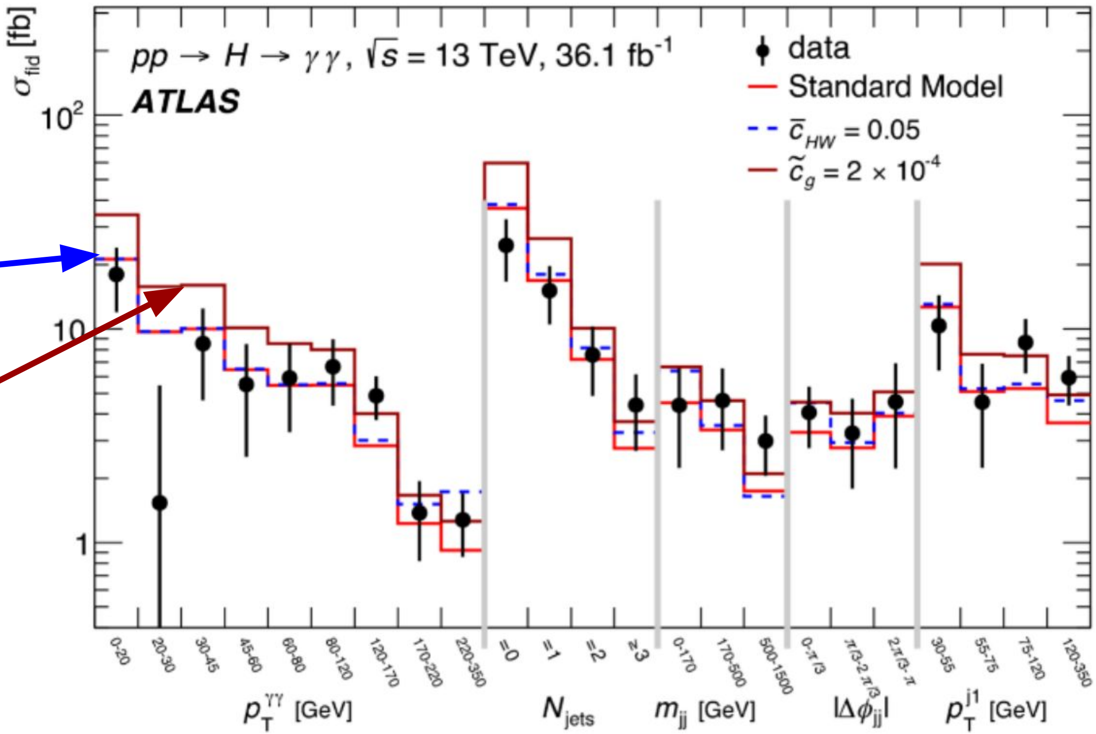


Differential cross sections as a gate to new physics

BSM physics contributions to the Higgs boson production process can modulate certain kinematical observables







Perform hypothesis tests to determine whether data fits better to SM predictions or BSM hypotheses

Introduction:

- **Decay rates and branching fraction:**
  - The **decay rate** is defined as the probability per unit time that a decay $X \rightarrow x_1 x_2$ will occur
    - Decay rate and mean lifetime of a particle are related via:

$$\tau = \frac{1}{\Gamma}$$

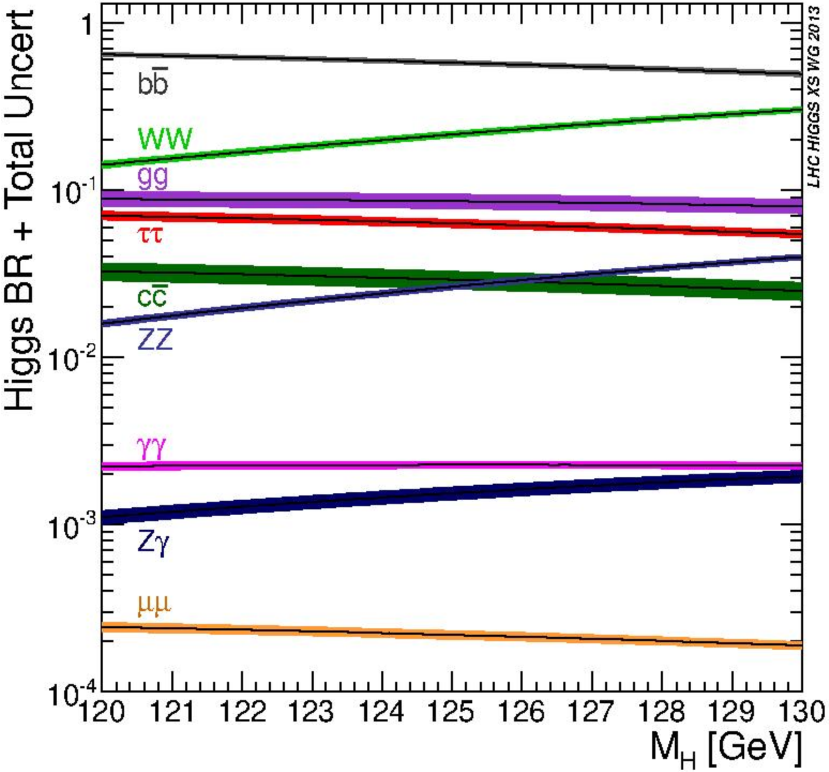
    - For particles with multiple decay modes, the total decay rate is the sum of the individual decay rates:

$$\Gamma_{\text{tot}} = \sum_{i}^{N} \Gamma_i \quad \Rightarrow \quad \tau = \frac{1}{\Gamma_{\text{tot}}}$$

  - The **branching ratio** of a decay $X \rightarrow x_1 x_2$ is defined via:

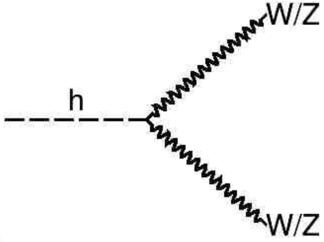
$$BR(X \rightarrow x_1 x_2) = \frac{\Gamma_i}{\Gamma_{\text{tot}}}$$

Higgs boson decays



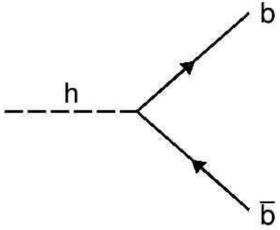
Decay mode	Branching fraction [%]
$H \to b\bar{b}$	57.5 ± 1.9
$H \to WW^*$	21.6 ± 0.9
$H \to gg$	8.56 ± 0.86
$H \to \tau\tau$	6.30 ± 0.36
$H \to c\bar{c}$	2.90 ± 0.35
$H \to ZZ^*$	2.67 ± 0.11
$H \to \gamma\gamma$	0.228 ± 0.011
$H \to Z\gamma$	0.155 ± 0.014
$H \to \mu\mu$	0.022 ± 0.001

Higgs boson decays

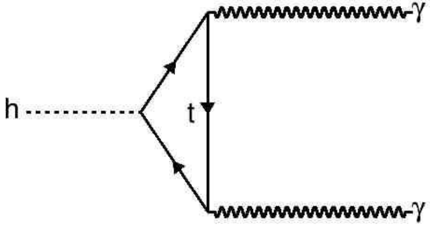


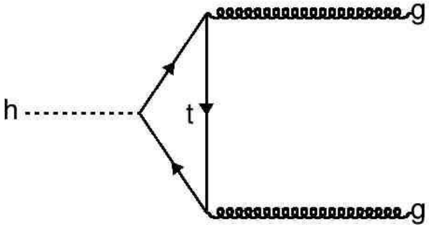
- Strength of the coupling between the Higgs boson and other particles is proportional to the particle mass:

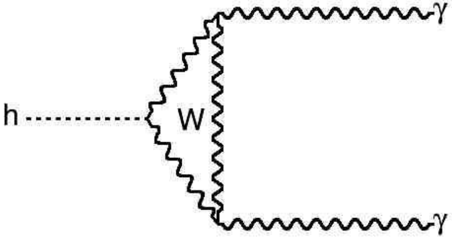
$$\mathcal{L}_{Hff} = -\frac{m_f}{v} h f \bar{f} \qquad \text{and} \qquad \mathcal{L}_{HVV} = \frac{1}{v}\left(2m_W^2 W_\mu^+ W^{-\mu} + 2m_Z^2 Z_\mu Z^\mu\right) h$$

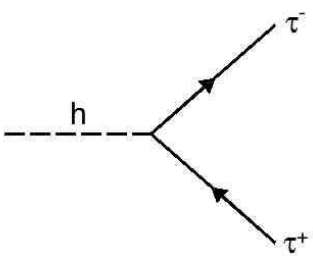


- Thus decays to massless particles such as photon or gluons is only possible via top quark (or W boson) loops
- The masses of the particles running in these loops are large and thus such decay modes can compete with decays to fermions or W and Z bosons









W/Z boson decays

- **Lepton universality:**
  - All three types of charged leptons interact in the same way with other particles.
  - The three lepton types are created equally often in particle transformations, or decays (once differences in their mass are accounted for)

Decay Mode	BR
$Z \to e^+e^-$	$(3.3632 \pm 0.0042)\%$
$Z \to \mu^+\mu^-$	$(3.3662 \pm 0.0066)\%$
$Z \to \tau^+\tau^-$	$(3.3696 \pm 0.0083)\%$
$Z \to$ invisible	$(20.000 \pm 0.055)\%$
$Z \to$ hadrons	$(69.911 \pm 0.056)\%$

Decay Mode	BR
$W \to e\nu$	$(10.71 \pm 0.16)\%$
$W \to \mu\nu$	$(10.63 \pm 0.15)\%$
$W \to \tau\nu$	$(11.38 \pm 0.21)\%$
$W \to$ hadrons	$(67.41 \pm 0.27)\%$

Quark decays

- **CKM matrix elements describe transition from one quark flavour to another:**
  - I.e. V_{ij} measures the coupling of quark i to quark j:
  - The CKM matrix is given via:

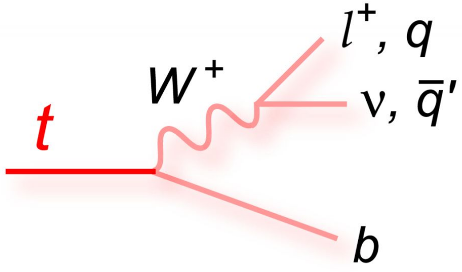
$$V_{\rm CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{tc} & V_{tb} \end{pmatrix}$$

  - The magnitudes of the matrix elements are:

$$\begin{pmatrix} 0.97446 \pm 0.00010 & 0.22452 \pm 0.00044 & 0.00365 \pm 0.00012 \\ 0.22438 \pm 0.00044 & 0.97359^{+0.00010}_{-0.00011} & 0.04214 \pm 0.00076 \\ 0.00896^{+0.00024}_{-0.00023} & 0.04133 \pm 0.00074 & 0.999105 \pm 0.000032 \end{pmatrix}$$

- Top quark decays almost exclusively via t → bW

$$|V_{tb}| \approx 1$$



Latest measurements of CKM matrix elements taken from: https://pdg.lbl.gov/2019/reviews/rpp2019-rev-ckm-matrix.pdf

Lepton decays

Decay Mode	BR
$\tau^- \rightarrow e^- \nu_e \nu_\tau$	$(17.83 \pm 0.04)\%$
$\tau^- \rightarrow \mu^- \nu_\mu \nu_\tau$	$(17.41 \pm 0.04)\%$
$\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$	$(25.52 \pm 0.09)\%$
$\tau^- \rightarrow \pi^- \nu_\tau$	$(10.83 \pm 0.06)\%$
$\tau^- \rightarrow \pi^- \pi^0 \pi^0 \nu_\tau$	$(9.30 \pm 0.11)\%$
$\tau^- \rightarrow \pi^- \pi^+ \pi^- \nu_\tau$	$(8.99 \pm 0.05)\%$
$\tau^- \rightarrow \pi^- \pi^+ \pi^- \pi^0 \nu_\tau$	$(2.74 \pm 0.07)\%$
$\tau^- \rightarrow \pi^- \pi^0 \pi^0 \pi^0 \nu_\tau$	$(1.04 \pm 0.07)\%$

Decay Mode	BR
$\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu$	100%

- **Electrons are stable**
- **Lifetimes:**
  - Muons: $2.2 \cdot 10^{-6}$ s
  - Taus: $290.6 \cdot 10^{-15}$ s
- **Neutrino oscillate:**
  - Will be discussed next semester

1.3.1 Feynman-diagrams